# Analyzing of SLR observations

## What do we do with the data?

Mathis Bloßfeld

Deutsches Geodätisches Forschungsinstitut at the Technical University of Munich (DGFI-TUM), Munich, Germany

SLR school 2019 – session 2: Data Analysis

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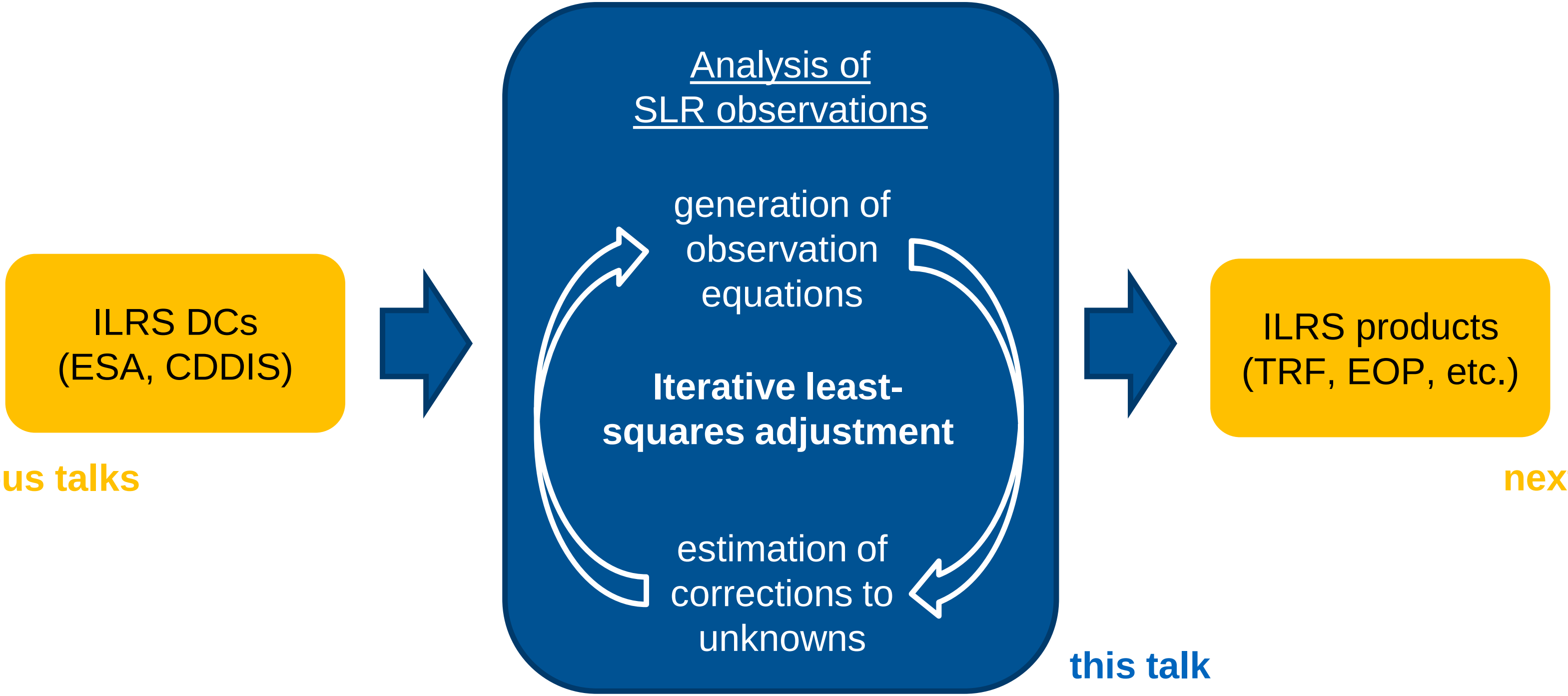
# General work/data flow



ILRS DCs
(ESA, CDDIS)

**previous talks**

Analysis of
SLR observations

generation of
observation
equations

**Iterative least-squares adjustment**

estimation of
corrections to
unknowns

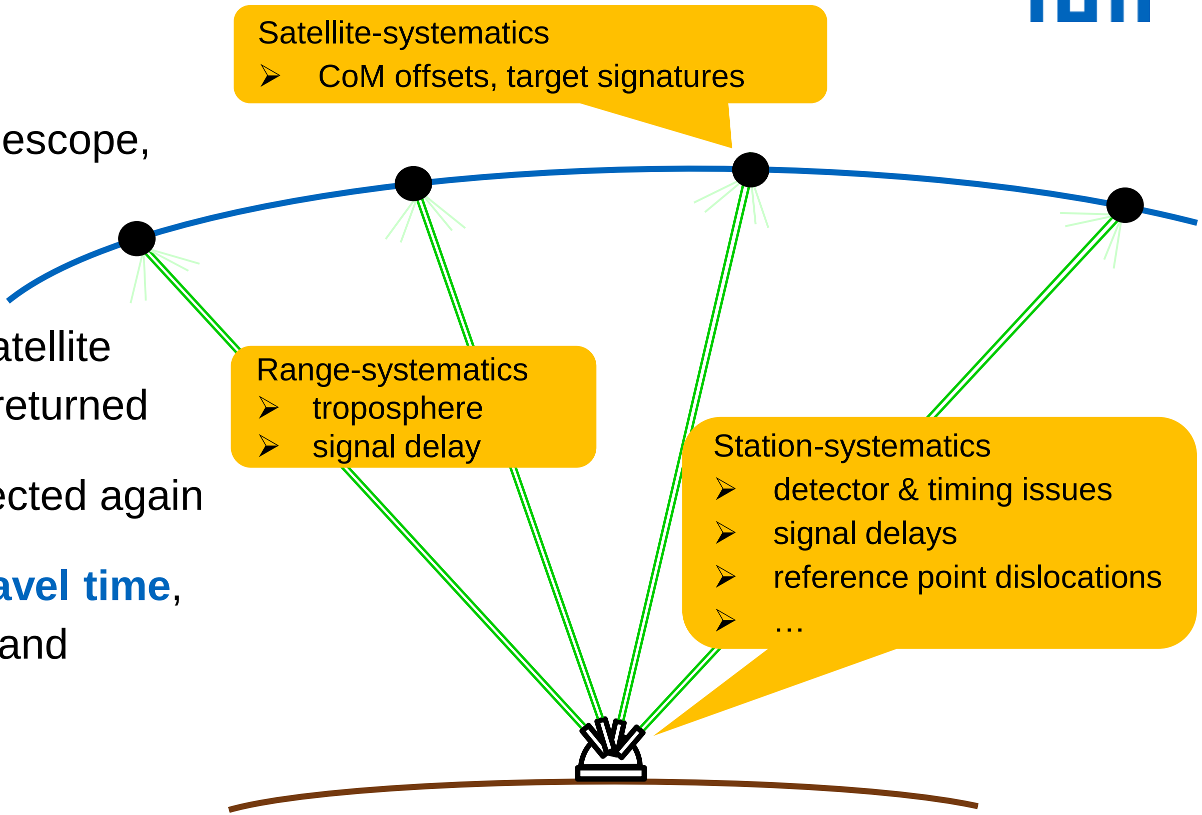
**this talk**

ILRS products
(TRF, EOP, etc.)

**next talk**

# The SLR observation

- When a satellite flies over a telescope, a laser pulse is emitted by the crust-fixed station
- This pulse is reflected at the satellite but only a part of the signal is returned
- At the station, the pulse is detected again
- The observation is the **light travel time**, not the range between station and satellite directly!
- Observations of a flyover
  → **satellite pass**



Satellite-systematics
- CoM offsets, target signatures

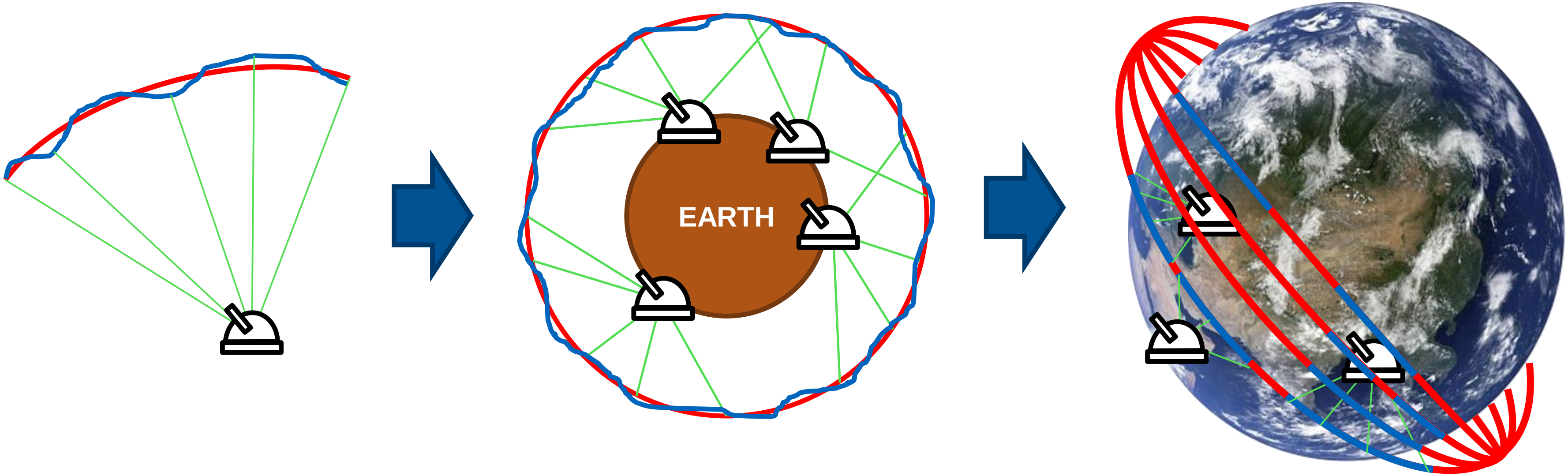
Range-systematics
- troposphere
- signal delay

Station-systematics
- detector & timing issues
- signal delays
- reference point dislocations
- …

**systematics/errors: next session**

# The SLR observation

- One station observes one pass during a flyover of the satellite
- Within one satellite revolution, multiple stations observe the satellite
- During a 7 day arc, multiple stations observe the satellite during multiple revolutions



e.g.: 7 day arc

# The SLR observation equation

$$\rho + \epsilon = \|\boldsymbol{r}_{\boldsymbol{sat}}(\boldsymbol{t}_{\boldsymbol{M}} + \Delta \boldsymbol{t}) - \boldsymbol{r}_{\boldsymbol{sta}}(\boldsymbol{t}_{\boldsymbol{M}} + \Delta \boldsymbol{t})\| + \Delta\rho + c_{trop}(1 + \Delta r) + c_{rel} + c_{sta} + c_{sta} + c_{mesc}$$

| | | |
|---|---|---|
| $\rho$ | … | one-way range measurement |
| $t_M$ | … | approximated epoch of the reflection |
| $\epsilon$ | … | measurement error |
| $\boldsymbol{r}_{\boldsymbol{sat}}$ | … | 3-D position of the satellite [GCRS] |
| $\boldsymbol{r}_{\boldsymbol{sta}}$ | … | 3-D position of the station [ITRS] |
| $\Delta t$ | … | time bias of the measurement |

| | | |
|---|---|---|
| $\Delta\rho$ | … | range bias of the measurement |
| $\Delta r$ | … | bias of tropospheric refraction |
| $c_{trop}$ | … | tropospheric range correction |
| $c_{rel}$ | … | relativistic range correction |
| $c_{sta}$ | … | station-dependent SLR correction |
| $c_{masc}$ | … | satellite-specific CoM correction |
| $c_{mesc}$ | … | SLR array-dependent correction |

# The SLR observation equation

$$\boxed{\underset{\substack{\uparrow \\ \text{observed range}}}{\rho} + \epsilon = \underbrace{\|\boldsymbol{r}_{sat}(\boldsymbol{t}_M + \Delta \boldsymbol{t}) - \boldsymbol{r}_{sta}(\boldsymbol{t}_M + \Delta \boldsymbol{t})\| + \Delta\rho + c_{trop}(1 + \Delta r) + c_{rel} + c_{sta} + c_{sta} + c_{mesc}}_{\text{theoretical (computed) range } m(\vec{p}_0)}}$$

- Residual = (observed – computed) range = $\rho - m(\vec{p}_0)$
- For the **theoretical range**, we compute the **instantaneous satellite position in the GCRS** and the **instantaneous station position in the ITRS**
- Models involved:

| | | | |
|---|---|---|---|
| **tropospheric delay** | Mendes & Pavlis (2004) | **Relativistic range correction** | IERS Conventions 2010 |
| **Horizontal gradients** | not applied | **All other range corrections** | **see next session** |

# Instantaneous satellite position

- Satellite orbit integration is done in the GCRS (quasi-inertial) system
- Models involved (**for some corrections can be estimated → partial derivatives**):

| | | | |
|---|---|---|---|
| **Earth's gravity field** | ITSG-Grace2018s | **Lunar gravity** | Konopliv et al., 2001 |
| **Solid Earth's tides** | IERS Conventions 2010 | **Solar radiation pressure** | constant TSI |
| **Permanent tide** | IERS Conventions 2010 | **Earth radiation pressure** | Albedo+infrared |
| **Ocean tides** | FES2014c + admittance | **Atmospheric drag** | JB2008 |
| **Solid Earth pole tide** | IERS Conventions 2010 | **General relativistic correction** | Schwarzschild, deSittter, Lense-Thirring |
| **Ocean pole tide** | Desai, 2002 | **Non-tidal grav. perturbation** | AOD1B-RL04 |
| **Atmospheric tides** | Bode & Biancale (2006) | **Thermal radiation** | not applied |
| **Third body effects** | DE-421 (Sun, moon, 5 panets) | **Satellite model** | cannonball |

# Instantaneous station position

- Station position is modelled in the ITRS
- Positions are rotated from the ITRS into the GCRS using the EOP
- Models involved (**for some corrections can be estimated → partial derivatives**):

| | | | |
|---|---|---|---|
| **TRF** | SLRF2014 | **Tidal atmospheric loading** | Ray & Ponte (2003) |
| **EOP** | IERS 14 C04 | **Solid Earth pole tide loading** | IERS Conventions 2010 |
| **Solid Earth tides** | IERS Conventions 2010 | **Ocean pole tide loading** | Desai (2002) |
| **Permanent tide** | IERS Conventions 2010 | **non-tidal loading** | not applied |
| **Ocean tides** | FES2014c | | |

# Minimization of residuals

- Residuals are minimized using an iterative least-squares adjustment algorithm
- Parameter vector $\vec{p}$ → computed range $m(\vec{p}_0)$ is based on known a priori values $\vec{p}_0$
- Linearization of this function in $\vec{p}_0$ leads to

$$\rho - m(\vec{p}_0) = grad\, m(\vec{p}_0) \cdot \Delta\vec{p}$$

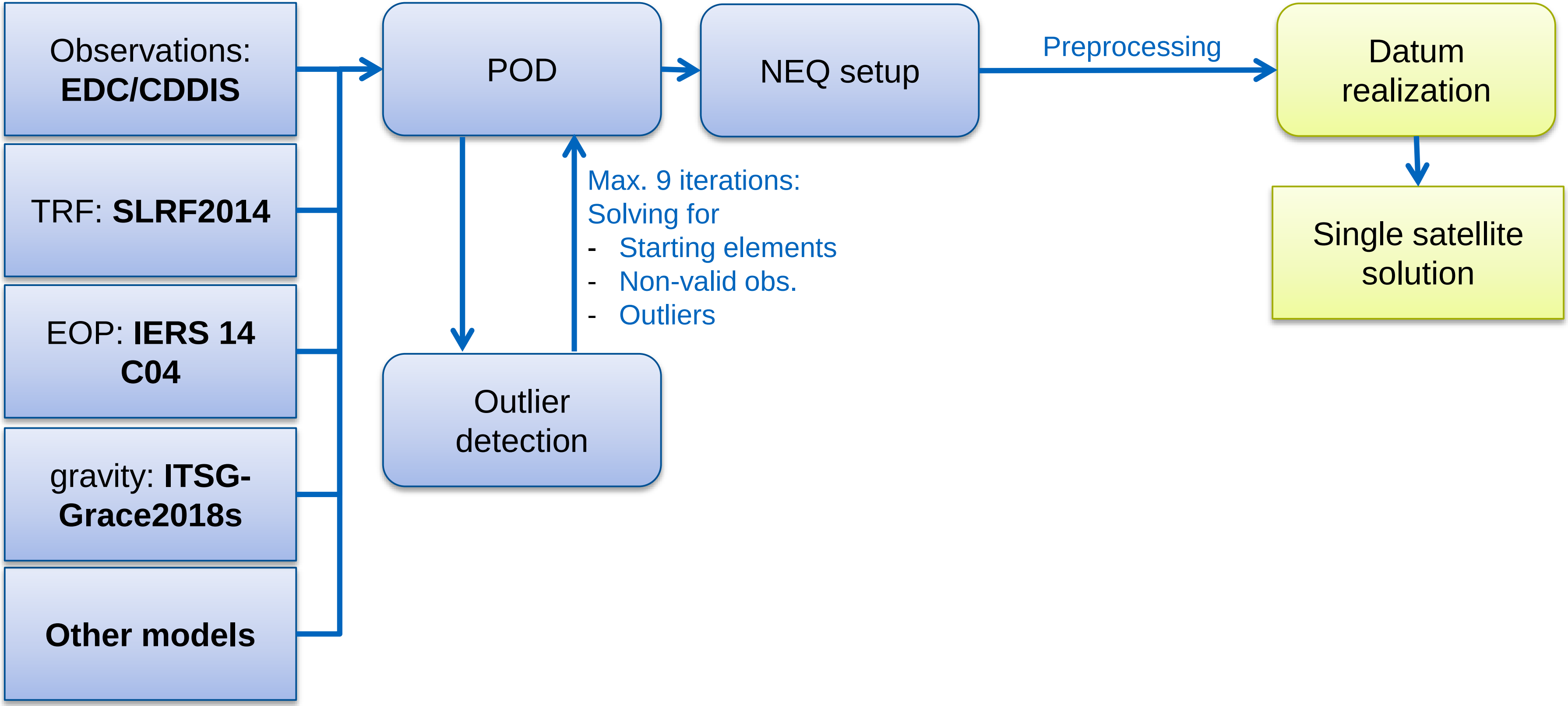
- Coefficients of the design matrix → partial derivatives of $m(\vec{p}_0)$ w.r.t. i free parameters

$$\rho - m(\vec{p}_0) = \sum_i \frac{\partial m}{\partial p_i}(\vec{p}_0) \cdot \Delta p_i$$

$$\frac{\partial m}{\partial p} = \boxed{\frac{\partial m}{\partial \vec{x}_{sat}(t)} \cdot \frac{\partial \vec{x}_{sat}(t)}{\partial p}} + \boxed{\frac{\partial m}{\partial \vec{x}_{sta}(t)} \cdot \frac{\partial \vec{x}_{sta}(t)}{\partial p}} + \boxed{\frac{\partial m}{\partial p}}$$

- Usually, only one of these derivatives is non-zero (e.g. dynamic parameter: $\frac{\partial \vec{x}_{sat}(t)}{\partial p} \neq 0$)

# Work/Data flow within s/w (DGFI-TUM setup!)



Observations: **EDC/CDDIS**

TRF: **SLRF2014**

EOP: **IERS 14 C04**

gravity: **ITSG-Grace2018s**

**Other models**

POD

NEQ setup

Preprocessing

Datum realization

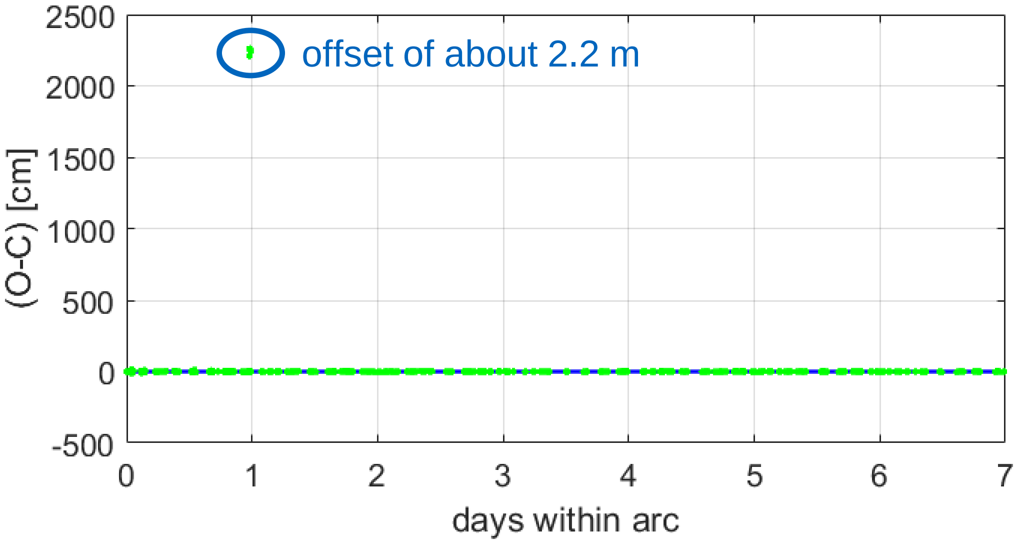
Single satellite solution

Outlier detection

Max. 9 iterations:
Solving for
- Starting elements
- Non-valid obs.
- Outliers

# POD - Example

- LAGEOS-1 7-day arc in October 2019
- 1833 observations (241 passes)
- 31 stations
- 1 erroneous pass detected during POD



# POD - Example

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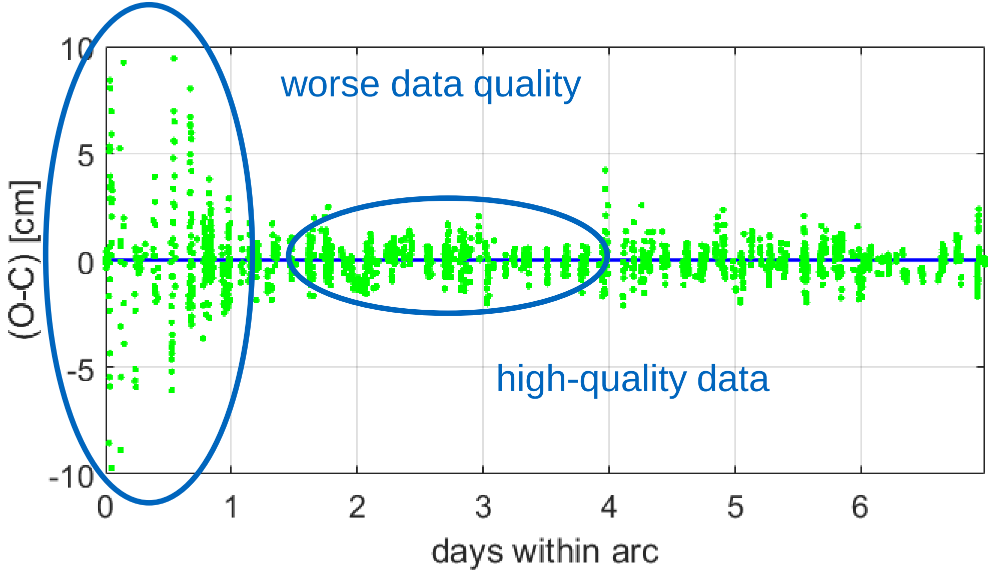
- RMS of residuals: 1.32 cm



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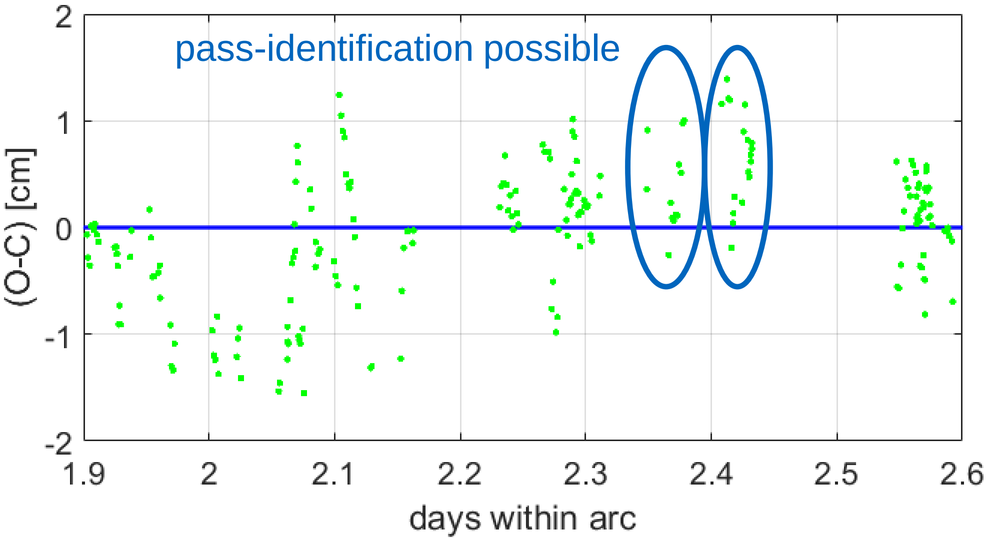

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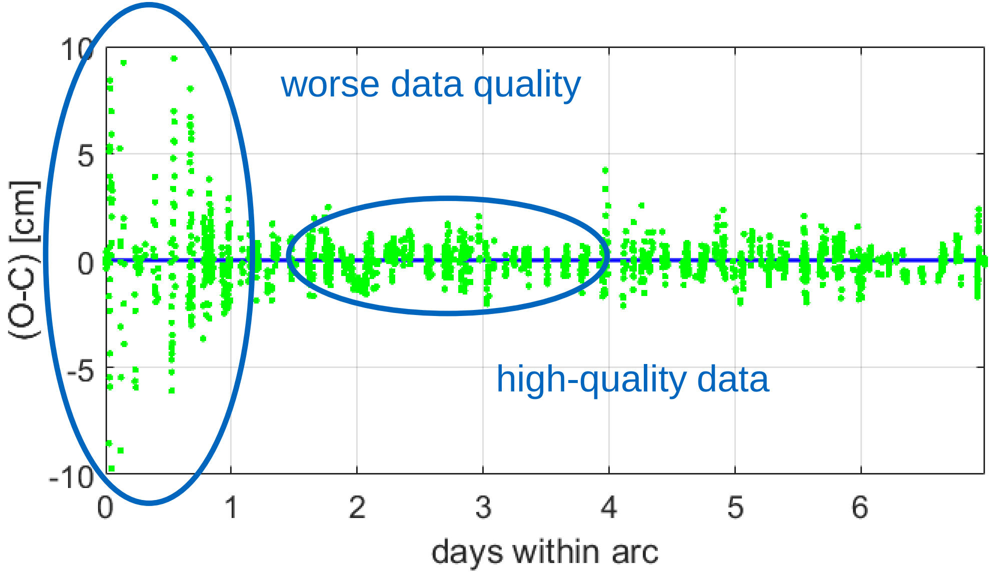
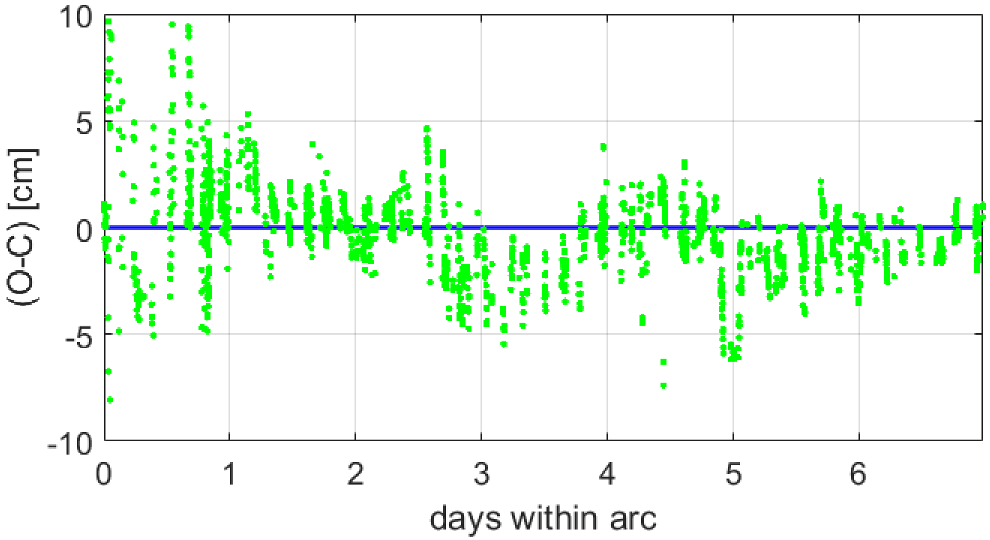


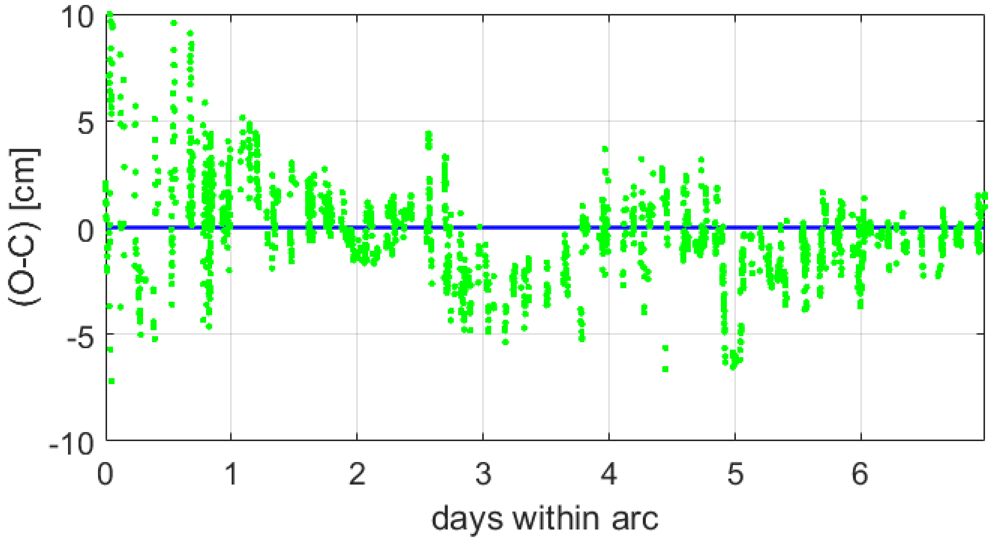
# POD - Example

- LAGEOS-1 7-day arc in October 2019
- 1833 observations (241 passes)
- 31 stations
- 1 erroneous pass detected during POD
- RMS of residuals: 2.36 cm
- Decrease of solution quality of less parameters are estimated
- No TRF corrections



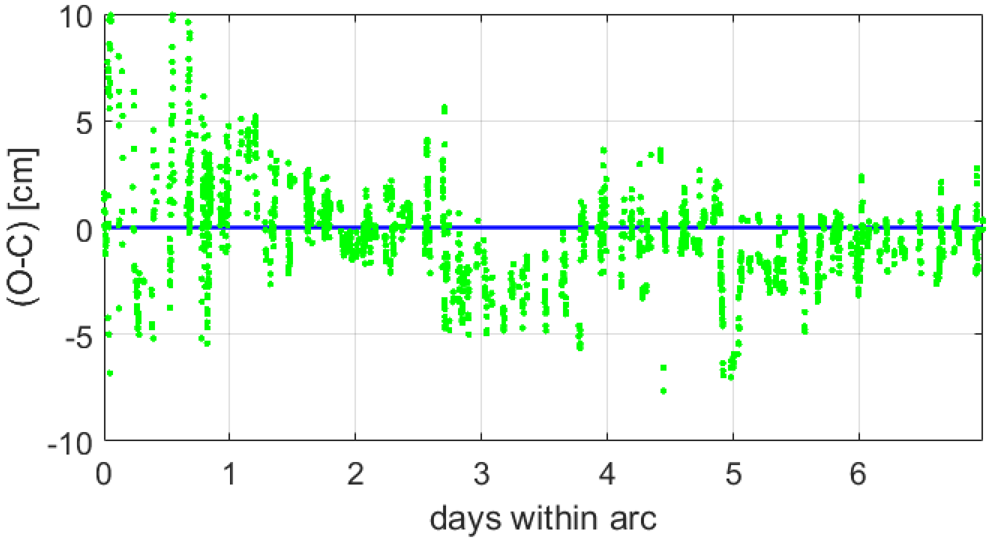
# POD - Example

- LAGEOS-1 7-day arc in October 2019
- 1833 observations (241 passes)
- 31 stations
- 1 erroneous pass detected during POD
- RMS of residuals: 2.45 cm
- Decrease of solution quality of less parameters are estimated
- No TRF and no EOP corrections



# POD - Example

- LAGEOS-1 7-day arc in October 2019
- 1833 observations (241 passes)
- 31 stations
- 1 erroneous pass detected during POD
- RMS of residuals: 2.55 cm
- Decrease of solution quality of less parameters are estimated
- No TRF and no EOP corrections
- No empirical accelerations estimated



# POD - final remarks

- Usually, up to 2% of the observations are outliers
- If arc-length is increased to, e.g., monthly arcs, integration quality rapidly decreases due to accumulating model errors
- The less parameters are estimated, the worse is the scatter of the residuals
- Estimated parameter and obtained RMS of residuals:

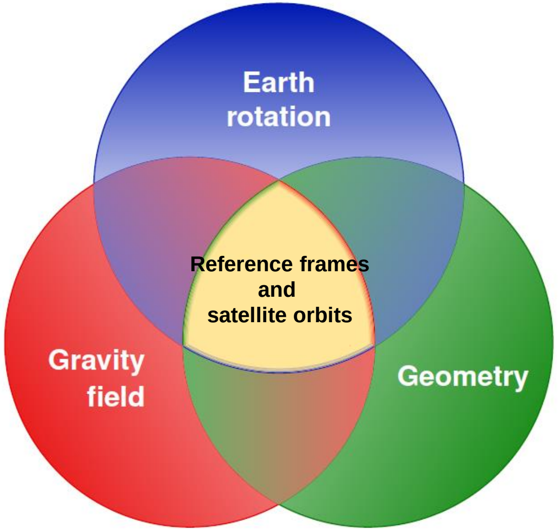
| state vector + TRF + EOP + EMP | state vector + EOP + EMP | state vector + EMP | state vector |
|---|---|---|---|
| 1.32 cm | 2.36 cm | 2.45 cm | 2.55 cm |

# SLR parameter space

SLR observations can be used to estimate

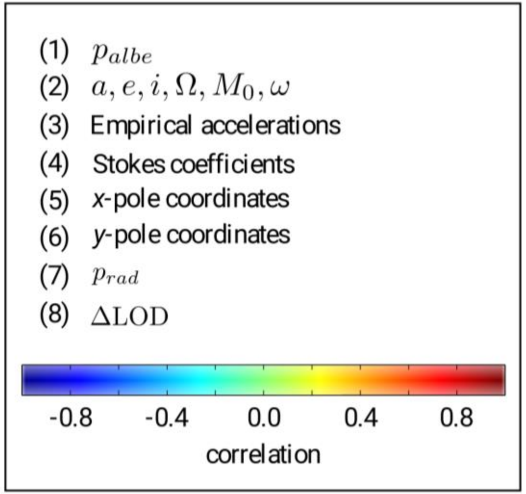
- **Initial state vector** of the satellite
- **Dynamic parameters** (which directly affect the orbit) such as
  - tidal coefficients, scaling factors, empirical accelerations
  - gravity field
- **Non-dynamic parameters** such as
  - TRF and
  - EOP

Three pillars of geodesy!

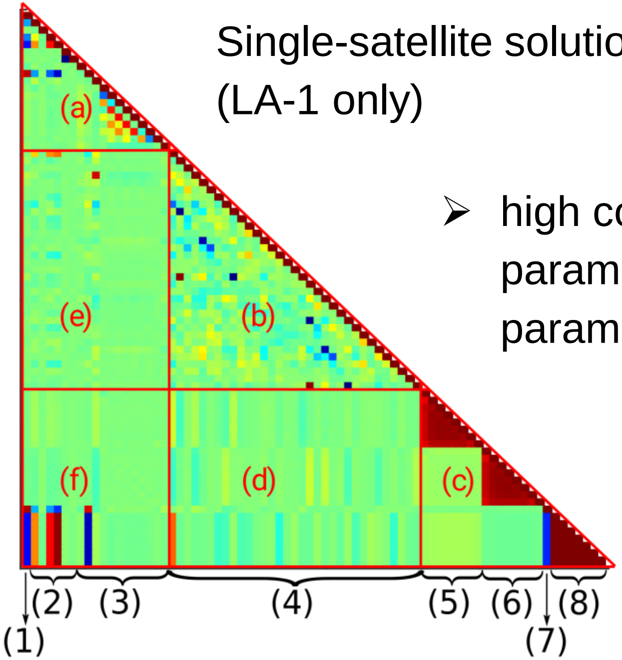


# SLR parameter space

Correlation matrix of combined solution comprising orbit parameters (LA-1 only), GFC, and EOP



(a) orbital elements and $p_{\mathrm{albe}}$
(b) Stokes coefficients
(c) EOP and SRP scaling factor ($p_{\mathrm{rad}}$)
(d), (e), (f) correlations between parameter groups

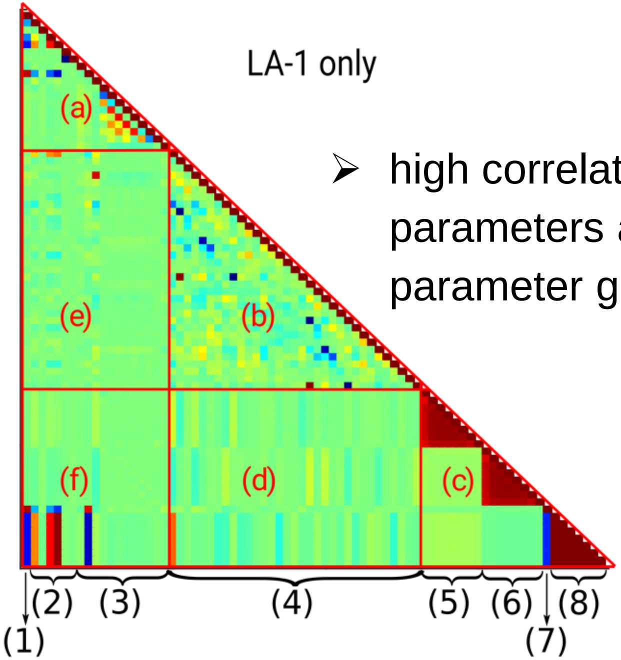


Single-satellite solution
(LA-1 only)

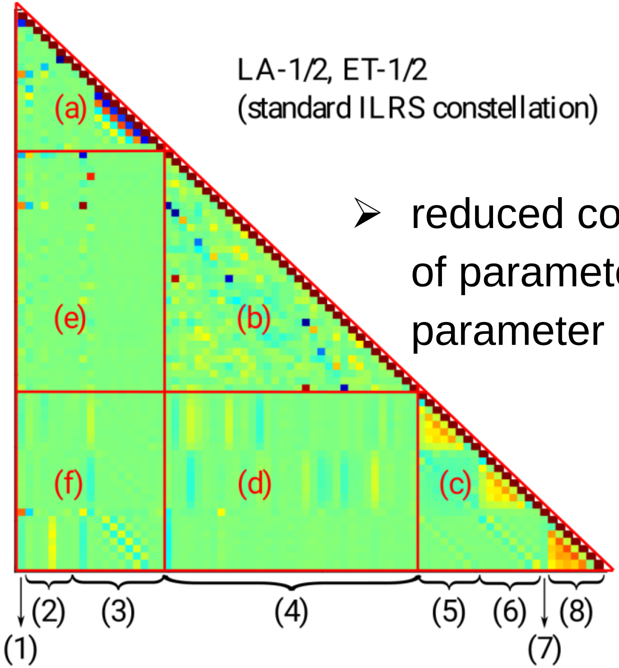
- high correlations of parameters and parameter groups

# SLR parameter space

Combination of multiple satellites to decorrelate the estimated parameter corrections

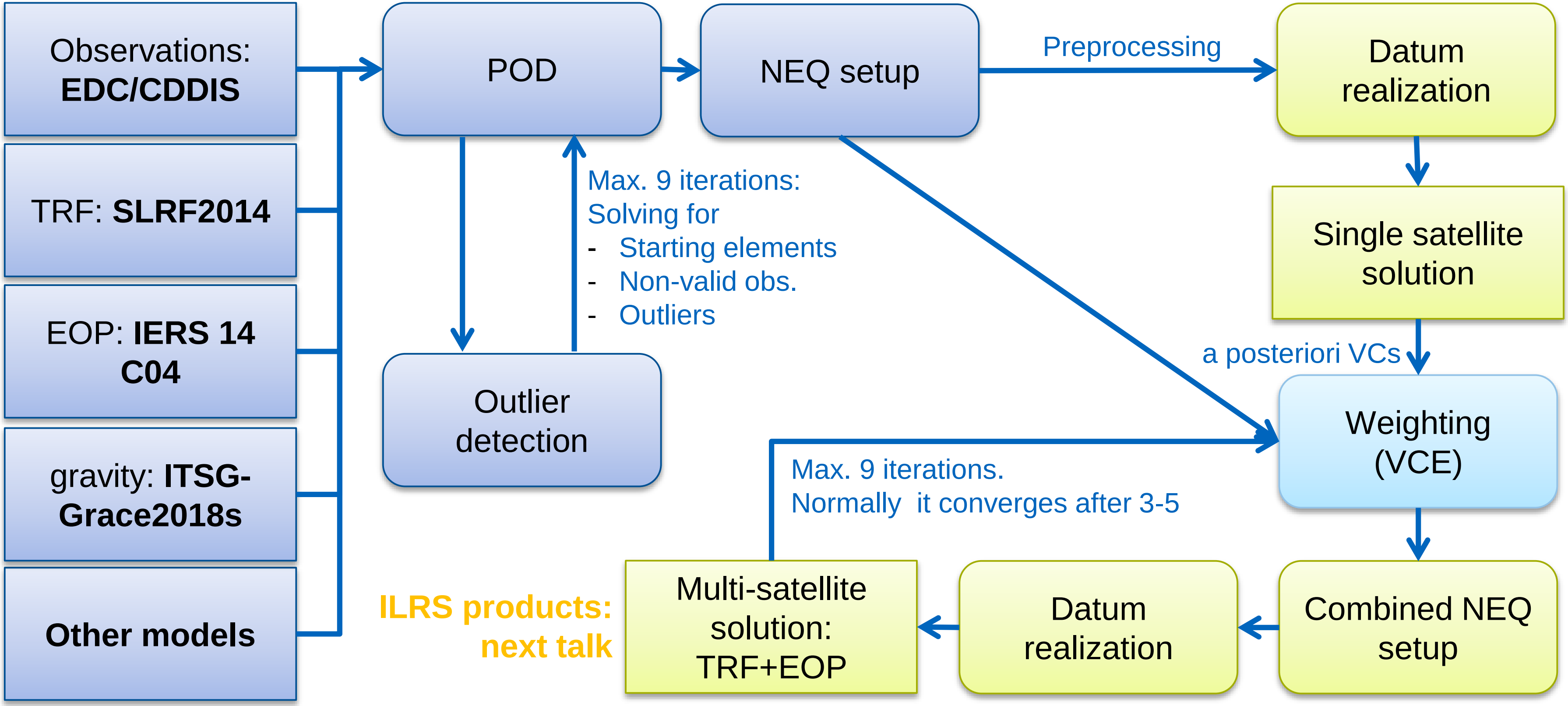


- high correlations of parameters and parameter groups



- reduced correlations of parameters and parameter groups

# Work/Data flow within s/w (DGFI-TUM setup!)



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POD

Outlier detection

Max. 9 iterations:
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- Outliers

NEQ setup

Preprocessing

Datum realization

Single satellite solution

a posteriori VCs

Weighting (VCE)

Max. 9 iterations.
Normally  it converges after 3-5

Combined NEQ setup

Datum realization

Multi-satellite solution: TRF+EOP

**ILRS products: next talk**

# Analyzing of SLR observations

## What do we do with the data?

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