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LAGEOS and LARES satellites attitude determination with the LASSOS spin model

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Summary

LAser RAnged Satellites Experiment

- Why modeling the Spin evolution
- Previous models
- The LASSOS model
- Comparison of LASSOS with observations
- Preliminary results for the thermal thrust effects
- Conclusions

Why modeling the Spin evolution

The importance of a model for the evolution of the **Spin** of the older **LAGEOS** satellite was initially highlighted by **Rubincam** in 1987/1988 in order to better model some **non-gravitational perturbations** (**NGP**):

- **Thermal drag/thrust effects arise as a consequence of the thermal radiation anisotropically emitted from the satellite's surface**
- **The radiation sources are the Earth (Yarkovsky or Rubincam effect) and the Sun (Yarkovsky-Schach effect)**

Therefore, the anisotropic distribution of temperature on the surface of the satellite and the ensuing recoil effects are function of the Spin-vector evolution, both in its orientation and rate, with respect to an inertial reference frame.

Rubincam, D.P., 1987. *LAGEOS orbit decay due to infrared radiation from Earth*. J. Geophys. Res. 92, 1287–1294.
Rubincam, D.P., 1988. *Yarkovsky thermal drag on LAGEOS*. J. Geophys. Res. 93, 13805–13810.

Why modeling the Spin evolution

Initially, **Rubincam** (1987) suggested that a constant orientation of the rotation of **LAGEOS** was sufficient to explain the data well, although the magnetic torques associated with the parasitic (eddy) currents in the body of the satellite would have been able to gradually change the direction of the spin axis of the satellite.

- Indeed, in 1993 **Ries** et al. concluded that the assumption of a fixed orientation was no longer valid to match the data

Ries, J.C., **Eanes**, R.J., **Watkins**, M.M., 1993. *Spin vector influence on LAGEOS ephemeris*. Second Meeting of Special Study Group 2.130, Int. Assoc. Of Geod., Baltimore, Md.

Why modeling the Spin evolution

There are several aspects that require the knowledge of the Spin evolution of **LAGEOS-like** satellites, such as:

- **Improved models for the NGP**
  - **Thermal drag/thrust effects (Yarkovsky effect, Yarkovsky-Schach effect)**
  - **Asymmetric reflectivity (LAGEOS, LAGEOS II)**
- **Range correction: “reflecting point” position with respect to the satellite CoM**

With consequences in the space geodesy products (ITRF, geocenter) and applications of **SLR** data for fundamental physics measurements in the field of the Earth.

Previous models

Bertotti, B., **Iess**, L., 1991. *The rotation of LAGEOS*. J. Geophys. Res. 96, 2431–2440.
Farinella, P., **Vokrouhlicky**, D., **Barlier**, F., 1996. *The rotation of LAGEOS and its long-term semimajor axis decay: a self-consistent solution*. J. Geophys. Res. 101, 17861–17872.
Vokrouhlicky, D., 1996. *Non-gravitational effects and LAGEOS' rotation*. Geophys. Res. Lett. 23, 3079–3082.
Andrés, J.I., **Noomen**, R., **Bianco**, G., **Currie**, D.G., **Otsubo**, T., 2004. *Spin axis behavior of the LAGEOS satellites.* J. Geophys. Res. 109, B06403.

The successful models developed in the past were all based on averaged equations for the torques acting on the **LAGEOS** and **LAGEOS II** satellites:

- **Bertotti and Iess (1991)**
- **Farinella, Vokrouhlicky and Barlier (1996)**
- **Vokrouhlicky (1996)**
- **Andres, Noomen, Bianco, Currie and Otsubo (2004)**

The main torques considered where:

- **The magnetic torque due to eddy (Foucault) currents**
- **The gravitational torque due to the oblateness of the satellites**
- **The torque due to the difference in the reflectivity of the two hemispheres**
- **The torque due to the possible non coincidence between the CoM and the geometrical center**

Previous models

Among the cited models, the most successful was that of Andrés (2007) in the final version of his PhD Thesis, named **LOSSAM** (**L**age**OS S**pin **A**xis **M**odel). This model is mainly based:

- **On the use of the four torques previously introduced**
- **A more detailed model for the complex structure of the satellites (starting from Slabinski (1996))**
- **A full consideration of all the available observations (orientation and rate) of the satellites spin**
- **A tune of some of the model parameters to best fit the observations**

Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.

Slabinski, V.J., 1996. *A numerical solution for LAGEOS thermal thrust: the rapid-spin case*. Celestial Mech. Dyn. Astron. 66, 131–179.

Previous models

Unfortunately, a Spin model based on **average equations** is valid only for a **fast spinning** satellite. Consequently, because of the **slowing down** of the rate produced by the magnetic torque, none of these models is reliable nowadays for the two **LAGEOS** satellites. Indeed, the rotational dynamics is dictated by the ratios of the following characteristic times:

- **The rotational period of the satellite** $\boldsymbol{T_{rot}}$
- **The orbital period of the satellite** $\boldsymbol{T_{orb}}$
- **The Earth rotational period** $\boldsymbol{T_{\oplus}}$
- **The thermal inertia of the surfaces of the satellite, in particular of the CCR** $\boldsymbol{\tau}$

Previous models

To overcome this problem, it is necessary to solve the entire set of **Euler equations**, so as to replace the averaged equations for the torques. Two (unsuccessful) tentative in this direction were made by:

- **Habib et al. (1994)**
- **Williams (2002)**

Here our work begins.

Habib, S., **Holz**, D.E., **Kheyfets**, A., **Matzner**, R.A., **Miller**, W.A., **Tolman**, B.W., 1994. *Spin dynamics of the LAGEOS satellite in support of a measurement of the Earth's gravitomagnetism*. Phys. Rev. D 50, 6068–6079.
Williams, S.E., 2002. *The Lageos satellite: A comprehensive spin model and analysis*, Ph.D. thesis, NCSU, 2002

The LASSOS model

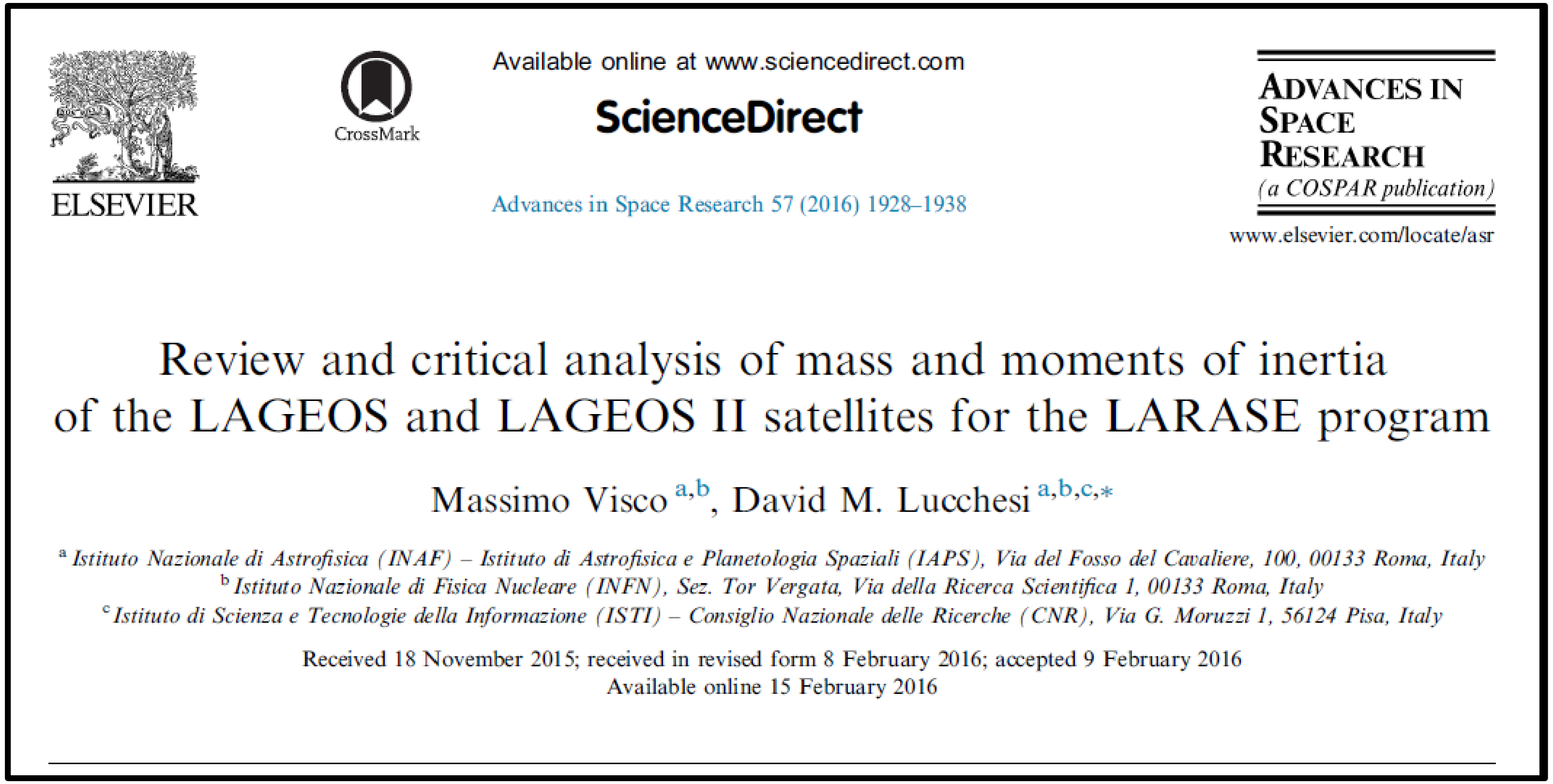
Our goal was to provide a new **general spin model**, i.e. valid also in the **slow-spin regime**, solving these three main issues:

1. **Use of the correct structure (both internal and external) for the considered satellites (the two LAGEOS and LARES)**
2. **Solve under a more general point of view the interaction of the satellite with the Earth's magnetic field**
3. **Solve Euler general equations for all the torques**

$$
\begin{aligned}
I_x \dot{\omega}^b_{sx} - \omega^b_{sy}\omega^b_{sz}(I_y - I_z) &= M_x \\
I_y \dot{\omega}^b_{sy} - \omega^b_{sx}\omega^b_{sz}(I_z - I_x) &= M_y \\
I_z \dot{\omega}^b_{sz} - \omega^b_{sx}\omega^b_{sy}(I_x - I_y) &= M_z
\end{aligned}
$$

The LASSOS model

The first issue was reached in 2016



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Review and critical analysis of mass and moments of inertia of the LAGEOS and LAGEOS II satellites for the LARASE program

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The LASSOS model

The first issue was reached in 2016, in particular:

- We reconstruct information about the structure, the material used and the moments of inertia of the two LAGEOS
- We built a 3D-CAD model of the satellites structure useful for finite element-based analysis
- We also solve for contradictions and overcome several misunderstanding present in the historical literature of the older LAGEOS (carefully re-analyzing the earlier technical documents)

LAGEOS

LARES

The LASSOS model

From the analysis of all the documentation that we have been able to collect, we concluded that:

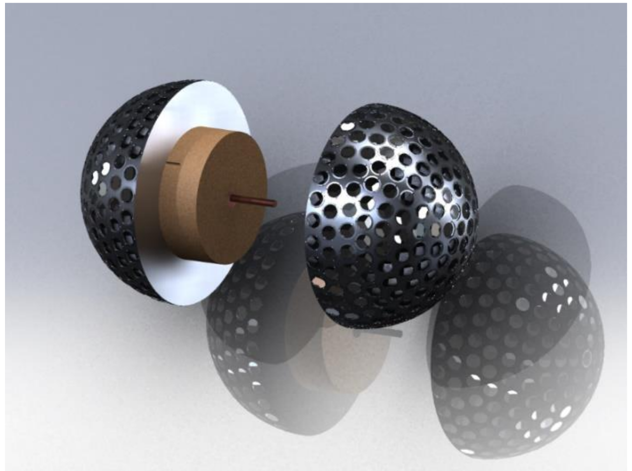
- **the two LAGEOS have been built using almost identical working drawings**
- **and, if we exclude the different mounting of the Ge CCRs, the two satellites are almost identical (twins), being (slightly) different for manufacturing tolerances and material alloys**

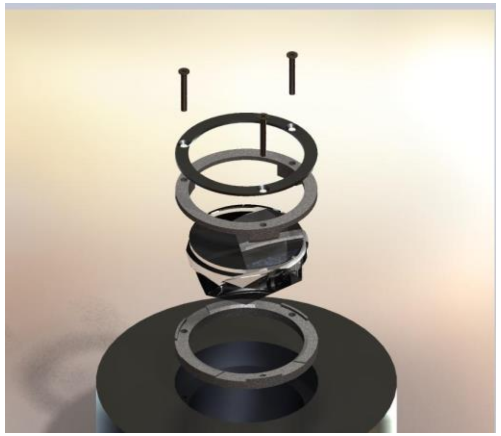
Therefore, we have been able to build a complete **3D model** of **LAGEOS** and **LAGEOS II** with **SOLIDWORKS** using:

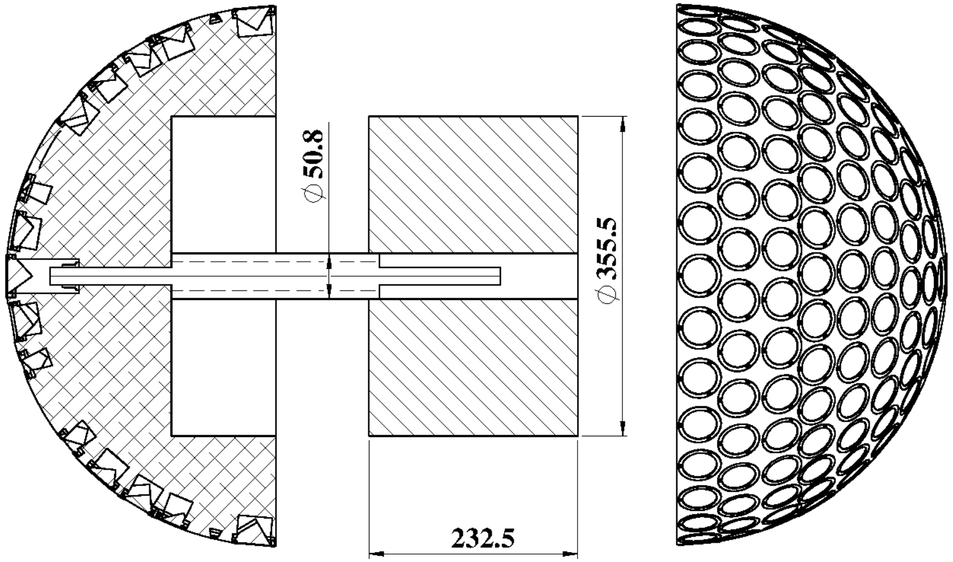
- **the working drawings of Minott et al. (1993)**
- **the information about the involved materials as reported in Cogo (1988)**

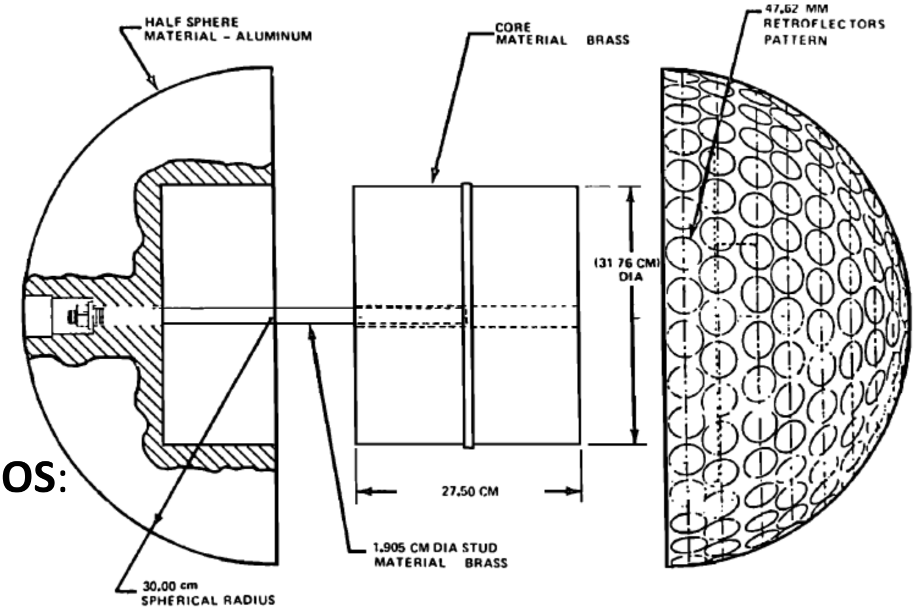
Minott, P.O., **Zagwodzki**, T.W., **Varghese**, T., **Seldon**, M., 1993. *Prelaunch Optical Characterization of the Laser Geodynamic Satellite (LAGEOS 2)*. Technical Report 3400. NASA Technical Paper 3400.
Cogo, F., 1988. *Weight discrepancy analysis between LAGEOS 1 and LAGEOS 2 satellites*. Technical Report LG-TN-AI-035. Aeritalia.

The LASSOS model









- **The core is made of BRASS**
- **The stud is made of BERYLLIUM and COPPER**

An example are the correct dimensions for the stud and the core of **LAGEOS**: for instance, those reported in **Cohen** and **Smith** 1985 are wrong

Cohen, S.C., **Smith**, D.E., 1985. *LAGEOS scientific results – Introduction*. J. Geophys. Res. 90, 9217–9220.

The LASSOS model

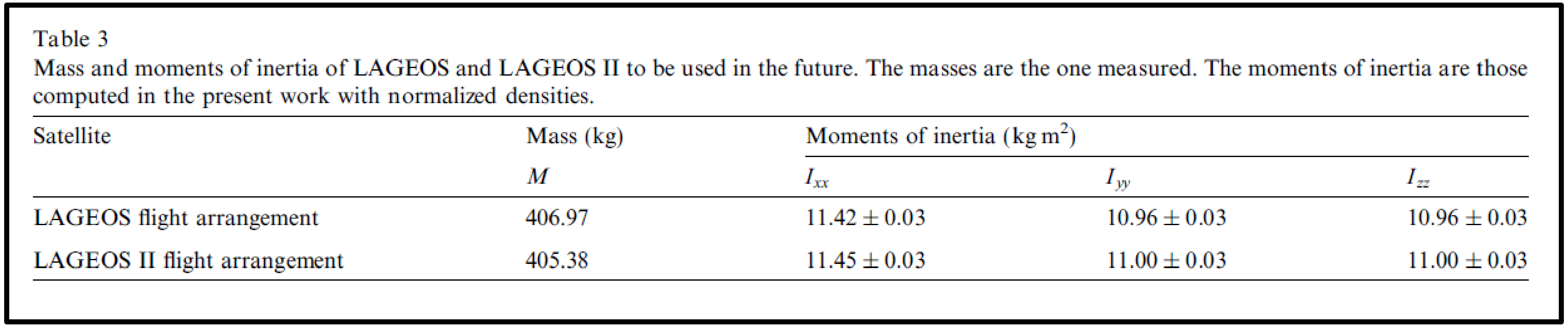


Table 3
Mass and moments of inertia of LAGEOS and LAGEOS II to be used in the future. The masses are the one measured. The moments of inertia are those computed in the present work with normalized densities.

Satellite	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement	406.97	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II flight arrangement	405.38	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03

This work was also extended to **LARES**:

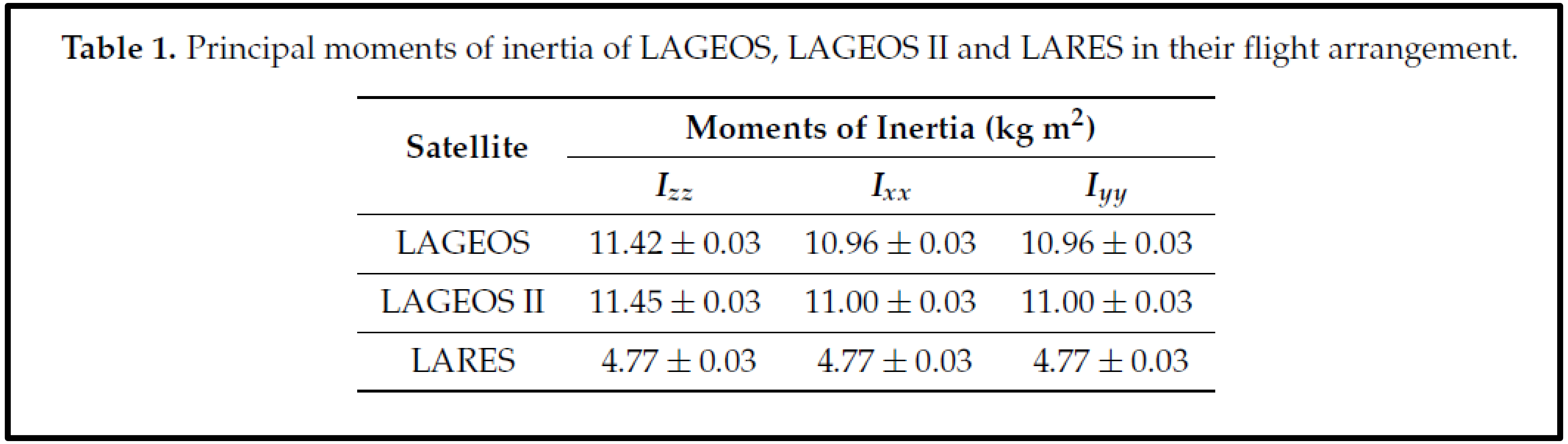


Table 1. Principal moments of inertia of LAGEOS, LAGEOS II and LARES in their flight arrangement.

Satellite	Moments of Inertia (kg m^2)		
	I_{zz}	I_{xx}	I_{yy}
LAGEOS	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03
LARES	4.77 ± 0.03	4.77 ± 0.03	4.77 ± 0.03

- The two **LAGEOS** have almost the same oblateness of about 0.04
- **LARES** is practically spherical in shape, even if an oblateness as small as 0.002 is however possible

The LASSOS model

The model for the magnetic torque. Since we are working with conductive satellites moving and rotating in the Earth's magnetic field $\boldsymbol{B}$, a magnetic moment $\boldsymbol{m}$ will be induced in their body and, consequently, a torque $\boldsymbol{M}_{mag}$ will be applied:

$$\boldsymbol{M}_{mag} = \boldsymbol{m} \times \boldsymbol{B}$$

In previous works, **LAGEOS** was modeled as a conducting sphere rotating in a **static magnetic** field

- **The value of the constant magnetic field was computed averaging the magnetic field over the entire orbit of the satellite**

The LASSOS model

This solution, which is completely valid in a **quasi-stationary** field, can be suitably used as long as the rotation period of the satellite is much shorter than its orbital period as well as of the Earth's rotation period, but it could produce wrong results when is used in slow-spin conditions.

$$\boldsymbol{T_{rot} \ll T_{orb} \qquad T_{rot} \ll T_{\oplus}}$$

In order to obtain a more general expression of the magnetic torque we faced the problem to find an easily integrable expression for the torque acting on a conducting sphere rotating in an **alternating magnetic** field.

The LASSOS model

$$\begin{aligned}\mathbf{M}^{E}_{\mathrm{mag}} = {} & V\sum_{i=0}^{8}\frac{|\mathbf{B_i}|^2}{2|\boldsymbol{\omega}_\mathbf{s}|}\{-A''_i[1+\cos(2\omega_i t+2\varphi_i)]+D'_i\sin(2\omega_i t+2\varphi_i)\}\boldsymbol{\omega}^{\mathbf{E}}_{\mathbf{s}} \\ & + V\sum_{i=0}^{8}\frac{\mathbf{B_i}\cdot\boldsymbol{\omega}_\mathbf{s}}{2|\boldsymbol{\omega}_\mathbf{s}|^2}\{[\alpha'(\omega_i)-A'_i][1+\cos(2\omega_i t+2\varphi_i)]-[D''_i+\alpha''(\omega_i)]\sin(2\omega_i t+2\varphi_i)\}(\boldsymbol{\omega}^{\mathbf{E}}_{\mathbf{s}}\times\mathbf{B_i}) \\ & + V\sum_{i=0}^{8}\frac{\mathbf{B_i}\cdot\boldsymbol{\omega}_\mathbf{s}}{2|\boldsymbol{\omega}_\mathbf{s}|}\{A''_i[1+\cos(2\omega_i t+2\varphi_i)]-D'_i\sin(2\omega_i t+2\varphi_i)\}\mathbf{B_i}, \qquad (7)\end{aligned}$$

where

$$A'_i = \frac{\alpha'(\omega^E_s-\omega_i)+\alpha'(\omega^E_s+\omega_i)}{2} \qquad D'_i = \frac{\alpha'(\omega^E_s-\omega_i)-\alpha'(\omega^E_s+\omega_i)}{2}$$

$$A''_i = \frac{\alpha''(\omega^E_s-\omega_i)+\alpha''(\omega^E_s+\omega_i)}{2} \qquad D''_i = \frac{\alpha''(\omega^E_s-\omega_i)-\alpha''(\omega^E_s+\omega_i)}{2}. \qquad (8)$$

$$\begin{aligned}\omega_0 &= 0\\ \omega_1 &= \omega_2 = \omega_\oplus - 2n\\ \omega_3 &= \omega_4 = \omega_\oplus + 2n\\ \omega_5 &= \omega_6 = 2n\\ \omega_7 &= \omega_8 = \omega_\oplus,\end{aligned}$$

$$\varphi_i = \begin{cases} -\frac{\pi}{2} & \text{for i} = 2,4,6,8\\ 0 & \text{for i} = 0,1,3,5,7.\end{cases}$$

$\alpha(\omega) = \alpha' + j\alpha''$ Complex Fourier Transform of the magnetic polarizability per unit of volume

$\omega_\oplus$ Earth's angular rate

n the satellite mean motion

$$\mathbf{B} = \sum_{i=0}^{8}\mathbf{B_i}\cos(\omega_i t+\varphi_i),$$

The LASSOS model

R	radius of the satellite
σ	electrical conductivity
μ_r	relative magnetic permeability
ω	angular frequency
β’, β’’	dimensioless constants

$$\alpha(\omega) = \alpha' + j\alpha''$$
$$= \frac{3}{8\pi}\left\{\frac{2\mu_r[1 - k\cdot\cot(k)] + [1 - k^2 - k\cdot\cot(k)]}{\mu_r[1 - k\cdot\cot(k)] - [1 - k^2 - k\cdot\cot(k)]}\right\},$$

Perfect sphere: ~ LARES

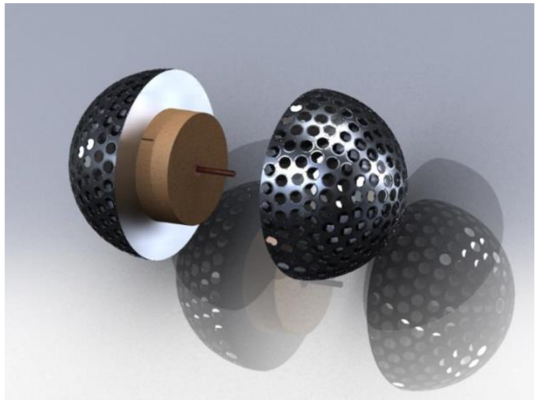
$$k(\omega) = \frac{R}{\delta(\omega)}(1 + j),$$

$$\delta(\omega) = \frac{c}{\sqrt{2\pi\omega\sigma\mu_r}}.$$

Penetration depth of
the magnetic field

$$\alpha(\omega) = \alpha' + j\alpha''$$
$$= \simeq \left[\frac{3}{4\pi}\frac{\mu_r - 1}{\mu_r + 2} - \frac{9}{350\pi}\frac{\mu_r(\mu_r + 9)}{(\mu_r + 2)^3}\left(\frac{R}{\delta}\right)^4\right]\beta'$$
$$+ j\frac{9}{20\pi}\frac{\mu_r}{(\mu_r + 2)^2}\left(\frac{R}{\delta}\right)^2\beta'',$$

Low frequency approximation: ~ LAGEOS, LAGEOS II



$$\frac{3}{4\pi}\frac{\mu_r - 1}{\mu_r + 2}$$ Residual polarizability at zero frequency

The LASSOS model

The involved torques

We considered in the case of the two LAGEOS satellites four torques:

1. **The magnetic torque (eddy currents)**

$$\mathbf{M}^{E}_{\mathrm{mag}} = V\sum_{i=0}^{8}\frac{|\mathbf{B_i}|^2}{2|\boldsymbol{\omega}_\mathbf{s}|}\{-A''_i[1+\cos(2\omega_i t+2\varphi_i)]+D'_i\sin(2\omega_i t+2\varphi_i)\}\boldsymbol{\omega}^{\mathbf{E}}_{\mathbf{s}}$$
$$+V\sum_{i=0}^{8}\frac{\mathbf{B_i}\cdot\boldsymbol{\omega}_\mathbf{s}}{2|\boldsymbol{\omega}_\mathbf{s}|^2}\{[\alpha'(\omega_i)-A'_i][1+\cos(2\omega_i t+2\varphi_i)]-[D''_i+\alpha''(\omega_i)]\sin(2\omega_i t+2\varphi_i)\}(\boldsymbol{\omega}^{\mathbf{E}}_{\mathbf{s}}\times\mathbf{B_i})$$
$$+V\sum^{8}\frac{\mathbf{B_i}\cdot\boldsymbol{\omega}_\mathbf{s}}{2|\boldsymbol{\omega}_\mathbf{s}|}\{A''_i[1+\cos(2\omega_i t+2\varphi_i)]-D'_i\sin(2\omega_i t+2\varphi_i)\}\mathbf{B_i},$$

2. **The gravitational torque**

$$M^{b}_{grav} = 3\omega^2_{\oplus}\left\{\ \hat{\boldsymbol{s}}^{\boldsymbol{b}}\times\left[I_x(\hat{\boldsymbol{s}}^{\boldsymbol{b}}\cdot\hat{\boldsymbol{x}}^{\boldsymbol{b}})\hat{\boldsymbol{x}}^{\boldsymbol{b}}+I_y(\hat{\boldsymbol{s}}^{\boldsymbol{b}}\cdot\hat{\boldsymbol{y}}^{\boldsymbol{b}})\hat{\boldsymbol{y}}^{\boldsymbol{b}}+I_z\left(\hat{\boldsymbol{s}}^{\boldsymbol{b}}\cdot\hat{\boldsymbol{z}}^{\boldsymbol{b}}\right)\hat{\boldsymbol{z}}^{\boldsymbol{b}}\right]\right\}$$

3. The asymmetric reflectivity torque (C_R differences)

$$M^{b}_{ar} \;=\; \nu\,\frac{2}{3}\rho^3\,\frac{\Phi}{c}\,\Delta\rho\,C_R\left(\hat{\boldsymbol{z}}^{\boldsymbol{b}}\times\hat{\boldsymbol{s}}^{\boldsymbol{b}}_{\odot}\right)\left|\hat{\boldsymbol{z}}^{\boldsymbol{b}}\times\hat{\boldsymbol{s}}^{\boldsymbol{b}}_{\odot}\right|$$

4. The CoM offset torque (with respect to the center of geometry)

$$M^{b}_{off} \;=\; \nu\pi\rho^2\,\frac{\Phi}{c}\,C_R\,\left(h^{b}\times\hat{s}^{b}_{\odot}\right)$$

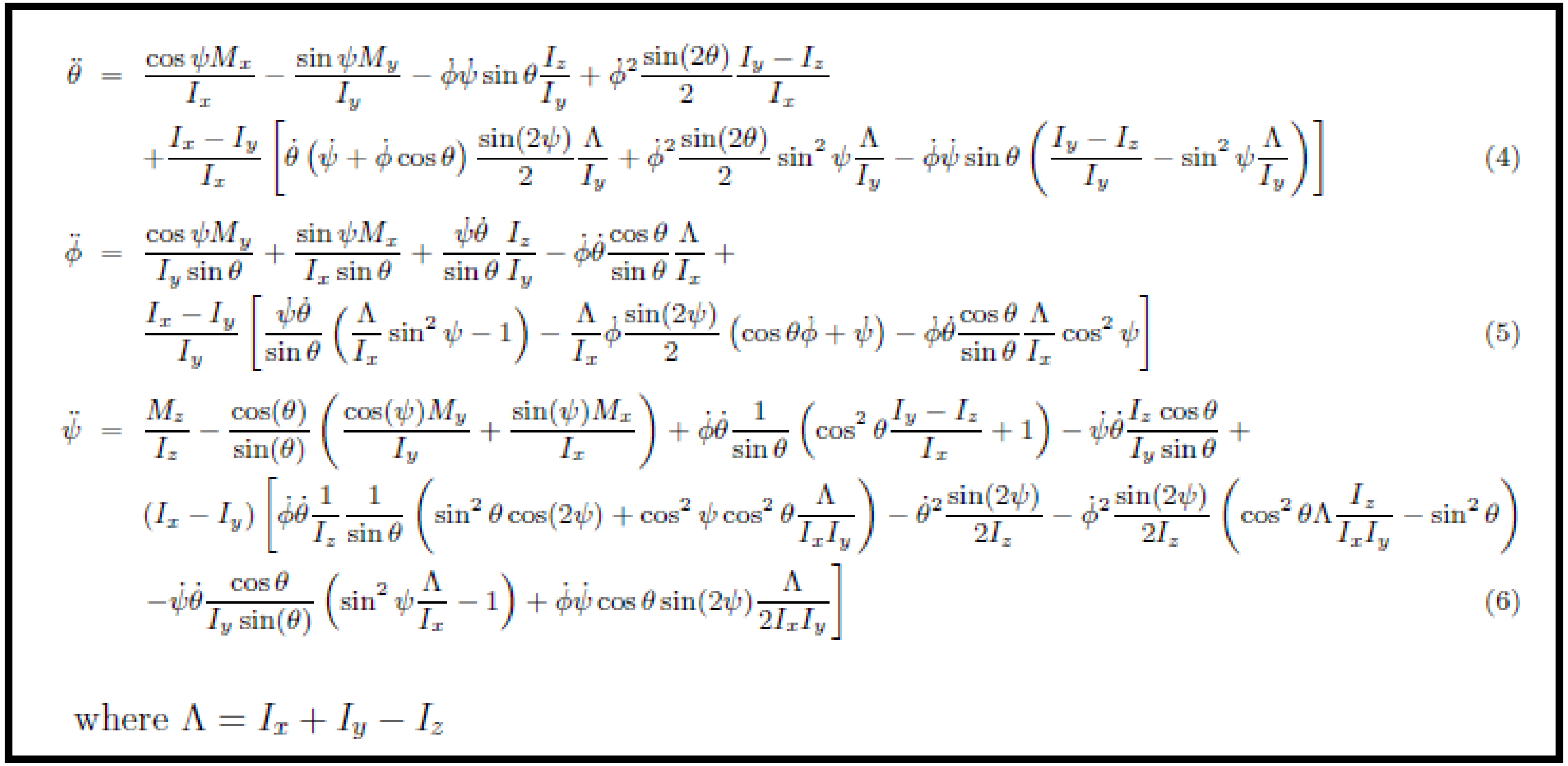
$$\boxed{\frac{d\boldsymbol{L}}{dt}=\underbrace{\boldsymbol{M}_{mag}+\boldsymbol{M}_{grav}}_{}+\boldsymbol{M}_{ar}+\boldsymbol{M}_{offset}}$$

Angular momentum evolution

The LASSOS model

We wrote the **Euler** equations in the body frame using the **Euler** angles with respect to the Earth Centered Inertial (ECI) reference frame:

$$I_x\dot{\omega}^b_{sx} - \omega^b_{sy}\omega^b_{sz}(I_y - I_z) = M_x$$
$$I_y\dot{\omega}^b_{sy} - \omega^b_{sx}\omega^b_{sz}(I_z - I_x) = M_y$$
$$I_z\dot{\omega}^b_{sz} - \omega^b_{sx}\omega^b_{sy}(I_x - I_y) = M_z$$



$$\begin{aligned}\ddot{\theta} &= \frac{\cos\psi M_x}{I_x} - \frac{\sin\psi M_y}{I_y} - \dot{\phi}\dot{\psi}\sin\theta\frac{I_z}{I_y} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\frac{I_y - I_z}{I_x} \\ &\quad + \frac{I_x - I_y}{I_x}\left[\dot{\theta}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)\frac{\sin(2\psi)}{2}\frac{\Lambda}{I_y} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\sin^2\psi\frac{\Lambda}{I_y} - \dot{\phi}\dot{\psi}\sin\theta\left(\frac{I_y - I_z}{I_y} - \sin^2\psi\frac{\Lambda}{I_y}\right)\right]\end{aligned} \tag{4}$$

$$\begin{aligned}\ddot{\phi} &= \frac{\cos\psi M_y}{I_y\sin\theta} + \frac{\sin\psi M_x}{I_x\sin\theta} + \frac{\dot{\psi}\dot{\theta}}{\sin\theta}\frac{I_z}{I_y} - \dot{\phi}\dot{\theta}\frac{\cos\theta}{\sin\theta}\frac{\Lambda}{I_x} + \\ &\quad \frac{I_x - I_y}{I_y}\left[\frac{\dot{\psi}\dot{\theta}}{\sin\theta}\left(\frac{\Lambda}{I_x}\sin^2\psi - 1\right) - \frac{\Lambda}{I_x}\dot{\phi}\frac{\sin(2\psi)}{2}\left(\cos\theta\dot{\phi} + \dot{\psi}\right) - \dot{\phi}\dot{\theta}\frac{\cos\theta}{\sin\theta}\frac{\Lambda}{I_x}\cos^2\psi\right]\end{aligned} \tag{5}$$

$$\begin{aligned}\ddot{\psi} &= \frac{M_z}{I_z} - \frac{\cos(\theta)}{\sin(\theta)}\left(\frac{\cos(\psi)M_y}{I_y} + \frac{\sin(\psi)M_x}{I_x}\right) + \dot{\phi}\dot{\theta}\frac{1}{\sin\theta}\left(\cos^2\theta\frac{I_y - I_z}{I_x} + 1\right) - \dot{\psi}\dot{\theta}\frac{I_z\cos\theta}{I_y\sin\theta} + \\ &\quad (I_x - I_y)\left[\dot{\phi}\dot{\theta}\frac{1}{I_z}\frac{1}{\sin\theta}\left(\sin^2\theta\cos(2\psi) + \cos^2\psi\cos^2\theta\frac{\Lambda}{I_xI_y}\right) - \dot{\theta}^2\frac{\sin(2\psi)}{2I_z} - \dot{\phi}^2\frac{\sin(2\psi)}{2I_z}\left(\cos^2\theta\Lambda\frac{I_z}{I_xI_y} - \sin^2\theta\right)\right. \\ &\quad \left. -\dot{\psi}\dot{\theta}\frac{\cos\theta}{I_y\sin(\theta)}\left(\sin^2\psi\frac{\Lambda}{I_x} - 1\right) + \dot{\phi}\dot{\psi}\cos\theta\sin(2\psi)\frac{\Lambda}{2I_xI_y}\right]\end{aligned} \tag{6}$$

where $\Lambda = I_x + I_y - I_z$

Comparison of LASSOS with observations

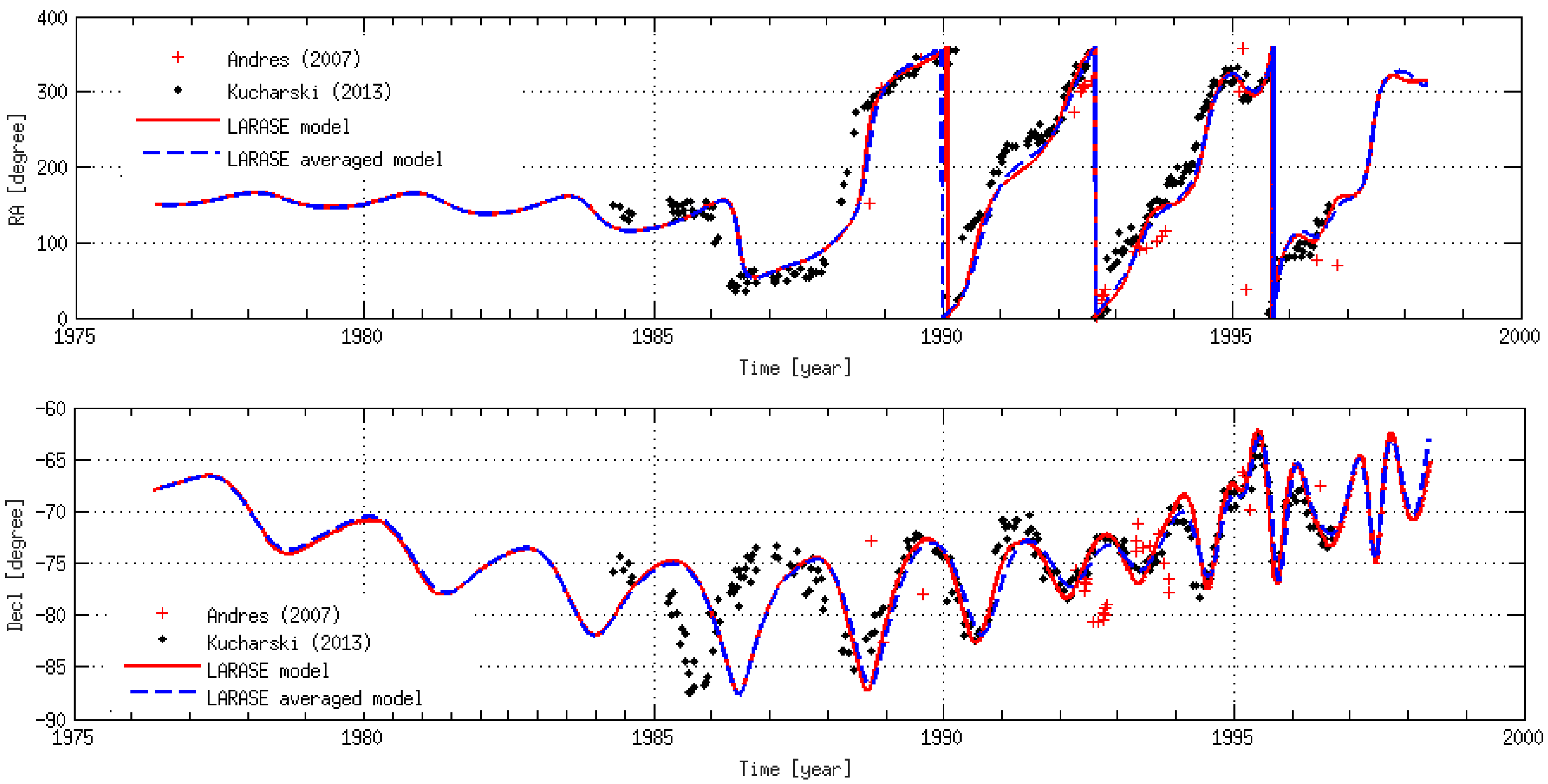
LASSOS Spin Model: results for LAGEOS

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Blue = LASSOS model for the rapid-spin
Red = LASSOS general model

Spin Orientation: α, δ

Andrés de la Fuente, J.I., 2007. ***Enhanced Modelling of LAGEOS Non-Gravitational Perturbations*** (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands. **Kucharski**, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. ***Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data***. Adv. Space Res. 52, 1332–1338.



Comparison of LASSOS with observations

LASSOS Spin Model: results for LAGEOS

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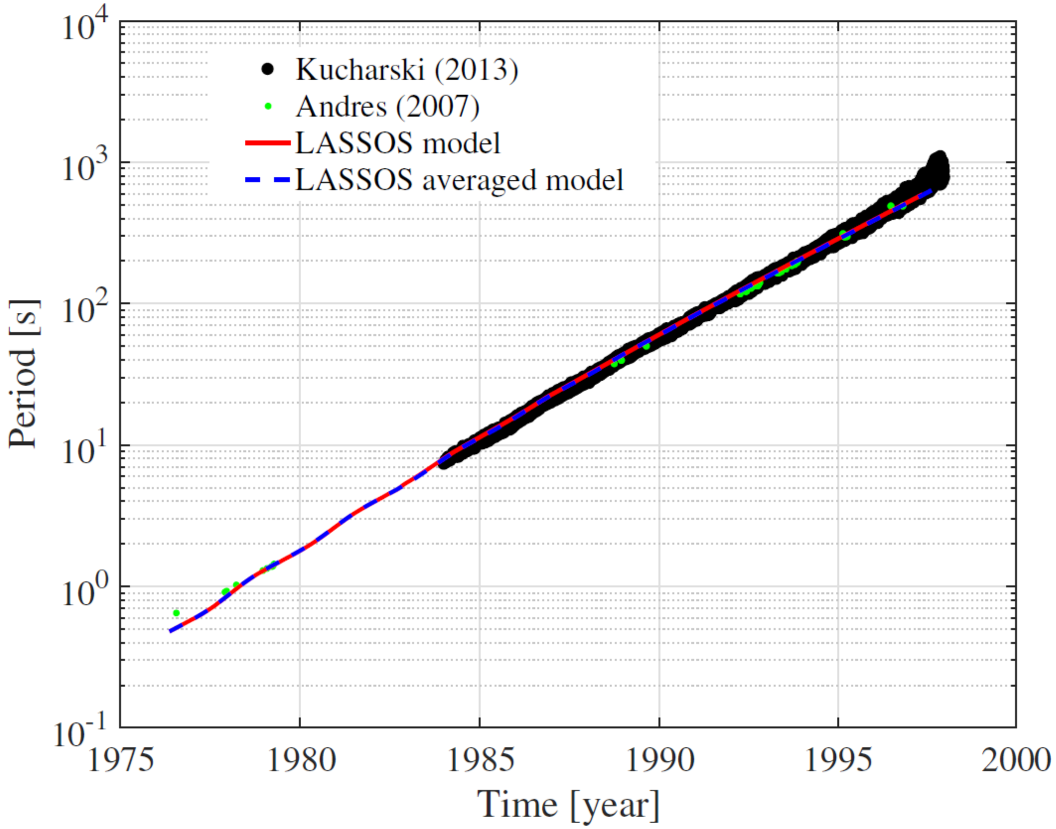


Comparison of LASSOS with observations

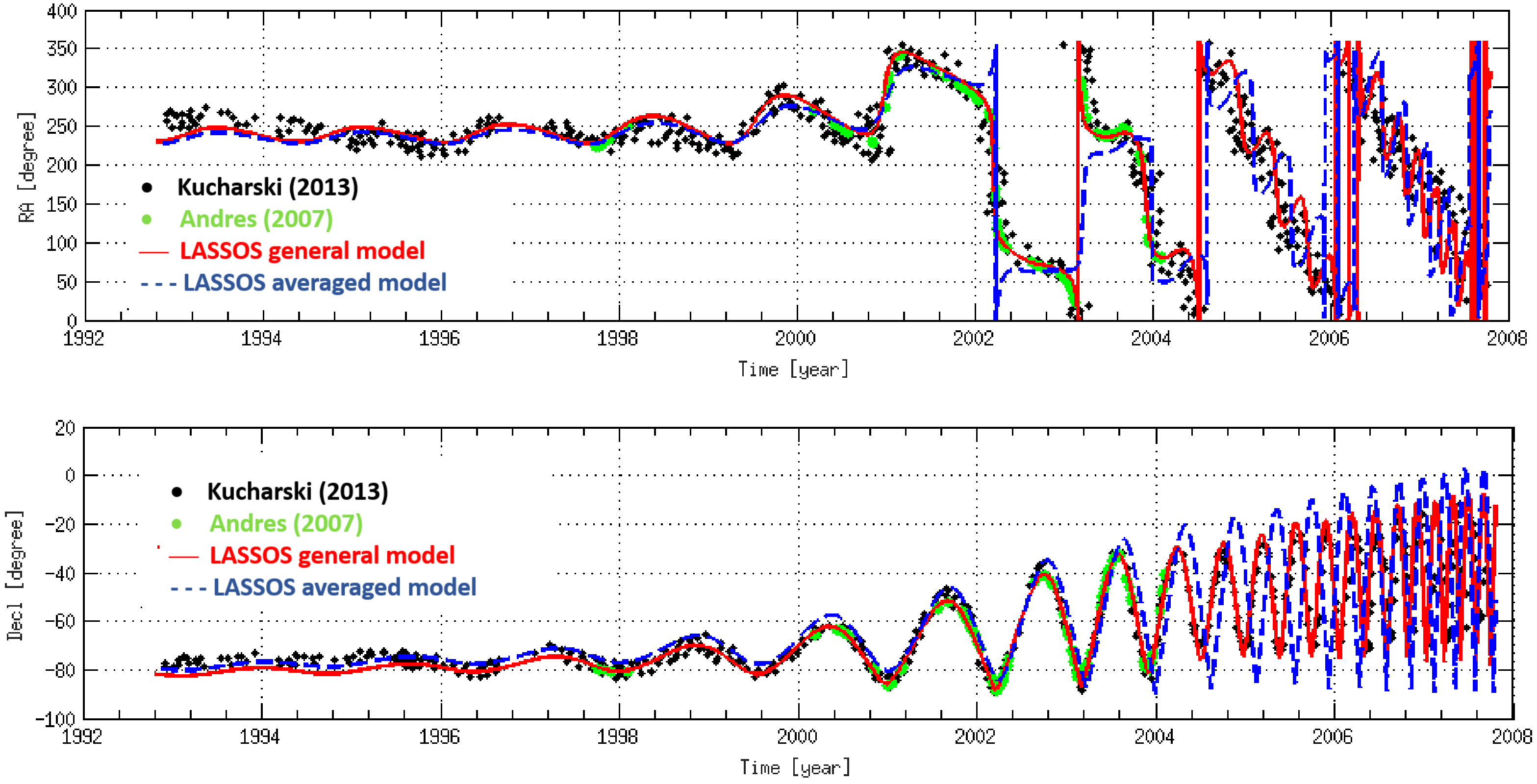
LASSOS Spin Model: results for LAGEOS II

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Blue = LASSOS model for the rapid-spin
Red = LASSOS general model

Spin Orientation: α, δ

Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.
Kucharski, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. *Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data*. Adv. Space Res. 52, 1332–1338.



Comparison of LASSOS with observations

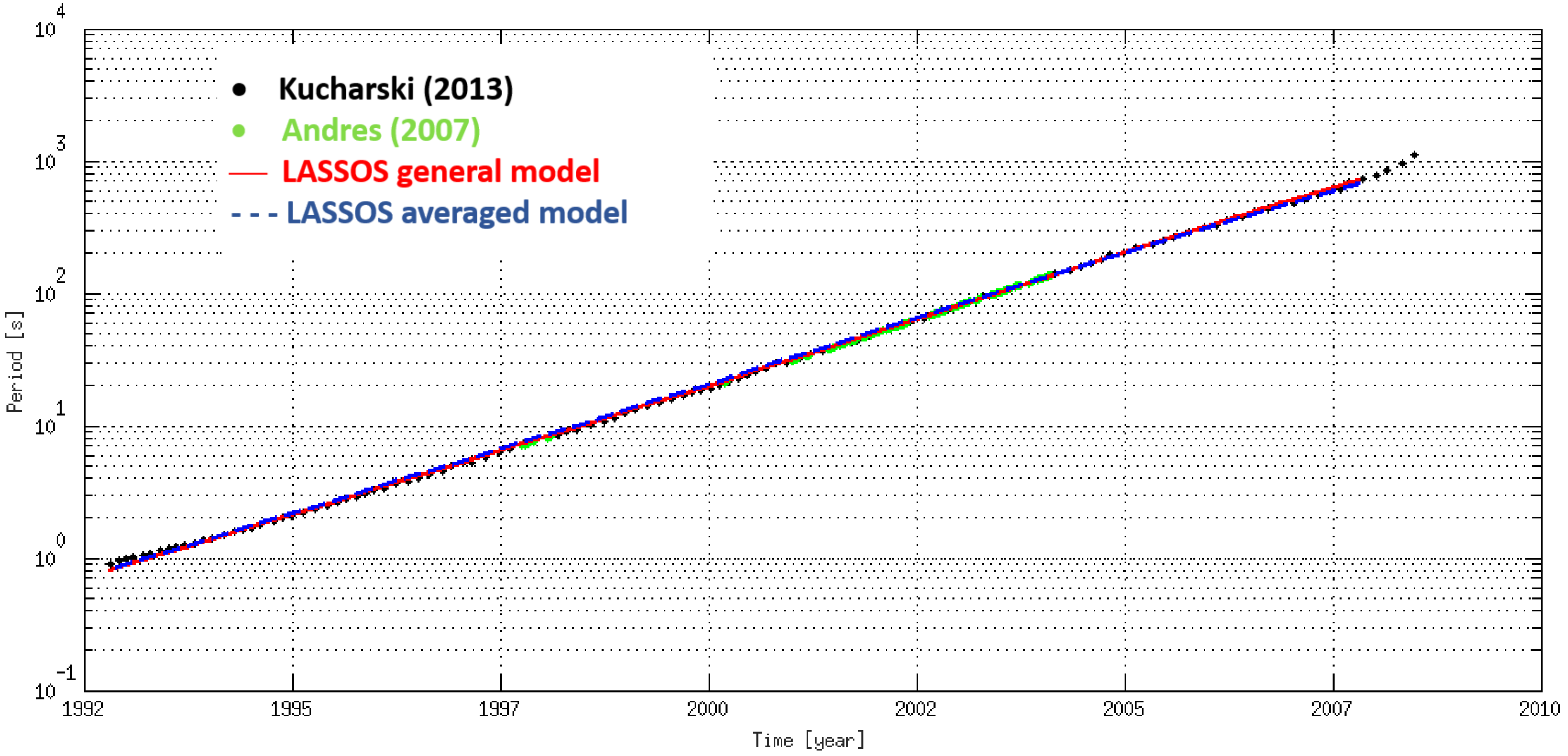
LASSOS Spin Model: results for LAGEOS II

LArase **S**atellites **S**pin m**O**del **S**olutions (**LASSOS**)

Blue = LASSOS model for the rapid-spin
Red = LASSOS general model

Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands. **Kucharski**, D., **Lim**, H.C., **Kirchner**, G., **Hwang**, J.Y., 2013. *Spin parameters of LAGEOS-1 and LAGEOS-2 spectrally determined from Satellite Laser Ranging data*. Adv. Space Res. 52, 1332–1338.

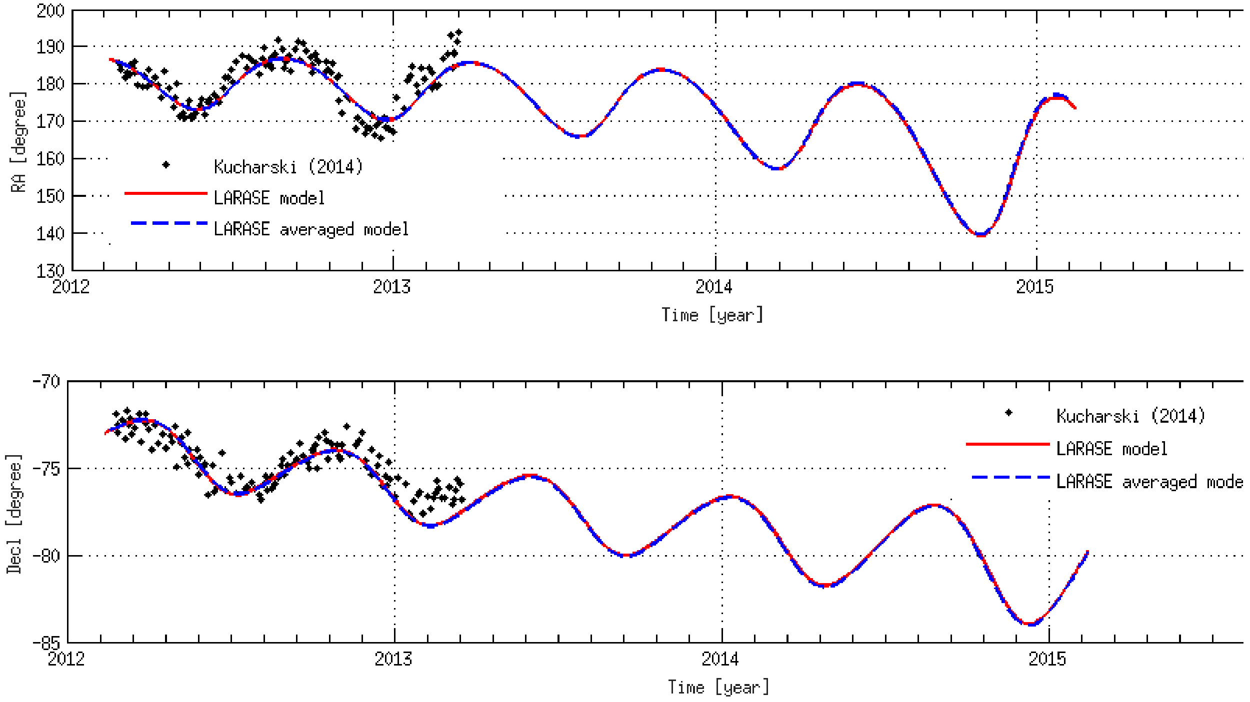
Rotational Period: P

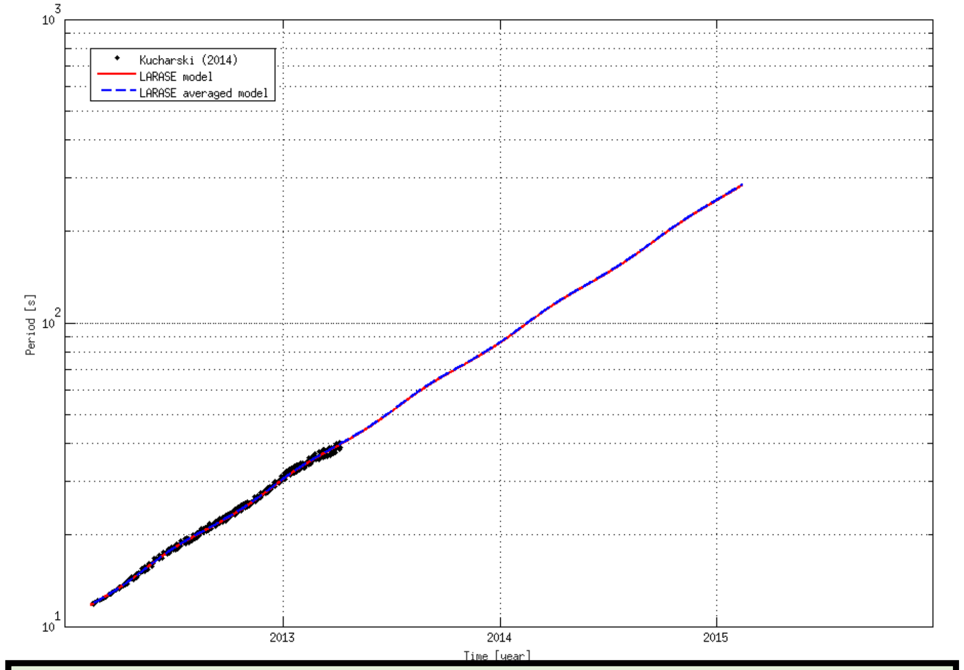


Comparison of LASSOS with observations

LASSOS Spin Model: preliminary results for LARES

Blue = LASSOS model for the rapid-spin
Red = LASSOS general model





Kucharski et al., IEEE Geos. Rem. Sens. Lett. 11, 2014

$T \approx P_{orb} \approx$ 115 min. after $\approx$ 5.9 years

$$T(s) \cong 11.8 \cdot e^{D/341} \quad \text{D [days]}$$

- The spin evolution is almost due to the magnetic torque
- The gravitational torque is almost null, we fit the data with an oblateness of about: $\frac{C-A}{C} < 10^{-4}$

Comparison of LASSOS with observations

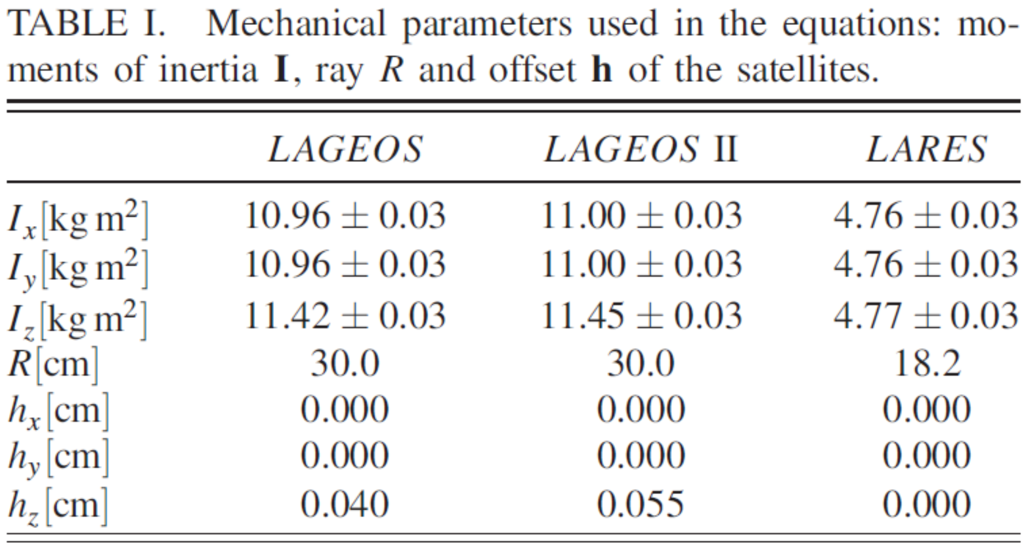


TABLE I. Mechanical parameters used in the equations: moments of inertia **I**, ray R and offset **h** of the satellites.

	LAGEOS	*LAGEOS* II	*LARES*
$I_x[\mathrm{kg\,m^2}]$	10.96 ± 0.03	11.00 ± 0.03	4.76 ± 0.03
$I_y[\mathrm{kg\,m^2}]$	10.96 ± 0.03	11.00 ± 0.03	4.76 ± 0.03
$I_z[\mathrm{kg\,m^2}]$	11.42 ± 0.03	11.45 ± 0.03	4.77 ± 0.03
$R[\mathrm{cm}]$	30.0	30.0	18.2
$h_x[\mathrm{cm}]$	0.000	0.000	0.000
$h_y[\mathrm{cm}]$	0.000	0.000	0.000
$h_z[\mathrm{cm}]$	0.040	0.055	0.000

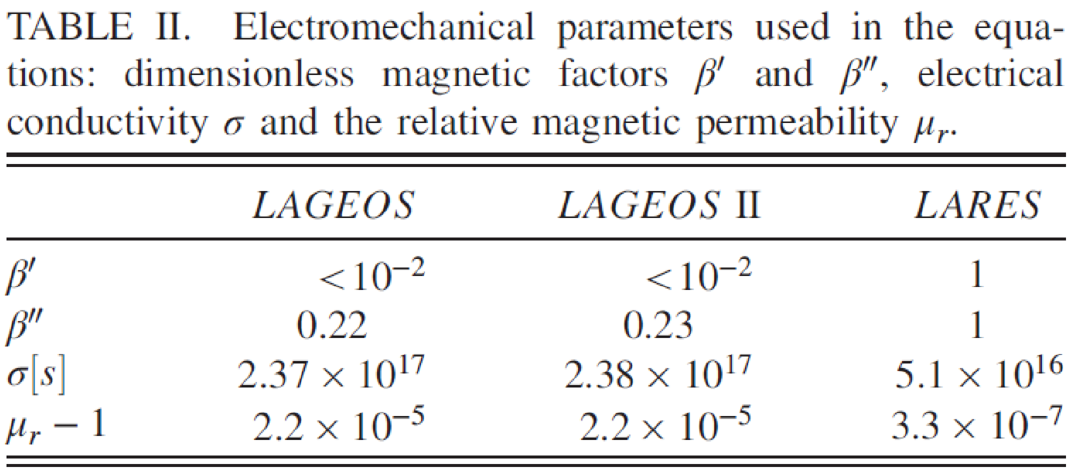


TABLE II. Electromechanical parameters used in the equations: dimensionless magnetic factors β' and β'', electrical conductivity σ and the relative magnetic permeability μ_r.

	LAGEOS	*LAGEOS* II	*LARES*
β'	$< 10^{-2}$	$< 10^{-2}$	1
β''	0.22	0.23	1
$\sigma[s]$	2.37×10^{17}	2.38×10^{17}	5.1×10^{16}
$\mu_r - 1$	2.2×10^{-5}	2.2×10^{-5}	3.3×10^{-7}

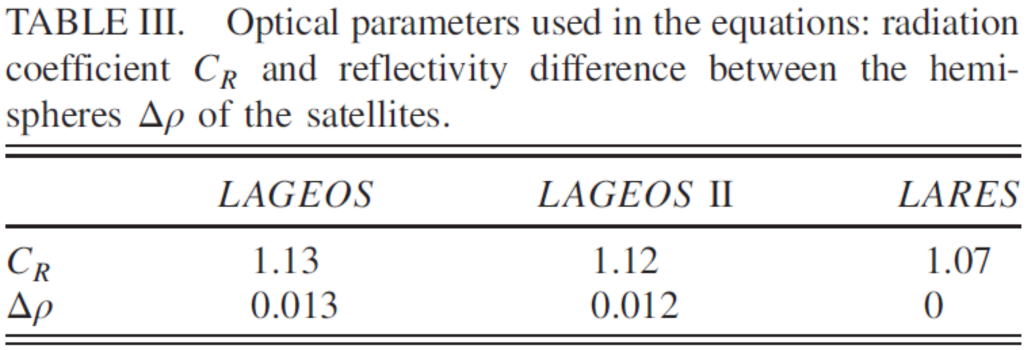


TABLE III. Optical parameters used in the equations: radiation coefficient C_R and reflectivity difference between the hemispheres $\Delta\rho$ of the satellites.

	LAGEOS	*LAGEOS* II	*LARES*
C_R	1.13	1.12	1.07
$\Delta\rho$	0.013	0.012	0

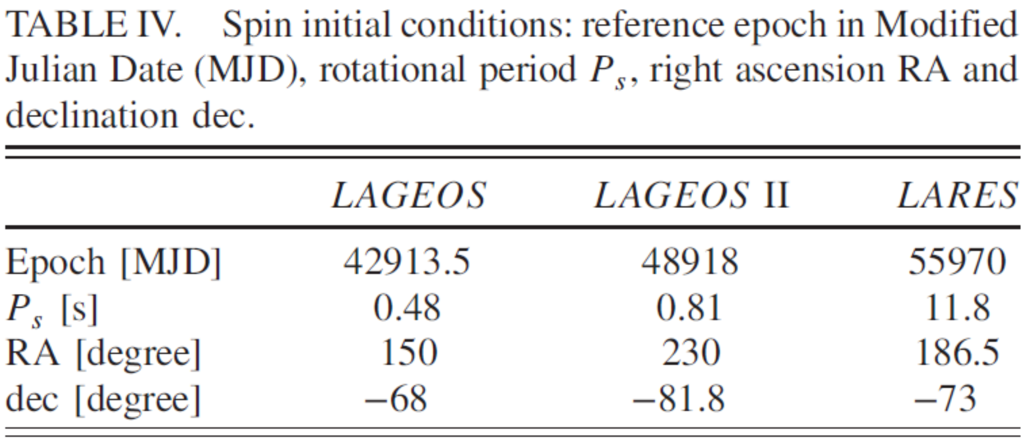
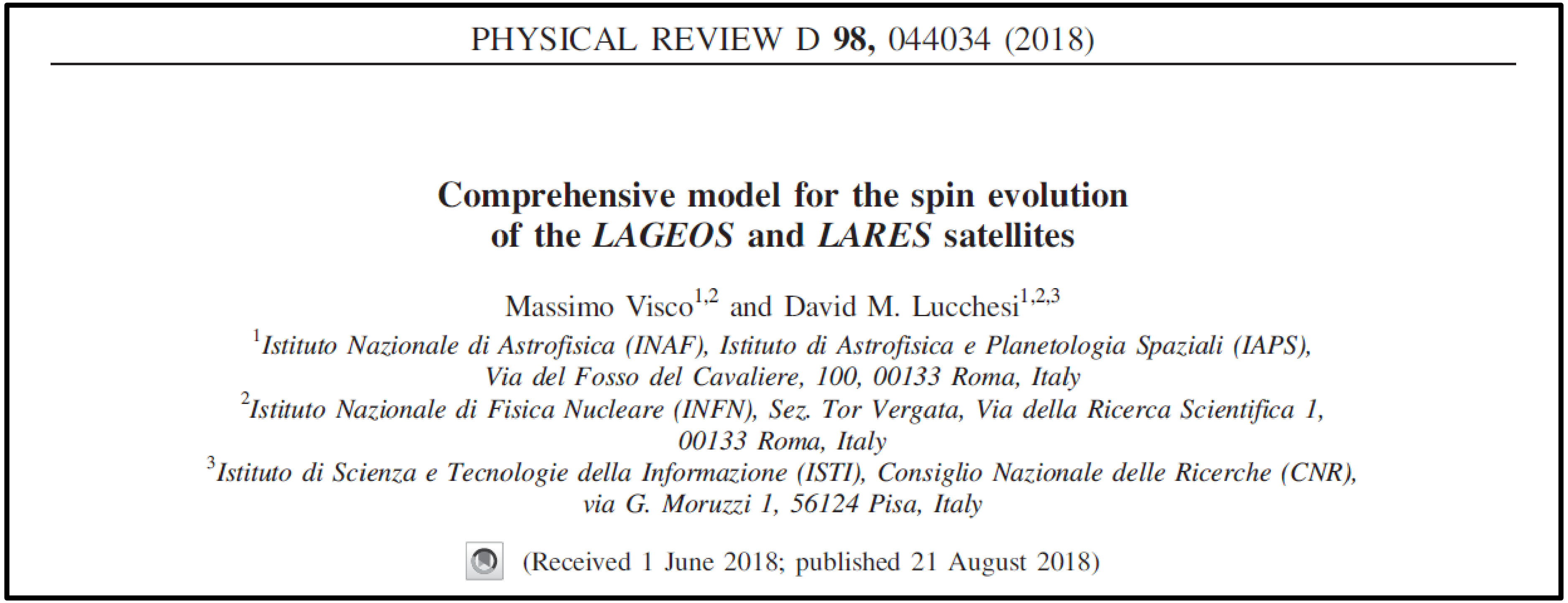


TABLE IV. Spin initial conditions: reference epoch in Modified Julian Date (MJD), rotational period P_s, right ascension RA and declination dec.

	LAGEOS	*LAGEOS* II	*LARES*
Epoch [MJD]	42913.5	48918	55970
P_s [s]	0.48	0.81	11.8
RA [degree]	150	230	186.5
dec [degree]	−68	−81.8	−73

Comparison of LASSOS with observations



PHYSICAL REVIEW D **98,** 044034 (2018)

Comprehensive model for the spin evolution of the *LAGEOS* and *LARES* satellites

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Preliminary results for the thermal thrust effects

Motivation:

Necessity of improved models for the NGP

- **Thermal drag/thrust effects (Yarkovsky effect, Yarkovsky-Schach effect)**
- **Asymmetric reflectivity (LAGEOS, LAGEOS II)**

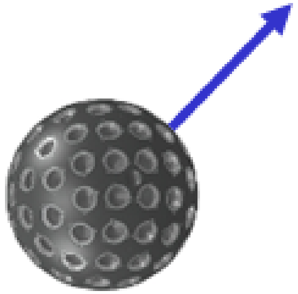
Previous models:

Rubincam, D.P., 1987. *LAGEOS orbit decay due to infrared radiation from Earth*. J. Geophys. Res. 92, 1287–1294.
Rubincam, D.P., 1988. *Yarkovsky thermal drag on LAGEOS*. J. Geophys. Res. 93, 13805–13810.
Rubincam, D.P., 1990. *Drag on the LAGEOS satellite*. J. Geophys. Res. 95, 4881–4886.
Farinella, P., **Nobili**, A.M., **Barlier**, F., **Mignard**, F., 1990. *Effects of thermal thrust on the node and inclination of LAGEOS*. Astron. Astrophys. 234, 546–554.
Farinella, P., **Vokrouhlicky**, D., 1996. *Thermal force effects on slowly rotating, spherical artificial satellites-I. Solar Heating.* Plan. Space Sci. 44, 1551–1561.
Vokrouhlicky, D., **Farinella**, P., 1996. *Thermal force effects on slowly rotating, spherical artificial satellites-II. Earth infrared heating*. Plan. Space Sci. 45, 419–425.
Slabinski, V.J., 1996. *A numerical solution for LAGEOS thermal thrust: the rapid-spin case*. Celestial Mech. Dyn. Astron. 66, 131–179.
Andrés de la Fuente, J.I., 2007. *Enhanced Modelling of LAGEOS Non-Gravitational Perturbations* (Ph.D. thesis). Delft University Press. Sieca Repro, Turbineweg 20, 2627 BP Delft, The Netherlands.

Preliminary results for the thermal thrust effects

The thermal thrust force:

$$\mathbf{dF_T} = -\frac{2}{3}\frac{\epsilon\sigma T^4 dA}{c}\mathbf{n}$$



The force, normal to each surface element *dA* depends from the temperature *T* and emissivity *ε* of the considered part.

It is necessary to know the temperature distribution inside the satellite and the satellite position with respect to the external heat sources (Sun and Earth).

Preliminary results for the thermal thrust effects

The thermal equations:

$$\frac{dT_i}{dt}\underset{\substack{\uparrow \\ \text{Thermal} \\ \text{capacity}}}{C_i} = \underbrace{\left(\Sigma_k P_{abs\ k} - P_{em\ i}\right)}_{\substack{\text{Difference between the total} \\ \text{Power absorbed and emitted}}} + \underbrace{\Sigma_j R_{i,j}\left(T_i^4 - T_j^4\right) + \Sigma_j C_{i,j}\left(T_i - T_j\right)}_{\substack{\text{Heat exchanged between the} \\ \text{different elements of the satellite} \\ \text{due to radiation and conduction}}}$$

The input to the system of differential equations are:

- **Attitude of the satellite (that we get from LASSOS model)**
- **Thermal and optical parameters of the satellite (get from technical documentation and tests) that contribute to the different constants in the system**

Preliminary results for the thermal thrust effects

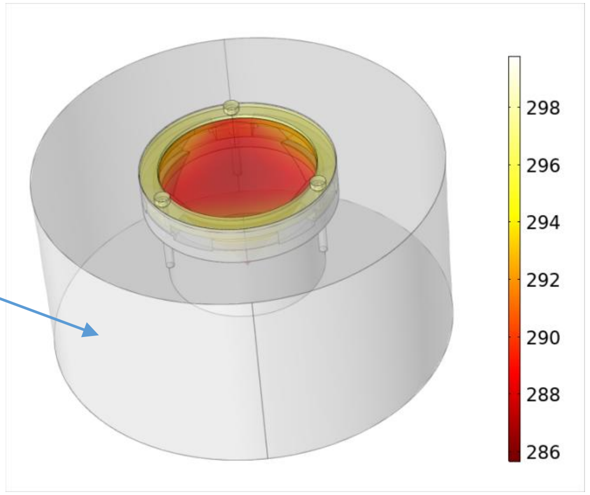
- The satellite is divided into several parts which are assumed to have no thermal gradient within them. For the two **LAGEOS**: the **CCRs**, the two **hemispheres** and the **core**. The **rings** that block the **CCRs** are considered isothermal to the hemispheres.

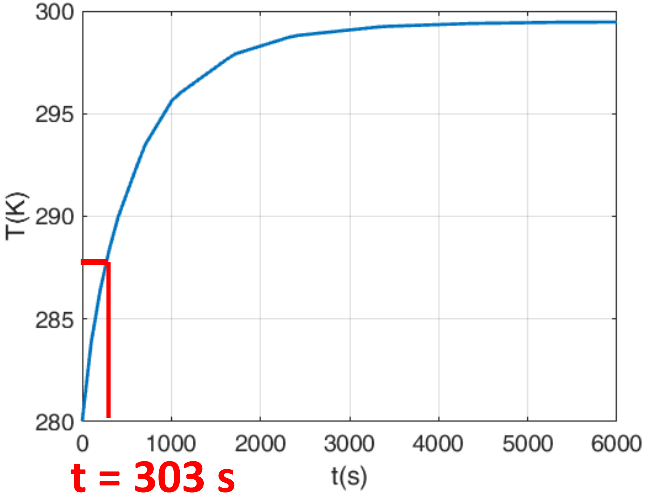
- The conduction constant between the **CCRs** and the hemisphere in which they are inserted was numerically calculated using a **FEM** model.

Coupling of a CCR with the structure

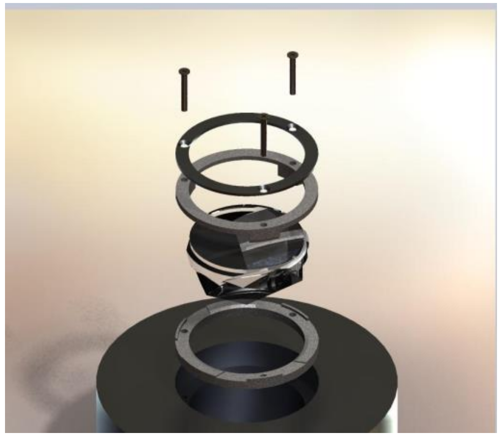
T = 300 K

t = 303 s





t = 303 s



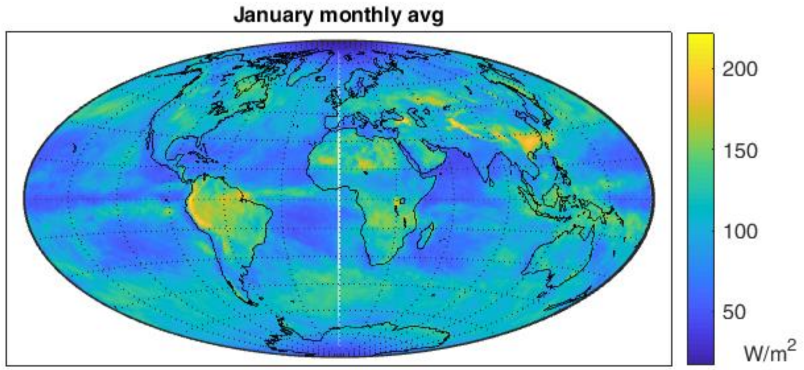
Preliminary results for the thermal thrust effects

We considered three external heat sources:

- **The direct Sun radiation – using the standard value of $\phi_{\odot} = 1360.8\,\frac{W}{m^2}$ at 1 A.U.**

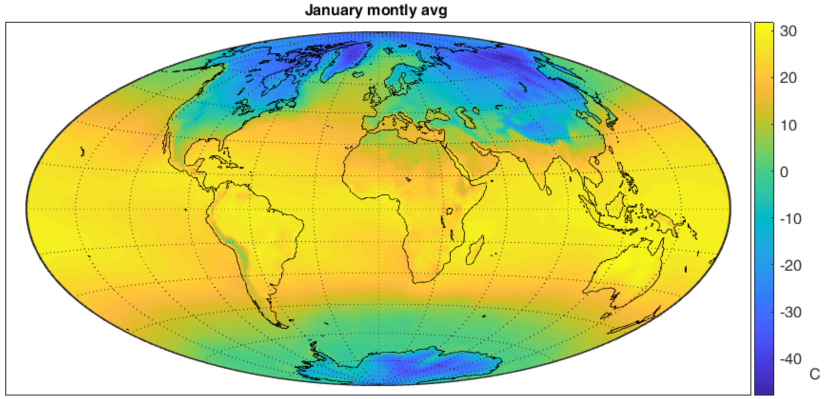
- **The Sun radiation reflected from Earth (Albedo)**

  We use CERES monthly averaged SW radiation data at the top of the atmosphere taking into account night-day alternance, satellite attitude and orbital position. The grid is 1°x1° Latitude-Longitude.



- **The infrared radiation from the Earth**

  We take into account the temperature of the different parts of the Earth using the monthly averaged data from Land + Ocean 1°x1° Latitude-Longitude grid from Berkeley Earth Organization. Attitude and orbit of the satellite are considered.



Preliminary results for the thermal thrust effects

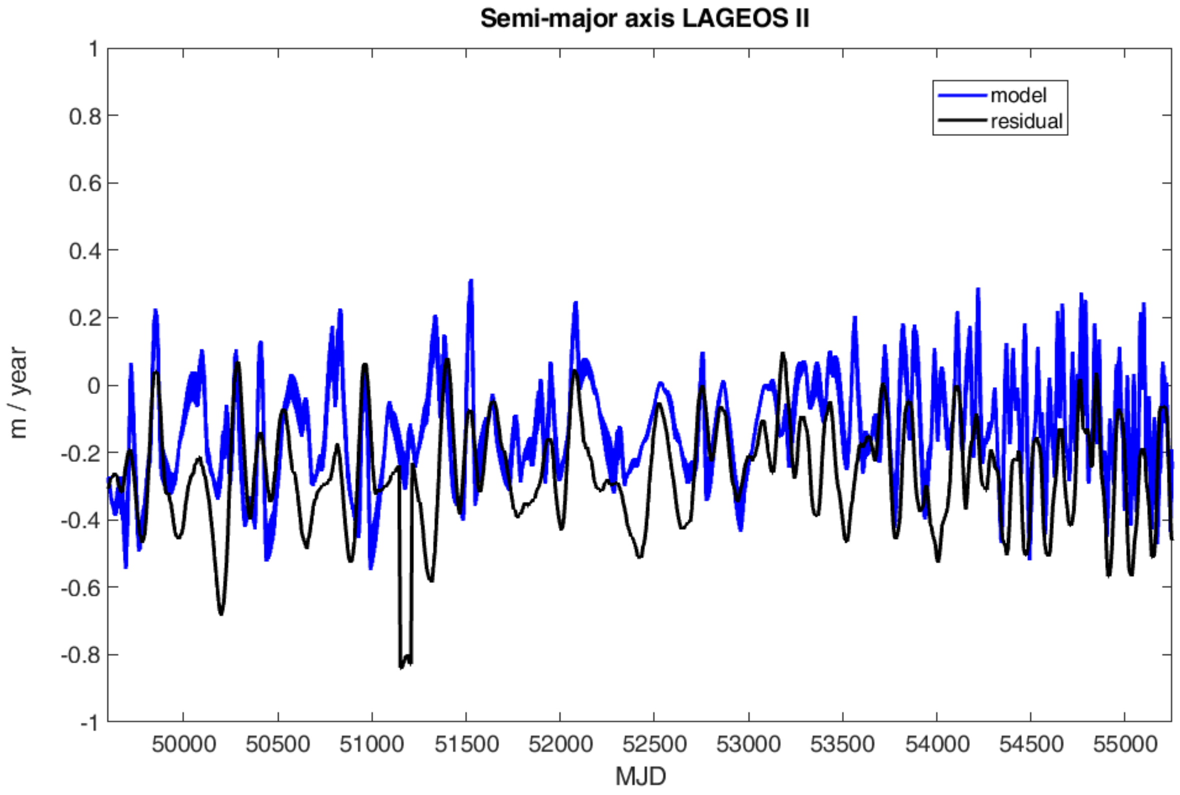
- We developed two versions of the model (**LATOS**), an averaged one, usable for fast-spin conditions, and a general one, not averaged, to be used when the spin is slow with respect to the orbital period.

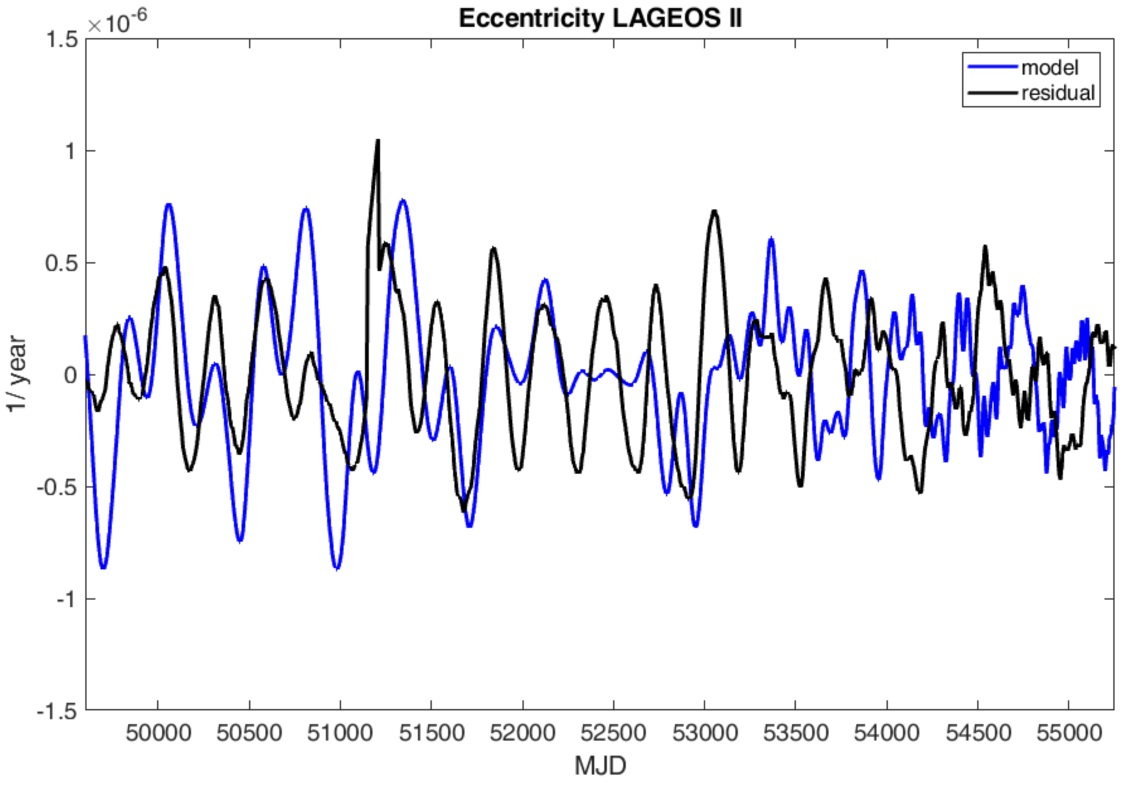
- By integrating the thermal equations we get the temperature distribution in the satellite and from this distribution we calculate the thermal thrust accelerations.

- We then calculated the effects of the thermal accelerations (via **Gauss** equations) on the rate of the Keplerian elements. The results can be compared with the corresponding rate residuals from a **precise orbit determination** (**POD**).

LArase Thermal mOdel Solutions (LATOS)

Preliminary results for the thermal thrust effects





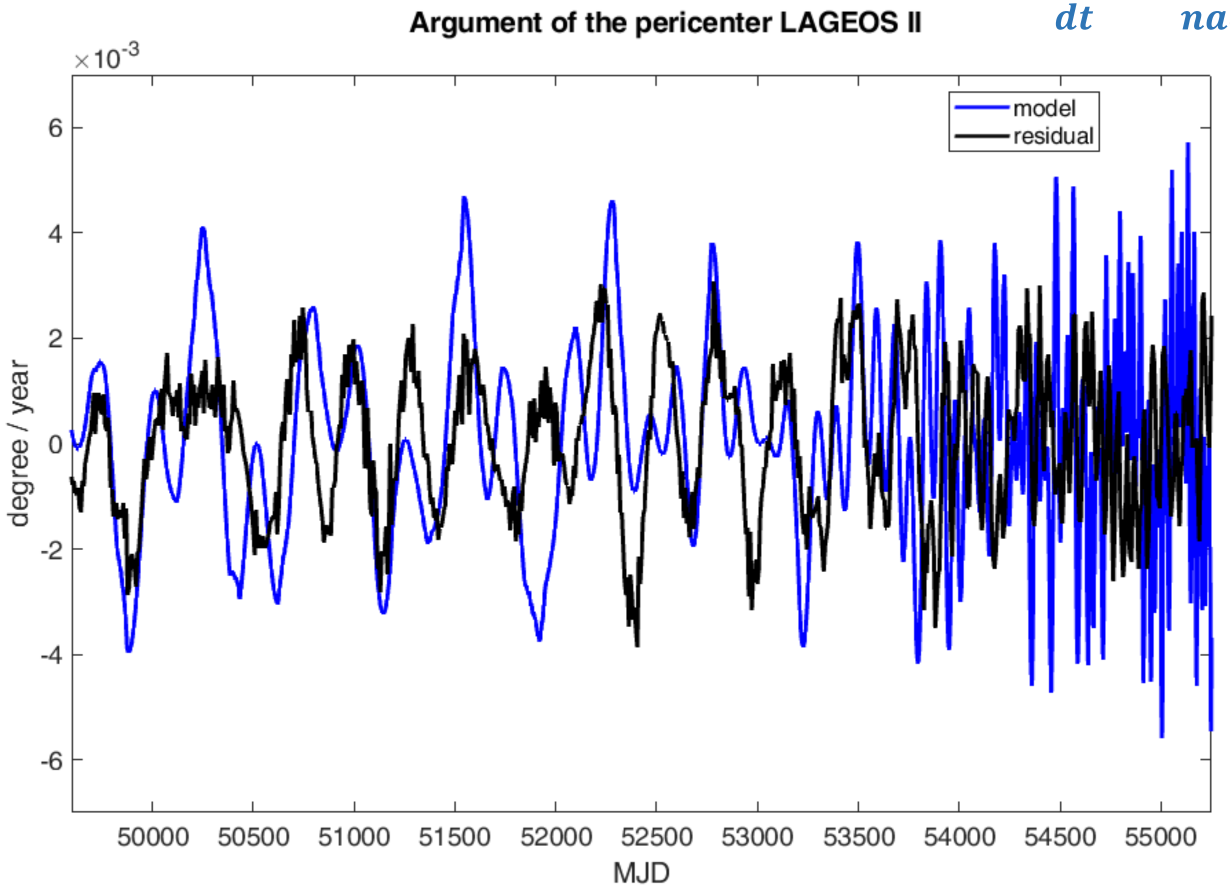
$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left[T + e(T\cos f + R\sin f)\right]$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left[R\sin f + T(\cos f + \cos u)\right]$$

About 27 years POD of LAGEOS II with GEODYN II

Preliminary results for the thermal thrust effects

$$\frac{d\omega}{dt} = \frac{\sqrt{1-e^2}}{nae}\left[-R\cos f + T\left(\sin f + \frac{\sin u}{\sqrt{1-e^2}}\right)\right] - \frac{W}{H\sin i} r\sin(\omega + f)\cos i$$



- Being able to clean up this parameter has a particular importance for us: it contains a secular effect from **General Relativity**, due to the **Gravitoelectric** field (M) and to the **Gravitomagnetic** field (J)

About 27 years POD of LAGEOS II with GEODYN II

Conclusions

- We developed a general model for the attitude (Spin evolution) of the satellites **LAGEOS**, **LAGEOS II** and **LARES**, based on the solutions of **Euler** equations
- The predictions of the model are in good agreement with the available observations
- A reliable model for the spin of these satellites represents an essential ingredient to model several **NGP**: thermal thrust effects and asymmetric reflectivity
- We presented the preliminary results for the thermal accelerations on **LAGEOS II** based on a new model, called **LATOS**
- These results are in good agreement with the orbital residuals
- Thermal accelerations determined from a reliable model may reduce the use of empirical accelerations in the satellites' **POD**, with possible improvements in
  - **Geophysical products**
  - **Fundamental physics measurements**

Many thanks for your kind attention

Backup slides

The LASSOS model

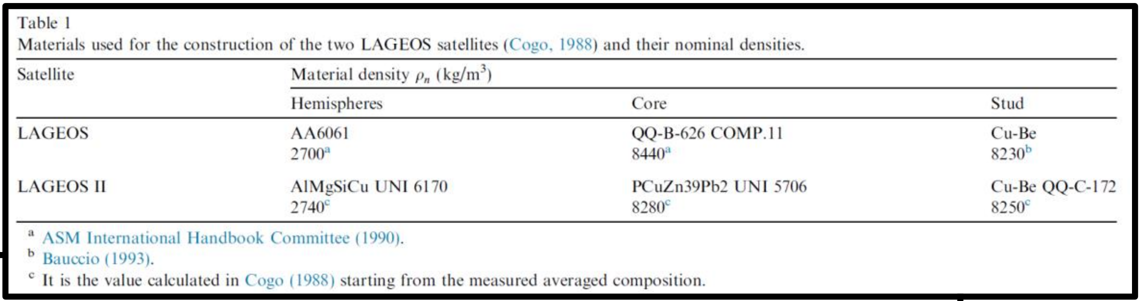


Table 1
Materials used for the construction of the two LAGEOS satellites (Cogo, 1988) and their nominal densities.

Satellite	Material density ρ_n (kg/m^3)		
	Hemispheres	Core	Stud
LAGEOS	AA6061 2700[a]	QQ-B-626 COMP.11 8440[a]	Cu-Be 8230[b]
LAGEOS II	AlMgSiCu UNI 6170 2740[c]	PCuZn39Pb2 UNI 5706 8280[c]	Cu-Be QQ-C-172 8250[c]

[a] ASM International Handbook Committee (1990).

[b] Baucco (1993).

[c] It is the value calculated in Cogo (1988) starting from the measured averaged composition.

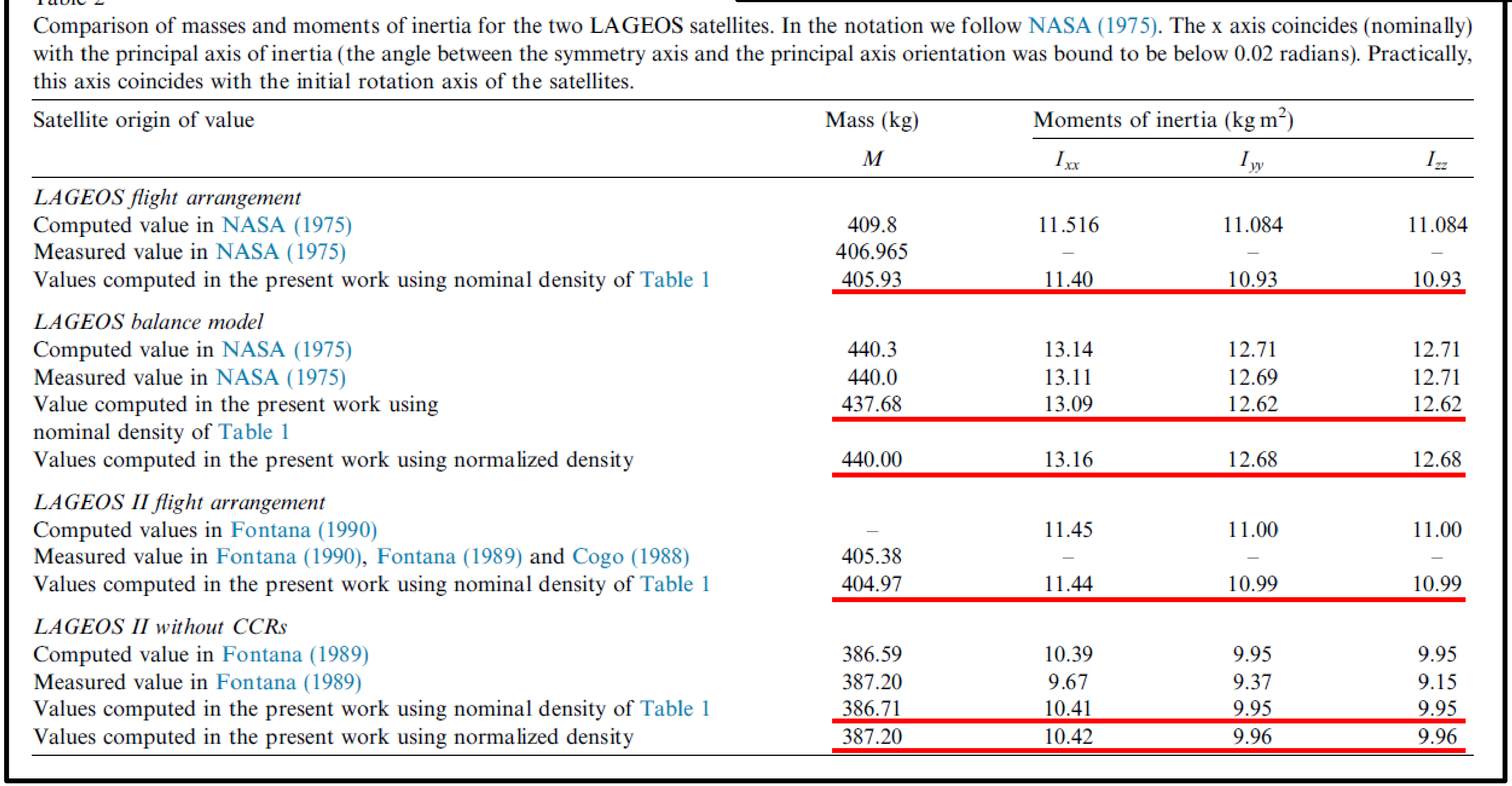
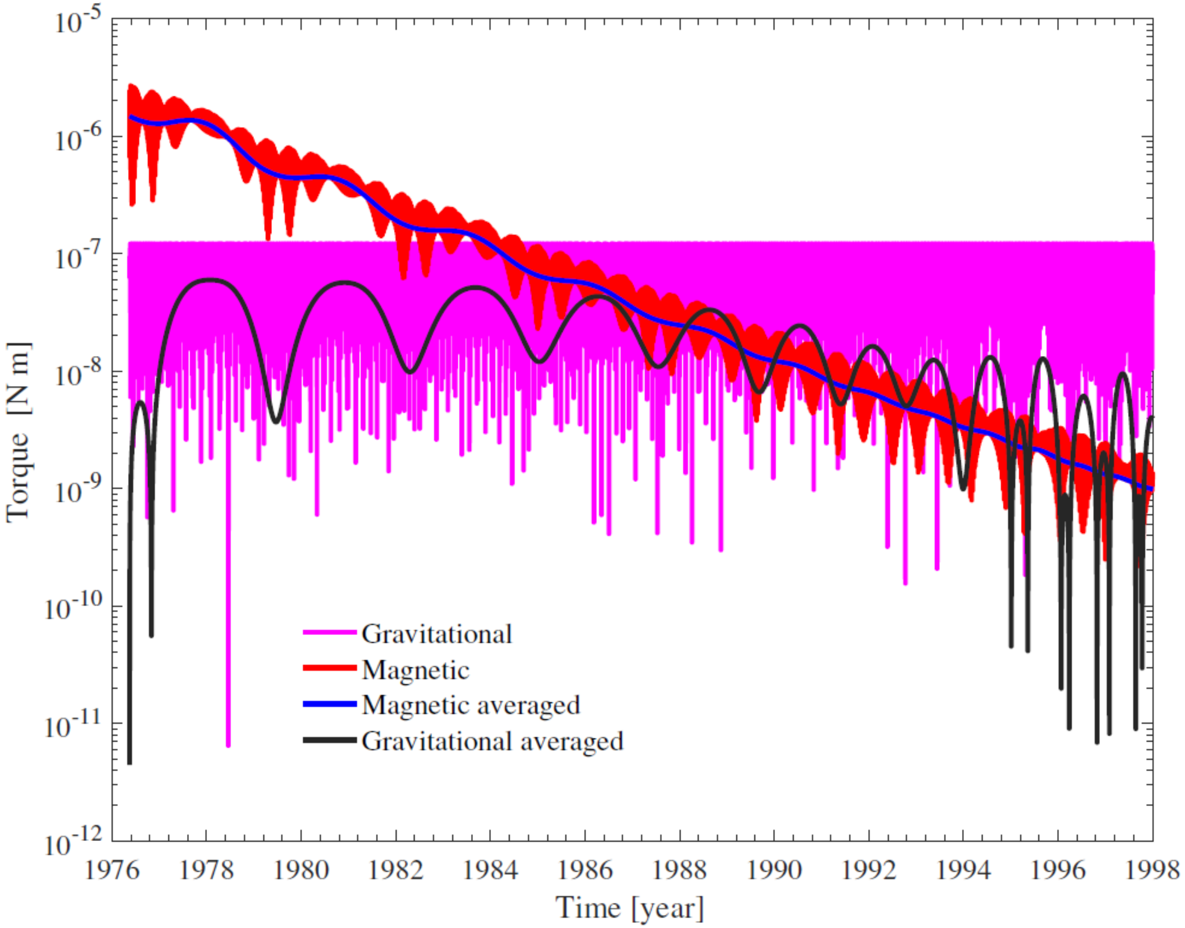


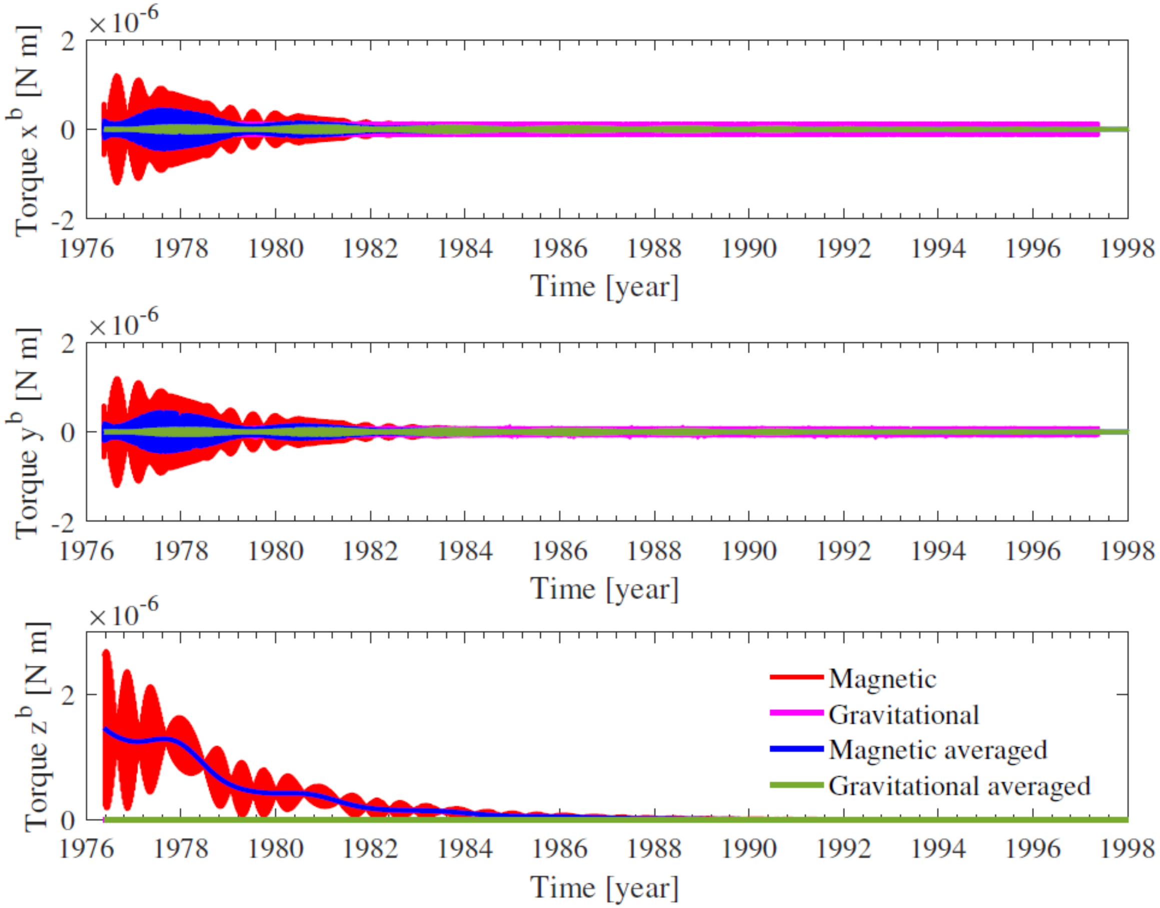
Table 2
Comparison of masses and moments of inertia for the two LAGEOS satellites. In the notation we follow NASA (1975). The x axis coincides (nominally) with the principal axis of inertia (the angle between the symmetry axis and the principal axis orientation was bound to be below 0.02 radians). Practically, this axis coincides with the initial rotation axis of the satellites.

Satellite origin of value	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement				
Computed value in NASA (1975)	409.8	11.516	11.084	11.084
Measured value in NASA (1975)	406.965	–	–	–
Values computed in the present work using nominal density of Table 1	405.93	11.40	10.93	10.93
LAGEOS balance model				
Computed value in NASA (1975)	440.3	13.14	12.71	12.71
Measured value in NASA (1975)	440.0	13.11	12.69	12.71
Value computed in the present work using nominal density of Table 1	437.68	13.09	12.62	12.62
Values computed in the present work using normalized density	440.00	13.16	12.68	12.68
LAGEOS II flight arrangement				
Computed values in Fontana (1990)	–	11.45	11.00	11.00
Measured value in Fontana (1990), Fontana (1989) and Cogo (1988)	405.38	–	–	–
Values computed in the present work using nominal density of Table 1	404.97	11.44	10.99	10.99
LAGEOS II without CCRs				
Computed value in Fontana (1989)	386.59	10.39	9.95	9.95
Measured value in Fontana (1989)	387.20	9.67	9.37	9.15
Values computed in the present work using nominal density of Table 1	386.71	10.41	9.95	9.95
Values computed in the present work using normalized density	387.20	10.42	9.96	9.96

Spin Model

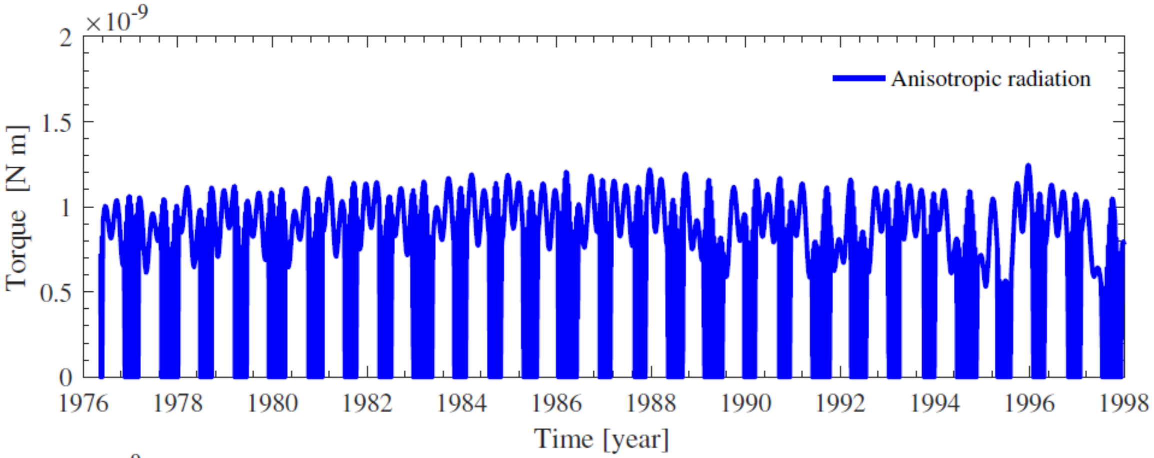
LAGEOS

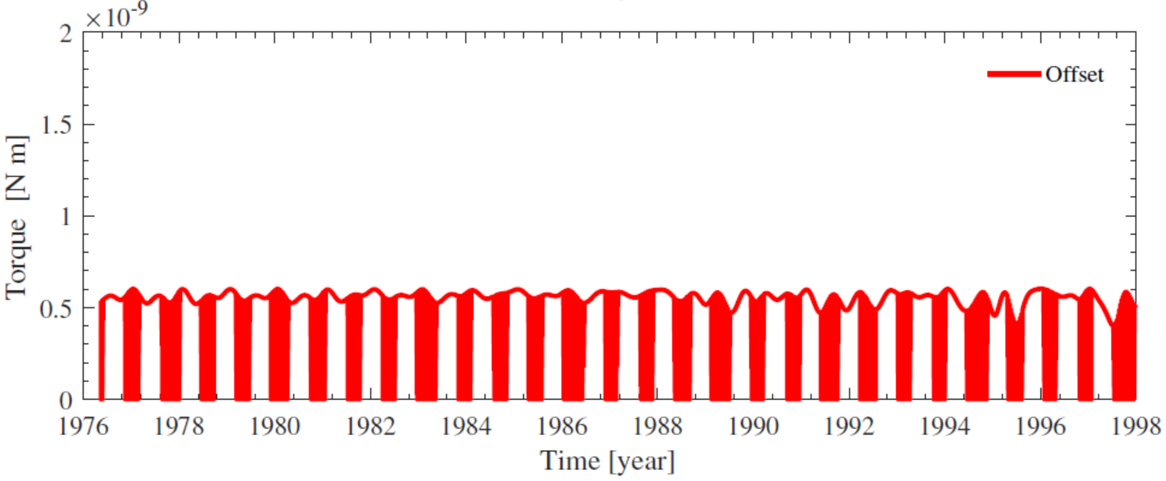


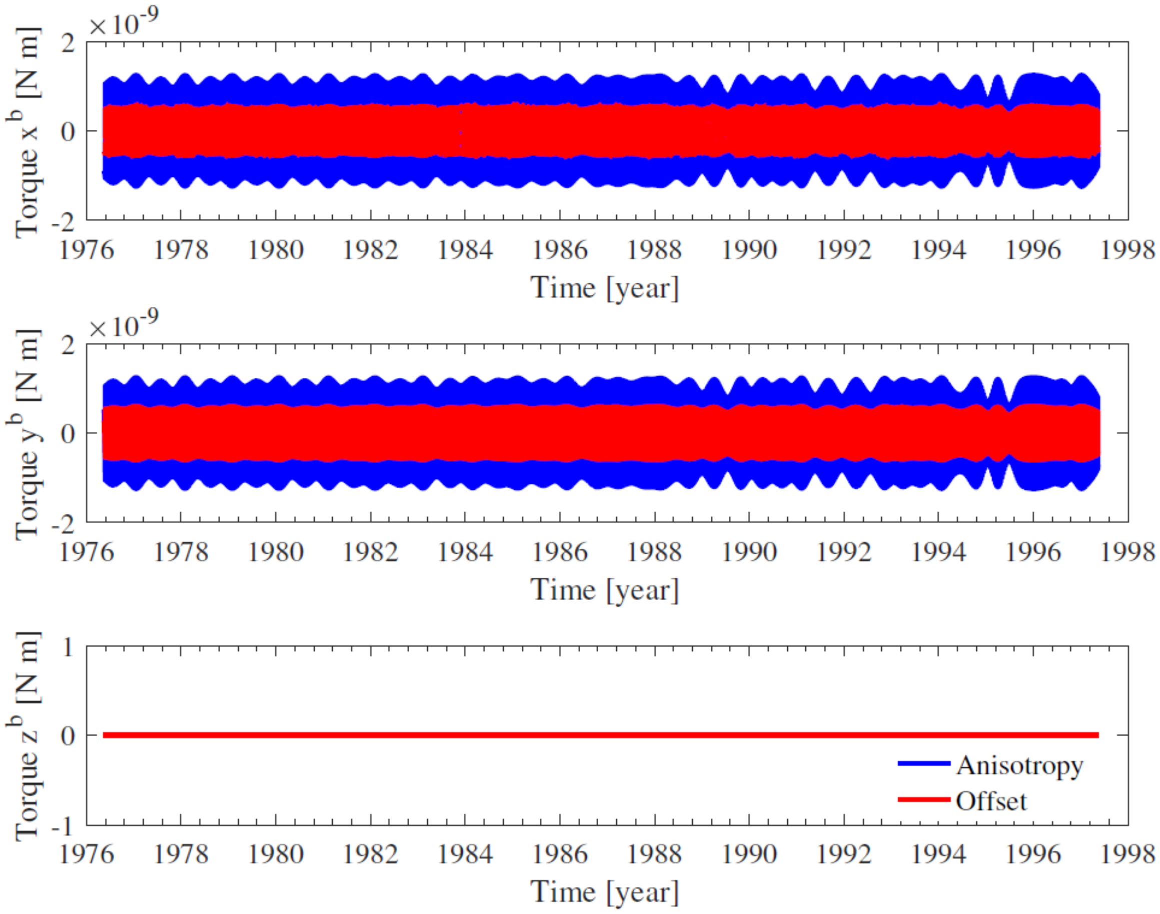


Spin Model

LAGEOS







Spin Model

LAGEOS/LAGEOS II

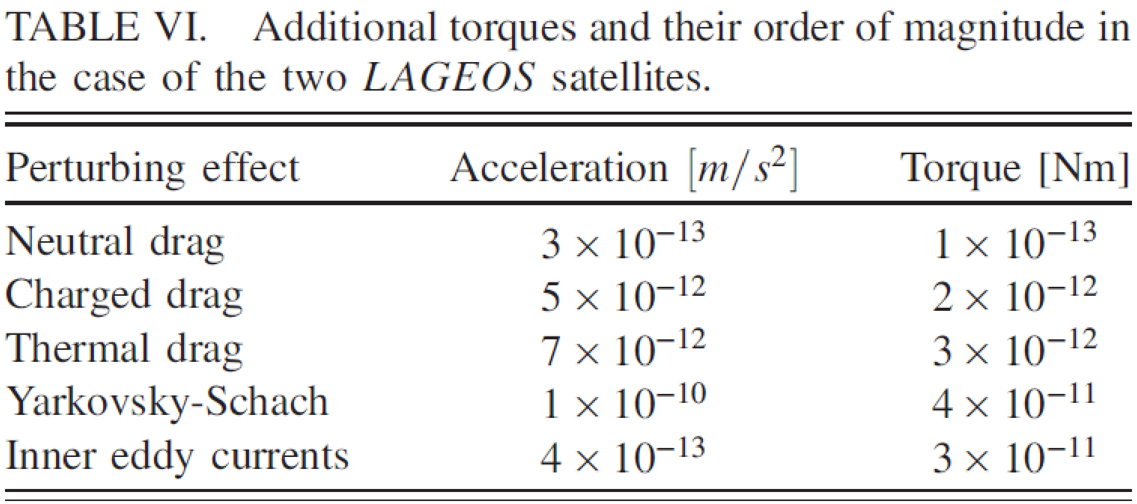


TABLE VI. Additional torques and their order of magnitude in the case of the two *LAGEOS* satellites.

Perturbing effect	Acceleration $[m/s^2]$	Torque [Nm]
Neutral drag	3×10^{-13}	1×10^{-13}
Charged drag	5×10^{-12}	2×10^{-12}
Thermal drag	7×10^{-12}	3×10^{-12}
Yarkovsky-Schach	1×10^{-10}	4×10^{-11}
Inner eddy currents	4×10^{-13}	3×10^{-11}

LARES

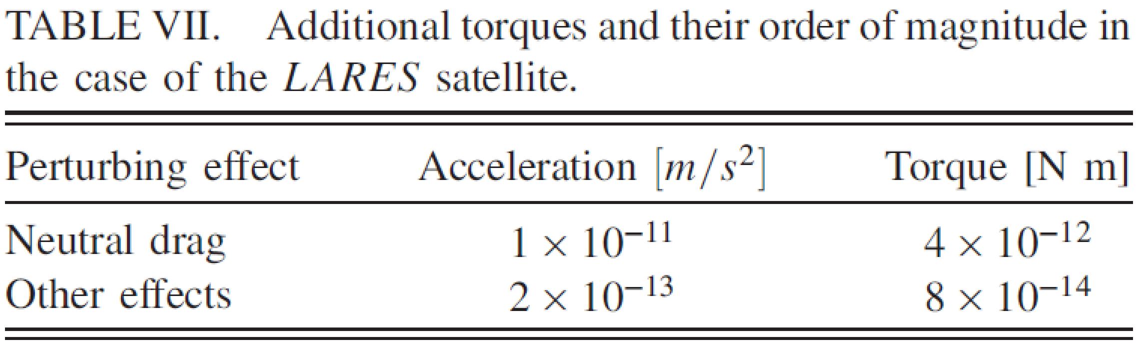
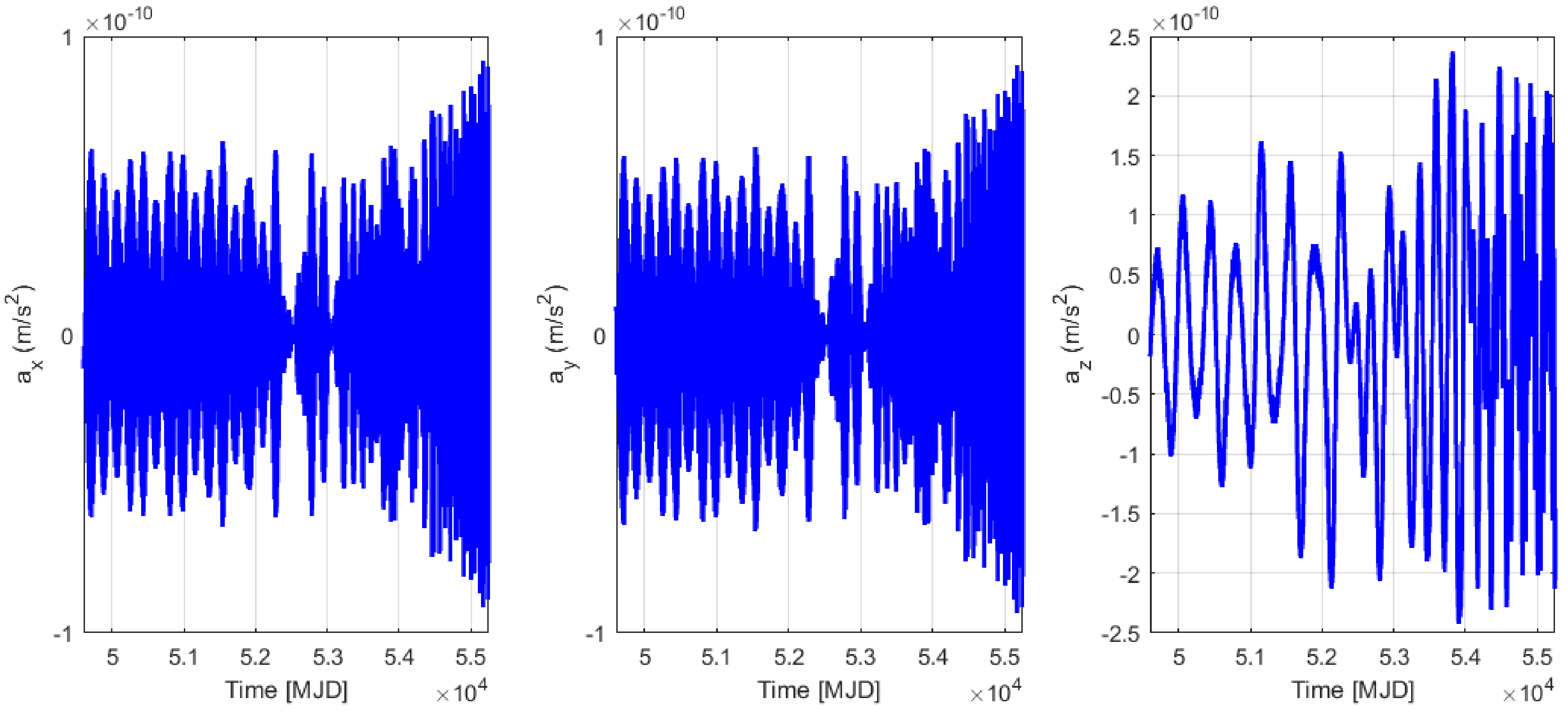


TABLE VII. Additional torques and their order of magnitude in the case of the *LARES* satellite.

Perturbing effect	Acceleration $[m/s^2]$	Torque [N m]
Neutral drag	1×10^{-11}	4×10^{-12}
Other effects	2×10^{-13}	8×10^{-14}

Thermal Model

LAGEOS II: accelerations in Gauss reference frame

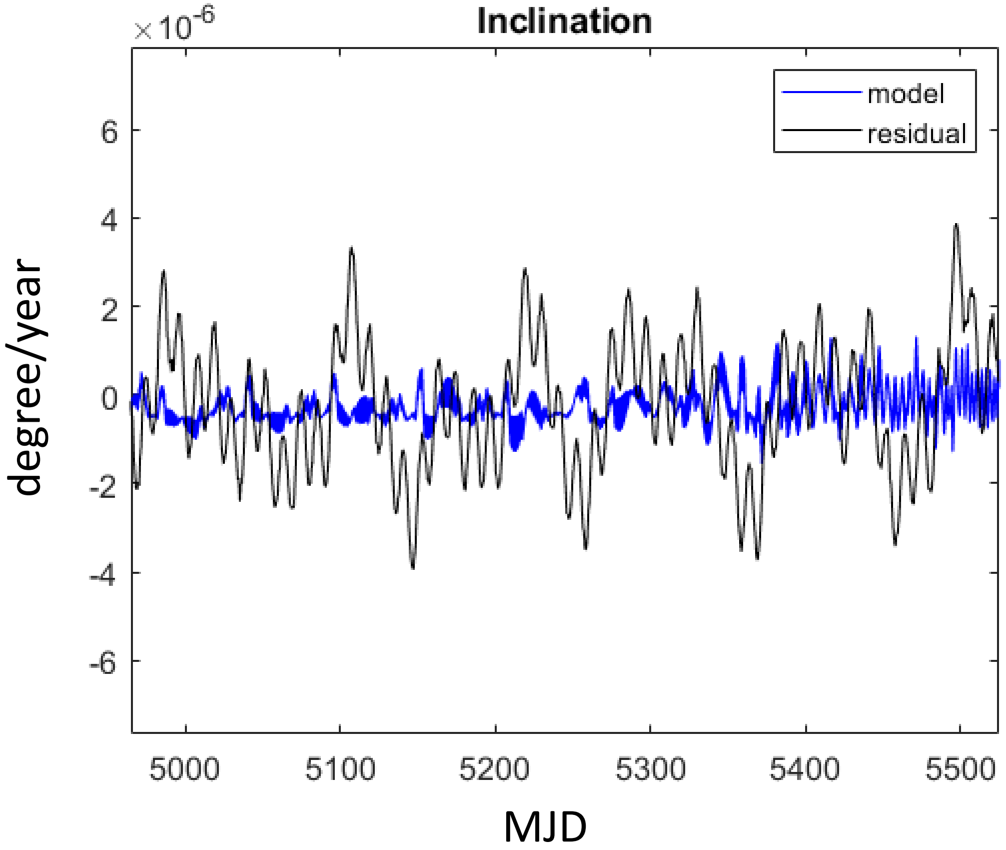


Radial **Transverse** **Out-of-plane**

Thermal Model

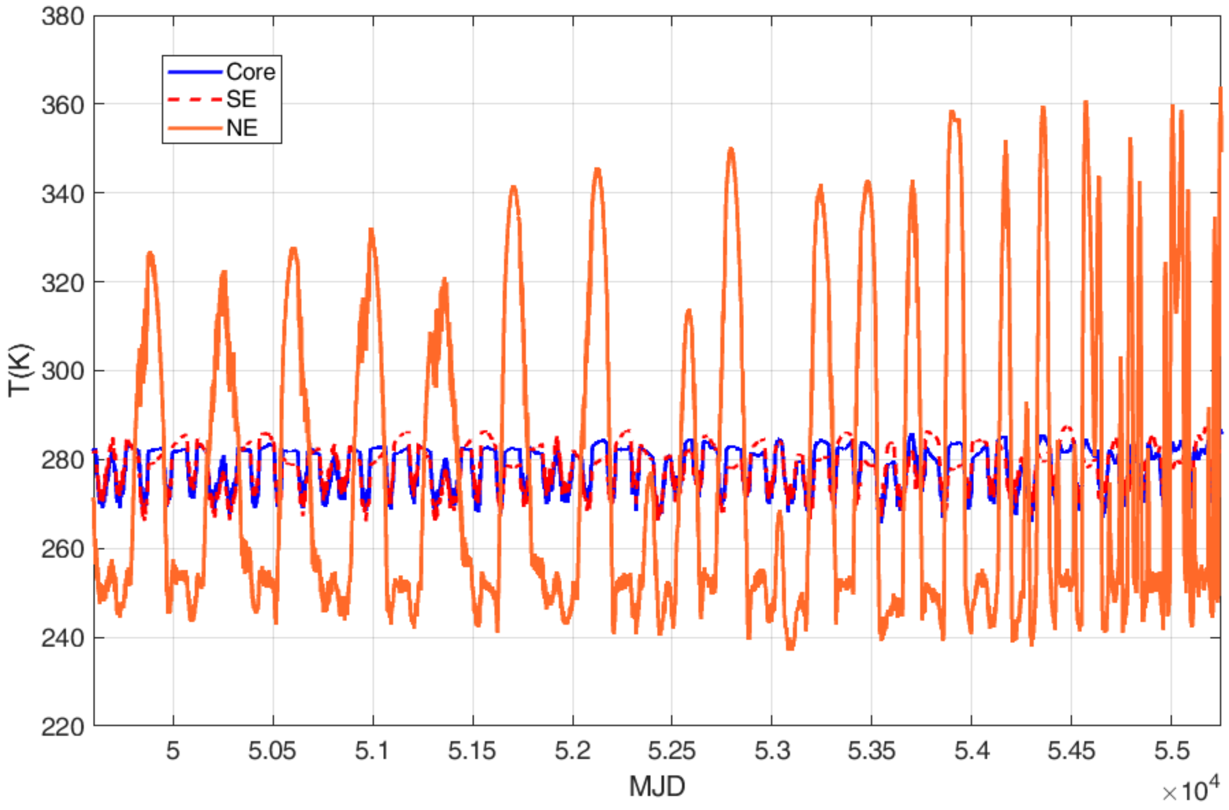
LAGEOS II: residuals vs thermal effects in the rate of the inclination

$$\frac{di}{dt} = \frac{W}{H} r \cos(\omega + f)$$



Thermal Model

LAGEOS II: Temperatures of core and of the hemispheres



Thermal Model

LAGEOS II: Temperature of CCR #1