Preliminary Estimate of the LARES Center of Mass Correction (CoM)

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Abstract

The center of mass correction of LARES is estimated for a laser ranging system working in single photoelectron mode assuming a Gaussian response of the system (distribution of the residuals for a flat target).

Method

The model used for estimating the satellite impulse response and the effective reflection plane are described in [1] and [2]. For this method the relative intensity of a cube corner as a function of the angle of inclination is needed. This function can be obtained theoretically regarding velocity aberration and diffraction [3]. Alternatively, it can be deduced empirically from the distribution of range residuals [4] using the empirical relation:

$$\text{Intensity} = a^{p} \cdot e$$

where "a" is the active aperture of the cube corner , "e" is the reflectivity and p is an empirical exponent. The active aperture and the reflectivity can be computed as a function of the angle of incidence and the azimuth using equations given in [6]. By some averaging process the theoretical distribution of range residuals can be computed and compared with the observations. Otsubo [4] got for LAGEOS p-values for best fit between 1.1 and 1.2. In the following we adopt p=1.2

For comparison we present data for p=2.0 also to give an idea how the results depend on this parameter.

The basic equation for the following is:

$$x(\alpha) = R \cdot \cos(\alpha) - L \cdot \sqrt{n^2 - \sin(\alpha)^2}$$

Where $x(\alpha)$ is the distance of the effective reflection plane of an individual cube corner from the satellite center and α is the angle of incidence.

For the index of refraction (n) the value describing the propagation of light pulses (group refractive index) is used in contrast to earlier work. For details on this question the reader is referred to [5].

The parameters of LARES are the following:

R = 178.5 mm	distance of the front faces from satellite center
L = 27.84 mm	vertex length of the cube corners
n_p = 1.4607	phase index of refraction @532 nm wavelength
n_{gr} = 1.4853	group index of refraction @532 nm wavelength

Results

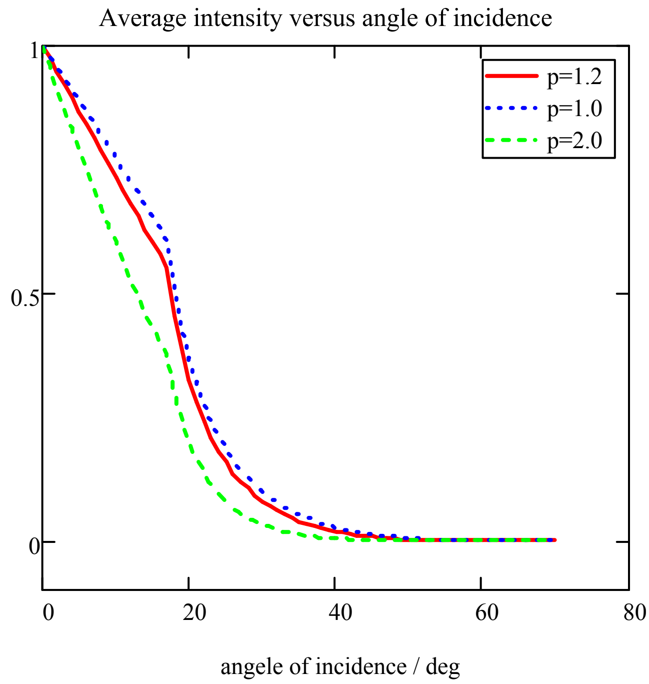

Fig.1: Average intensity of a cube corner versus angle of incidence. Averaging is done over all azimuth angles taking into account the loss of total internal reflection at some critical angle.
This curve can be converted into the impulse response of the whole satellite using equations given in Ref. [2] (s. Fig.2).

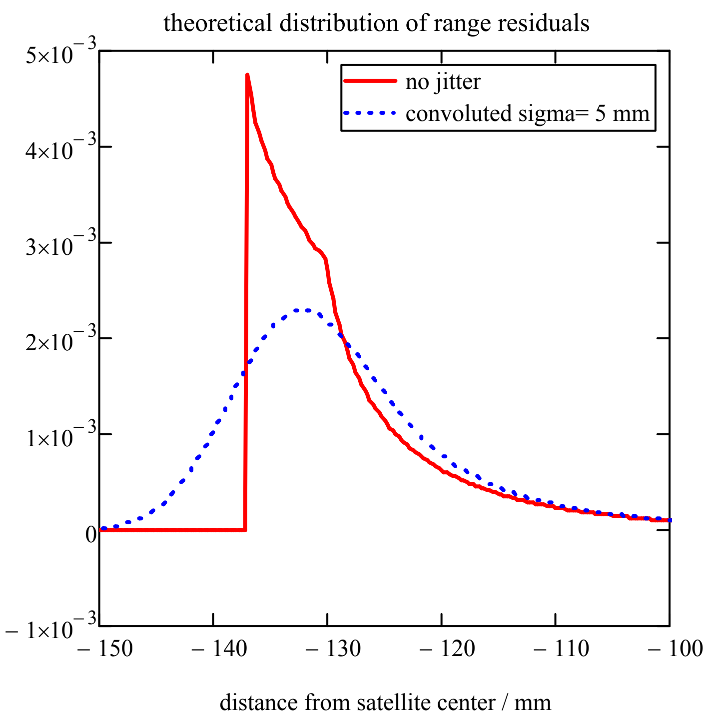

Fig.2: Theoretical distribution of range residuals ("impulse response") of LARES (p=1.2) for negligible jitter of the ranging system (red curve) and convoluted with a Gaussian (σ = 5 mm, blue dots)

The center of Mass correction is the 1st Moment of the distribution

CoM = 128.1 mm
(without any data clipping, independently from ranging noise)

It is common practice to form normal points by taking the average of the measurements within some time window. Therefore the effective range correction of the whole satellite is

equal to the 1st moment of the impulse response (red curve in Fig.2). This is true as long as the system response is symmetrical (e.g. Gaussian) and no data editing is applied. Unfortunately, the real system response of the ranging stations are not fully symmetrical and in addition some data editing (iterative clipping using some edit criteria) is applied to remove outliers. This makes the range correction system dependent.
We assume for the preliminary estimate a simple Gaussian system response, but even though the resulting distribution is skewed. Therefore the range correction depends on the edit criteria as well as from the width of the system response.

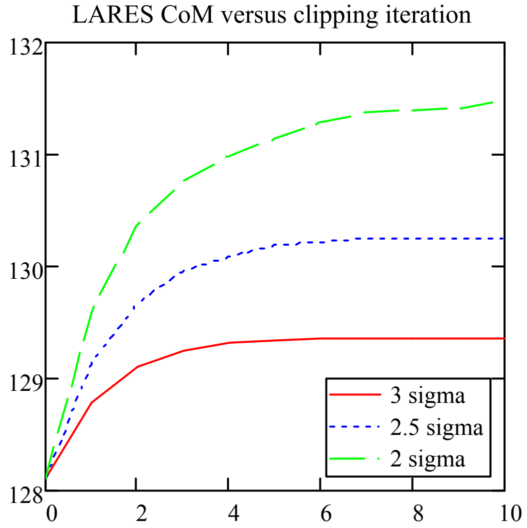

Fig.3: Center of Mass (mm) versus clipping iteration for different editing criteria.

Gaussian system response with 5 mm standard deviation

Table: LARES CoM for different system rms and 2.5-Sigma iterative editing

	Iterative clipping (10 iterations)			no clipping
System- σ/mm	1	5	10	
p = 1.2	131.3	130.3	129.3	128.1
p = 2.0	132.5	131.7	131.1	130.5

Conclusion:

Preliminary LARES-CoM from this study: (130 +/-2) mm
Leading edge: 137.1 mm

T.Otsubo estimated the LARES CoM incuding the effect of cube corner recession:
http://geo.science.hit-u.ac.jp/research-en/memo-en/lares-centre-of-mass-correction?set_language=en

this raises the value by about 1 mm.

To include leading edge detection a preliminary CoM correction for all systems of (133 ± 5) mm is proposed

References:

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