The Study on the Coefficients of Earth's Gravitational Field Using Scaled Sensitivity Matrix method

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Contents

Introduction

- The estimated monthly mean gravitational field parameters from one or two satellites only represent the linear combinations of a few primary spherical harmonic coefficients due to the limited sensitivity to the Earth's gravitational field.

- Using multiple satellites at various altitudes and inclinations can increase the sensitivity and mitigate the non-unique problem.

- The previous studies(Eanes,1995;Cheng,1989) are all according to the orbit node analysis to estimate the even zonal harmonics from SLR observations. Using this method, all the estimated linear combination coefficients are constants and only include the primarily zonal spherical harmonics.

Introduction

- These problems can restrict the overall understanding of the high correlation between the estimated parameters and other un-estimated parameters of the gravitational field, especially the study on the time-dependent correlation.

- We study the estimated monthly mean, uncorrelated gravitational field parameters($C_{20}, C_{30}, C_{21}, S_{21}$) from the Lageos1 and Lageos2 data for the 10 years period (2004-2013).

- Using the Scaled Sensitivity Matrix（SSM）method, we quantitatively investigate the contributions from the other 73 un-estimated gravitational coefficients for degrees and orders 2 through 8 to the 4 estimated gravitational parameters.

SSM Method(Dong et al.,2002)

- Assuming that the complete observation equation is

$$y = A_1X_1 + A_2X_2 + A_3X_3 + \varepsilon \qquad (1)$$

X_1: the estimated parameters,

X_3 : the considered parameters which cannot be resolved by the observations y due to high correlation.

X_2: the other parameters

The corresponding normal matrix is :

$$\begin{bmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \qquad (2)$$

- Because X_3 cannot be resolved directly, we actually solve the following normal equation

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (3)$$

SSM Method(Dong et al.,2002)

- According to the least squares solution :

$$\hat{X}_1 = X_1 + (N_{11} - N_{12}N_{22}{}^{-1}N_{21})^{-1}(N_{13} - N_{12}N_{22}{}^{-1}N_{23})X_3 \qquad (4)$$

The estimated values of X_1 are actually the linear combinations of X_1 and X_3 .

- The coefficients are the Scaled Sensitivity Matrix (SSM):

$$(N_{11} - N_{12}N_{22}{}^{-1}N_{21})^{-1}(N_{13} - N_{12}N_{22}{}^{-1}N_{23}) \qquad (5)$$

- In our study:

X_1 containt the four estimated parameters $C_{20}, C_{30}, C_{21}, S_{21}$

X_3 containt the ten satellites parameters and the other 73 gravitational field parameters for degrees and orders 2 through 8 .

X_2 is zero.

SLR measurement

Table1 The priori models, the constants and the reference frame used in the computations

The equatorial radius of Earth	6378137m
Flattening of earth	1/298.257222101
The reference system	ITRF2000
Precession and nutation	IERS2003 convention(McCarthy,2004)
Earth rotation parameter	IERS Bulletin C04
Gravitional field model	GGM001C
Ocean tide model	CSR3.0(Eanes,1995)
Solid tide model	IERS2003 convention(McCarthy,2004)
Planetary ephemeris	DE403/LE403
Center-of-mass offset	0.251m
Atmospheric refraction	Marini-Murray(1973)
Solar radiation pressure	Box-Wing
General relativistic corrections	IERS2003 convention（McCarthy,2004）

- We use SHORDE developed by SHAO to perform numerical integration of satellite orbits. The parameters included in each normal matrix :
- three satellite position and three velocity parameters,
- four along-track acceleration parameters solved for every 15 days,
- 77 monthly averaged normalized Stokes coefficients for degrees and orders 2 through 8.
- The monthly normal matrix include these 87 parameters

The recovery of the gravitational field parameters

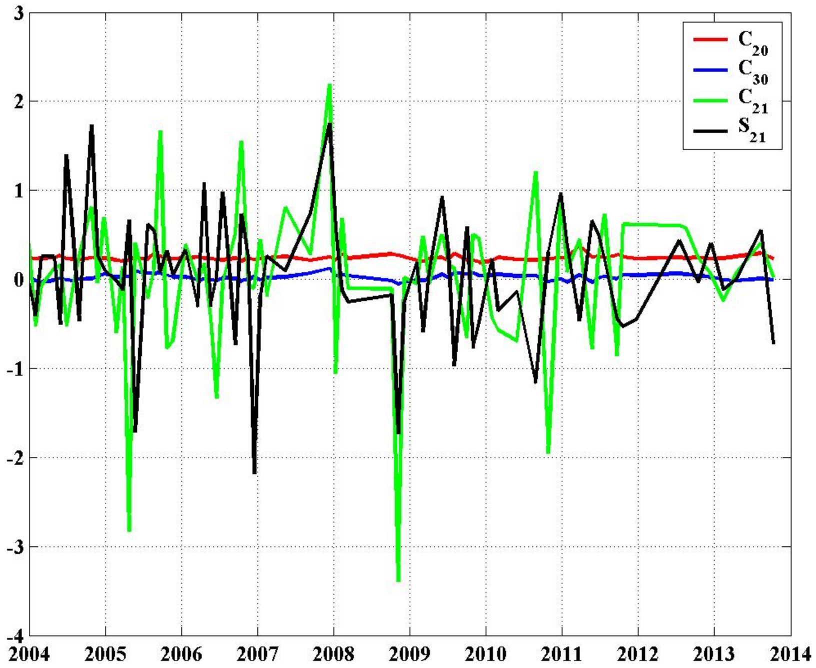

Fig1.The time series of the monthly-averaged values of the coefficients for the C_{40} term of the four parameters from two satellites during 2004-2013

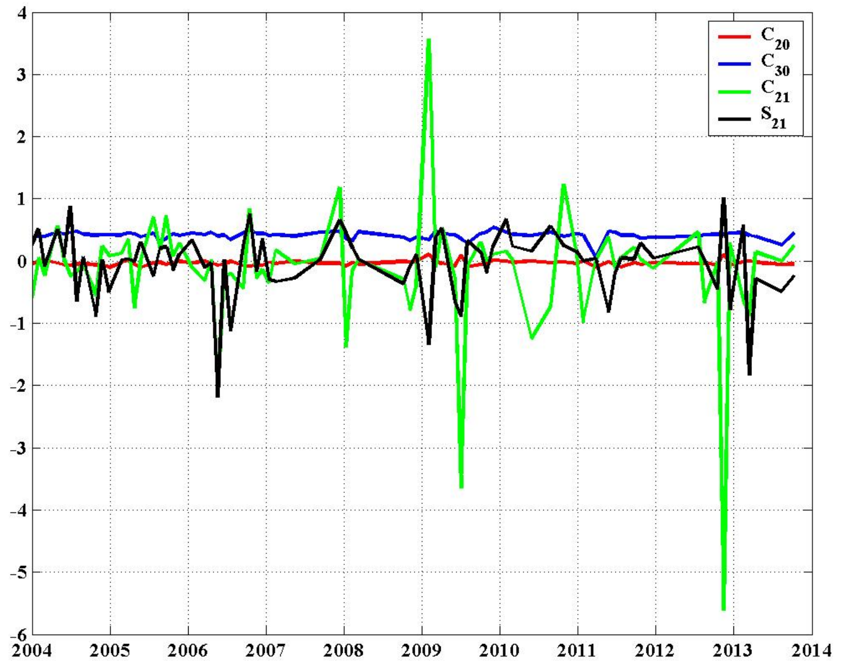

Fig2.The time series of the monthly-averaged values of the coefficients for the C_{50} term of the four parameters from two satellites during 2004-2013

The recovery of the gravitational field parameters

	C_{40}	C_{50}
C_{20}	0.2332 ± 0.0654	-0.0338 ± 0.0402
C_{30}	0.0212 ± 0.0352	0.4123 ± 0.0649
C_{21}	0.1712 ± 1.3234	-0.1536 ± 1.0152
S_{21}	0.1108 ± 0.9500	-0.0442 ± 0.5502

Table 2. Mean value of the monthly expansion coefficients for the C_{40} and C_{50} terms of $C_{20}, C_{30}, C_{21}, S_{21}$ from two satellites

Lageos1					
	C_{40}	C_{50}	C_{60}	C_{70}	C_{80}
C_{20}	0.4956 ± 0.006	0.0006 ± 0.0056	0.1271 ± 0.0023	0.0003 ± 0.0031	0.0105 ± 0.0005
C_{20} (Cheng,1989)	0.497		0.128		0.009
C_{20} (Eanes,1995)	0.496		0.1290		0.0184
C_{30}	-0.0020 ± 0.0102	-0.3336 ± 0.0039	-0.0019 ± 0.0098	-0.3195 ± 0.0023	-0.0010 ± 0.0054
C_{21}	-0.0145 ± 0.4713	-0.0019 ± 0.1839	-0.0042 ± 0.1753	0.0061 ± 0.1314	-0.0008 ± 0.0413
S_{21}	0.0012 ± 0.5642	0.0106 ± 0.2104	-0.0016 ± 0.1985	0.0106 ± 0.1513	0.0003 ± 0.0454
Lageos2					
C_{20}	0.0966 ± 0.0018	0.0014 ± 0.0095	-0.1052 ± 0.0004	0.0005 ± 0.0036	-0.0265 ± 0.0002
C_{30}	-0.0062 ± 0.0458	0.6715 ± 0.0028	-0.0021 ± 0.0139	-0.0354 ± 0.0005	0.0011 ± 0.0078
C_{21}	0.0214 ± 0.2705	0.0251 ± 0.1561	-0.0028 ± 0.0676	0.0011 ± 0.0170	-0.0025 ± 0.0236
S_{21}	-0.0184 ± 0.2959	-0.0085 ± 0.1647	-0.0033 ± 0.0730	0.0005 ± 0.0162	0.0023 ± 0.0266

Table 3. Mean value of the monthly expansion coefficients for the $C_{40}, C_{50}, C_{60}, C_{70}, C_{80}$ terms of $C_{20}, C_{30}, C_{21}, S_{21}$ from Lageos1 and Lageos2,respcetively

Summary

- Our results confirm the reasonableness of the previous zonal combination coefficients from the orbital node analysis.

- The dominant contributor to C_{20} is seen to be C_{40} that around 23.32% of C_{40} is absorbed by C_{20} and the dominant contributor to C_{30} is seen to be C_{50} that around 41.23% of C_{50} is absorbed by C_{30}

- For the first time, we obtain the combination coefficients of the estimated C_{21}, S_{21} . These results reveal time-dependent correlations between the estimated non-zonal parameters and the un-estimated gravitational coefficients.

Summary

- SSM analysis can provide the quantitative assessment of the influence on the recovered parameters from the highly correlative parameters which can not be estimated directly. This means SSM can provide important insight into the nature of the obtained solutions.

- In practice, the application of SSM is not restricted to the gravitational field. The problem of the estimated parameters contaminated by the other un-estimated parameters due to the high correlation exists widely among various disciplines , such as the high correlation of the sidebands with the main bands in Earth orientation variation.

Thank you for your attention