Modification of Laser Ranging Equation

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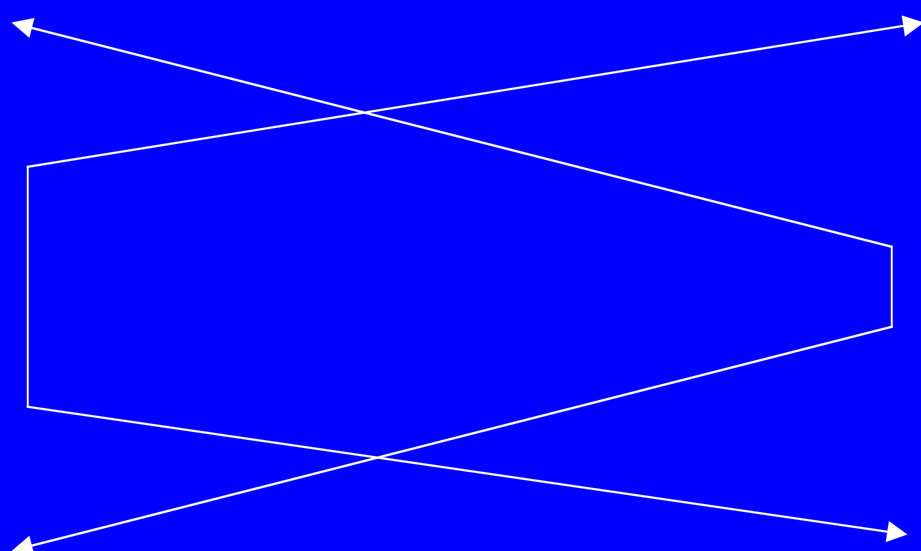
1. Classical Laser Ranging Equation

Returned photoelectron numbers *N* for one laser pulse transmission

$$\boxed{N = \frac{16 E N_0 A_m A_r T_a^2 T_t T_r \eta \alpha}{\pi^2 R^4 \theta_e^2 \theta_m^2}}$$



Considered:

- T_a atmospheric transmission, 0.5, amplitude attenuate

Unconsidered:

- 1. Atmospheric turbulence effects on laser beam propagation.
- 2. The distribution of the laser beam

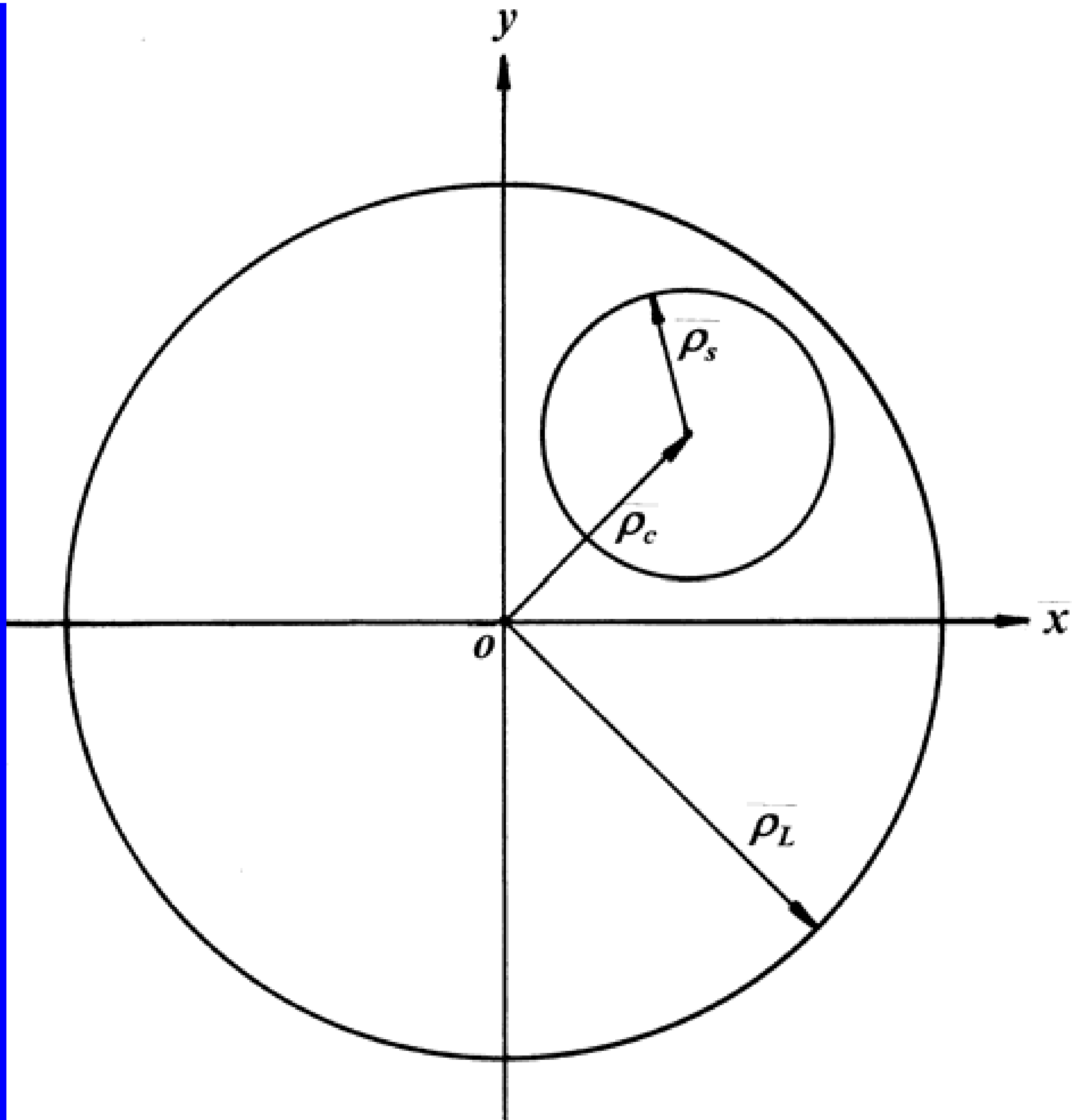
For Kunming station 1.2m laser ranging system on LLR:
$N = 0.17$ sub-single photon detection.

2. Atmospheric Turbulence Effects on Laser Beam Propagation

Random time delay, pulse spread, (<1 ps), negligible

Scintillation, variance of intensity fluctuation ≤ 0.02, now may be negligible

Beam wander and beam spread, focusing on the short-term beam wander



Laser beam at far-field

Short-term beam wander:

$$\langle \rho_C^2 \rangle = \frac{10.22 Z^2}{k^2 r_0^{\frac{5}{3}} D^{\frac{1}{3}}}$$

Short-term beam spread:

$$\langle \rho_S^2 \rangle = \frac{4Z^2}{k^2 D^2} + \frac{D^2}{4}\left(1 - \frac{Z}{F}\right)^2 + \frac{17.6 Z^2}{k^2 r_0^2}\left[1 - 0.48\left(\frac{r_0}{D}\right)^{\frac{1}{3}}\right]^{\frac{6}{5}}$$

Long-term beam spreading:

$$\langle \rho_L^2 \rangle = \frac{4Z^2}{k^2 D^2} + \frac{D^2}{4}\left(1 - \frac{Z}{F}\right)^2 + \frac{17.6 Z^2}{k^2 r_0^2}$$

- Here,
  k wave number, D laser transmitter diameter
  Z laser propagation axis and coordinate
  F radius of curvature of laser beam
  r_o Fried's coherence length, 5 ~ 20 cm

- Method:
  Maxwell wave equation → *Markov* approximation → the second moment and the four moment (approximation) of the field → mean square value of above terms

Changing ρ_C , ρ_S , and ρ_L to their correspond angle θ_C , θ_S , and θ_L

Angle deviation of laser beam at different r_o

	r_o=5cm	r_o=10cm	r_o=15cm
θ_L	2.″93	1.″48	0.″98
θ_S	2.″63	1.″27	0.″83
θ_C	1.″32	0.″74	0.″53

3. Atmospheric Turbulence Effects on Laser Ranging

3.1 Laser ranging accuracy

Consideration a random path deviation caused by the refractive index fluctuation for a round trip laser ranging, the accuracy of the laser ranging *ΔL* is:

$$\langle \Delta L^2 \rangle = \frac{3.127 C_n^2(0) L_0^{\frac{5}{3}} h_T}{SinE}$$

Here: C_n^2 turbulence structure parameter

L_o turbulence outer scale, 100m

E target elevation angle

h_T atmospheric scale height, 11km

Laser ranging accuracy at different turbulence

ΔL(mm)	$E=10^0$	$E=30^0$	$E=60^0$
$C_n^2 \sim 10^{-13}\, m^{-2/3}$	10.33	6.09	4.63
$C_n^2 \sim 10^{-15}\, m^{-2/3}$	0.83	0.45	0.37
$C_n^2 \sim 10^{-17}\, m^{-2/3}$	0.17	0.10	0.08

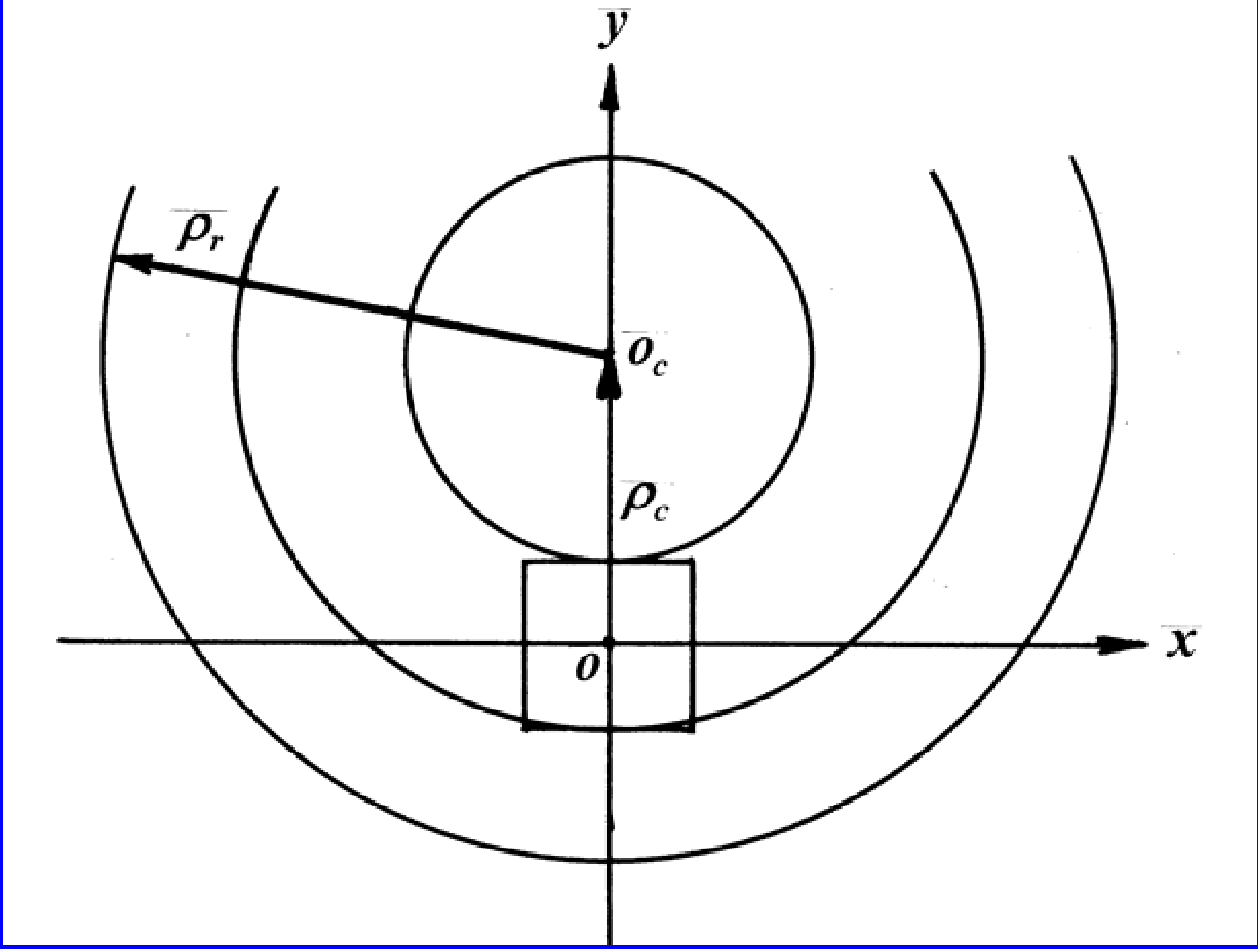
3.2 Returned laser photons

Need to be considered:

1. Short-term laser beam wander caused by the atmospheric turbulence
2. Gaussian distribution of the laser beam along radial:

$$\boxed{E(\rho) = \frac{E_0}{\pi \rho_e^2} \exp\left(- \frac{\rho^2}{\rho_e^2} \right)}$$

Calculation returned laser photons



Returned laser photoelectrons N_r
on the ground receiver for one laser pulse firing:

$$\boxed{N_r = \frac{4EN_0 A_m A_r T_a^2 T_t T_r \eta\alpha}{\pi^2\left(\theta_e^2+\theta_s^2\right)\theta_m^2 R^4}\exp\left(-\frac{\rho_C^2}{\rho_e^2+\rho_s^2}\right)}$$

<u>New form of Laser Ranging Equation</u>
<u>not unique, depend on how many turbulence terms to be concerned</u>

here: ρ_e laser beam radius at target, determined by laser divergence

ρ_c short-term beam wander

ρ_s short-term beam spread

- If tilt is removed, the correction factor for the laser ranging is:

$$\boxed{\frac{N_r}{N} = \frac{\theta_e^2}{4\left(\theta_e^2 + \theta_s^2\right)} \exp\left(-\frac{\theta_C^2}{\theta_e^2 + \theta_s^2}\right)}$$

- 1/40 ~ 1/6, depend on the turbulence

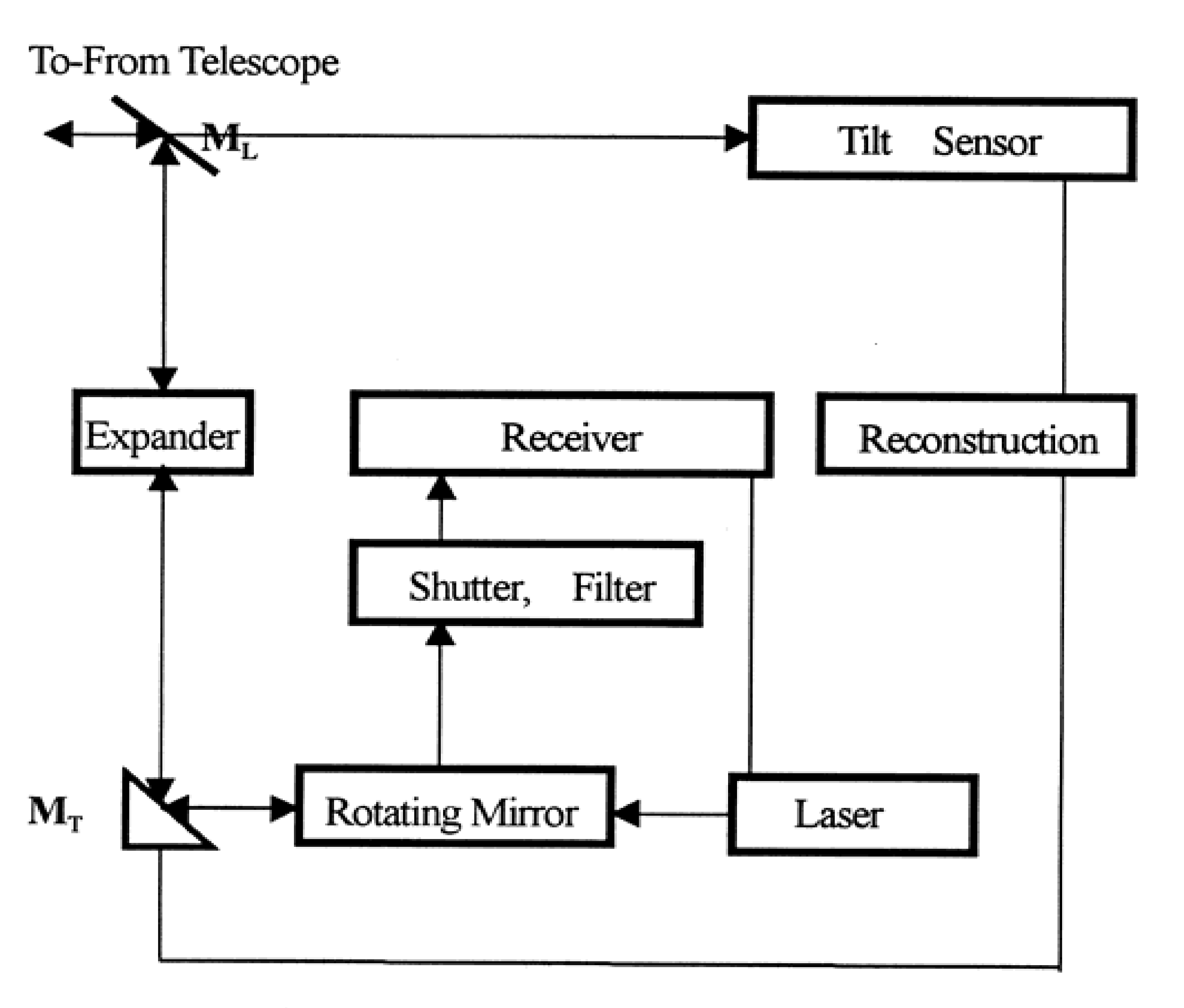
For Kunming station 1.2m laser ranging system:

N_r=0.17×(1/40 ~ 1/6)

More less than one photoelectrons!

4. Further Thoughts

- Real-time tip-tilt compensation for the laser beam wander on the LLR, low-order compensation
- Atmospheric tilt comes from the moon surface, the extended light source, using absolute differences algorithm to calculation the tilt.
- For all-order compensation, more complicated techniques are needed.



Optical Scheme of Kunming 1.2m LR System for Tilt Correction

Thanks