FIVE-TARGET SYSTEM CALIBRATION

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"The man who has one ground target KNOWS what his system delay is. The man who has two is never quite sure."

ABSTRACT
Stromlo SLR systems have five terrestrial calibration targets which are used in the MINICO method for verifying the assumptions made in calibrating the system delay. Four of the targets are mounted externally on pillars surrounding the System Reference Point (SRP) which is the telescope's intersection of axes. The fifth, which can act as a real-time internal target during satellite laser ranging, is the "Spider Retro" mounted on one of the vanes holding the secondary mirror. The full MINICO method involves ranging to all of these targets and estimating the horizontal coordinates (East & North) of the SRP, the distance between SRP and the Spider Retro, and the current system delay.

From data kindly supplied by Geoscience Australia, the stability of the solutions over the 3½ year period from July 1999 to January 2003 will be presented, and application of the method in the new Stromlo-III system will be discussed.

Figure 1: View of the EOS Space Research Centre on Mount Stromlo, taken from the North-East calibration pillar. The 1.0-metre SLR telescope is inside the Typhoon-III dome atop the main building. The large Icestorm dome houses a prototype 1.8-metre research telescope. The slender tower to the right contains a Differential Image Motion Monitor. The Fiducial Monument is to the left, with a GPS/GLONASS antenna on it.

INTRODUCTION
The EOS Space Research Centre including the new Stromlo-III SLR system is shown in Figure 1. One of the calibration targets can be seen in Figure 2. Their layout is depicted in Figure 3, and their positions derived from a comprehensive collocation survey performed in

December 2003 by Geoscience Australia *(Dawson et.al, 2004*) are summarized in Table 1. The pillars on which the ground targets are mounted are the same as those used for Stromlo-I

Figure 2: View from the DIMM Tower over the SLR building's roof to the North-East calibration pillar, to the left of the burnt-out Oddie building (centre).

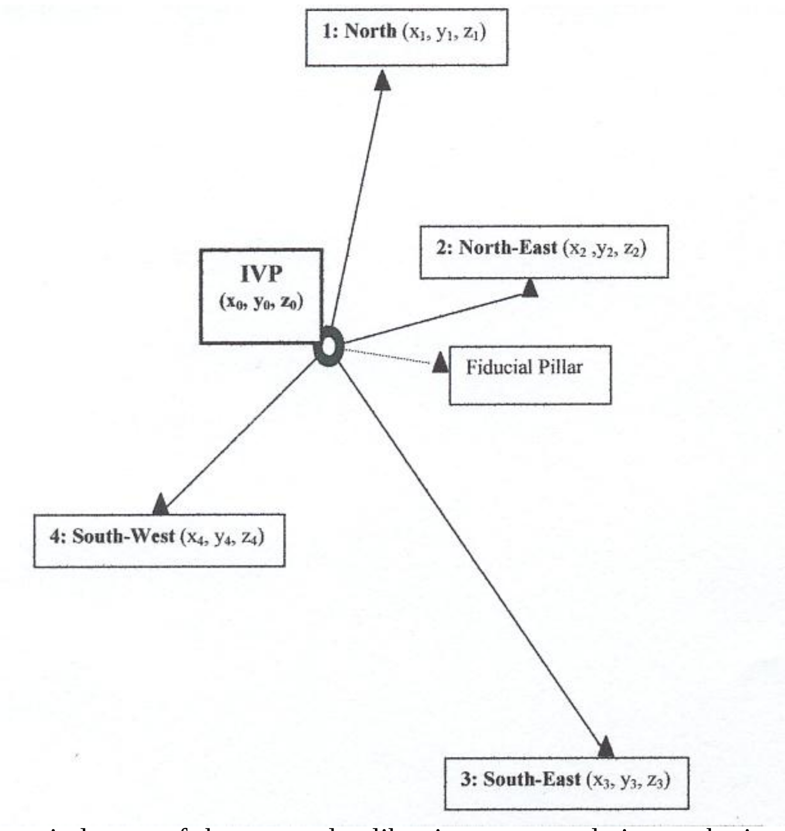


Figure 3: Planimetric layout of the external calibration targets relative to the intersection of axes of the SLR 1.0-metre telescope.

from 1998 to 2003 - they survived the firestorm. Of particular interest is the arrangement on top of the North pillar, shown in Figure 4, which holds the IGS GPS antenna as well as the target routinely used for pre-/post-calibration.

Table 1: Coordinates of the calibration targets relative to the System Reference Point (SRP). They refer to the targets' optical zeros (Effective Reflection Points).

Target	**East** (m)	**North** (m)	**Up** (m)	**Range** (m)	**Azimuth** (deg)	**Elevat'n** (deg)	**Notes**
North	15.6582	67.6123	-5.1257	69.5908	13.019	-4.225	Official survey results, 28/5/04
North-East	50.6242	19.8455	2.2953	54.4235	68.574	2.416	
South-East	74.2873	-90.6180	-3.3310	117.2234	140.637	-1.629	
South-West	-42.4571	-32.0138	-10.3644	54.1748	232.965	-11.029	
Spider				1.4562			1[st]guess

Figure 4: Example of very close collocation, between calibration target retroreflector, GPS antenna and survey monument plate on top of the North pillar.

A feature of laser ranging from Canberra stations since 1987 has been the use of internal calibrations from "Spider Retros", viz. retroreflectors mounted on the secondary mirrors' spider vanes *(Luck, 1992; Degnan, 1985; Luck & Johnston, 2000*). Given the right timing unit such as epoch timers and congenial detector gating characteristics, they can be performed simultaneously with satellite range measurements, hence constitute "real-time internal calibrations".

Using 3 or more ground targets to position the telescope is essentially a resection process, and was proposed in (*Greene, 1986*). The Keystone Network in Japan established a ground network of four calibration targets surrounding the SRP at each station, more-or-less equally spaced in azimuth (*Katsuo et al, 1999*). This concept was adopted at Stromlo, augmented by the Spider Retro.

The North Pier target is the standard used for routine pre/post system calibration during normal operations. The Spider Retro is an alternate standard for real-time system calibration. The primary purpose of weekly MINICO measurements to all 4 or 5 targets is to verify that the ranges to the North Pier and Spider Retro targets are correct, and hence that the inferred values of system delay are accurate. The solutions also reveal whether the horizontal coordinates of the SRP are constant with respect to the adopted coordinates of the targets, which can yield useful information about the system and its immediate environment. Finally, use of four targets provides the priceless commodity of redundancy in the determination of the system delay.

ESTIMATION OF SYSTEM DELAY

With 5 calibration targets available, there are several ways of estimating, applying or even eliminating the system delay, which represents all the optical and electronic delays on the laser/receiver side of the SRP. The system delay may legitimately be changed at any time (although preferably not while ranging!) and may fluctuate during ranging, so it must be calibrated against universal constants (or at least against local constants), at or very close to the time of ranging. These constants are, in practice, the surveyed distances from the SRP to the calibration targets. But they could themselves vary, so they need to be monitored.

Notation

Let: (x_0, y_0, z_0) be the local (East, North, Up) coordinates of the SRP;

(x_i, y_i, z_i), $i = 1,..,4$ be the local coordinates of the four ground targets;

r_i, $i = 1,..,4$ be the distances from SRP to the four ground targets;

g be the distance from SRP to Spider Retro optical zero projected parallel to the optical axis of the telescope;

T_i, $i = 1,..,4$ be the raw measurements of the ranges to the four ground targets;

Q_i, $i = 1,..,4$ be the system delay while ranging to the four ground targets;

S_i, $i = 1,..,4$ be the raw range measurements to Spider Retro during ranging to the four ground targets;

δT_i, $i = 1,..,4$ be known corrections to the ranges, such as atmospheric corrections and the additional delay when ranging through the dome's glass window;

δS_i, $i = 1,..,4$ be known corrections to the Spider Retro measurements, such as the additional delay through the prism and through any ND filters;

c be the speed of light in vacuum;

D be the reduced two-way range to a satellite, and $d = (c/2)D$.

The System Delay in Satellite Ranging

The raw measurement T to a satellite is given by:

$$T = D + Q + \delta T + \varepsilon \tag{1}$$

where δT is the sum of the known corrections such as extra delay through the dome window glass, and ε is random measurement error (which will be suppressed hereinafter). The system delay is Q. The reduced measurement D used to construct normal points is thus:

$$D = T - \delta T - Q \tag{2}$$

The problem is to determine Q (equivalently $q = (c/2)Q$) as near to the time of the satellite range measurement as possible.

General Solution for Five-Target Calibration Ranging

The full set of observation equations during ground target ranging including internal target is:

$$\begin{aligned} T_i - \delta T_i &= (2/c) r_i + Q_i, \quad i = 1,..,4 \\ &= (2/c)\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} + Q_i \\ S_i - \delta S_i &= (2/c) g + Q_i, \quad i = 1,..,4 \end{aligned} \tag{3}$$

where it is assumed that each observation is subject to zero-mean random measurement errors. It is customary to assume that the system delays are the same during one MINICO session, i.e. $Q_i = Q$ for all *i* - any variations will be included in the measurement random errors. If we convert to linear measurements where necessary by putting:

$$t_i = (c/2)[T_i - \delta T_i], \quad s_i = (c/2)[S_i - \delta S_i], \quad q_i = (c/2) Q_i = q, \quad i = 1,..,4$$

then the observation equations become simply:

$$t_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} + q \tag{4}$$

$$s_i = g + q, \quad i = 1,..,4 . \tag{5}$$

They can then be solved by least squares for the 5 parameters x_0, y_0, z_0, g, q . (They can also be solved if there are only 3 ground targets.) In practice, the elevation angle range is completely inadequate for determining the height z_0 so its survey value is adopted. Hence there are 8 observations in 4 unknowns, and a solution is possible. In particular, the solution includes both the Spider Retro distance *g* and the system delay *q*. The Keystone systems (*Katsuo et al, 1999*) used upwards-facing retroreflectors at ground level to enable height determination as well.

SRP and Spider Retro Stability

For geodetic monitoring of the stability of the SRP, as opposed to measuring the system delay, it is convenient to eliminate the system delay by subtracting equation (5) from equation (4). Thus:

$$\Delta t_i \equiv t_i - s_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} - g \tag{6}$$

leaving 4 observations in the 3 unknowns $x_0, y_{0,} g$ which include a good determination of the Spider Retro distance g, which can be a beast to measure directly by survey or engineering methods. But it does not yield a value for the system delay *q*.

Single Ground Target Pre/Post System Delay Calibration

Normal pre/post system calibration in the absence of Spider Retro is routinely accomplished from just equation (4) expressed as:

$$t_1 = r_1 + q + \varepsilon \tag{7}$$

using just one target $i = 1$.This can only be solved for *q* if the survey value for r_1 is adopted. This procedure, which is standard at most stations, does not have the advantage of observational redundancy.

Only Real-Time Spider Retro Calibration

With no ground target, only equation (5) is available which can only be solved for *q* if *g* is adopted. The value of *g* can be obtained either from survey (which as mentioned above is difficult) or from equation (6). In this case the reduced satellite measurement from equation (2) becomes:

$$D = (T - \delta T) - (S - \delta S) + (2/c)g \tag{8}$$

in which the system delay Q has been eliminated in favour of real-time measurements S. In the "feedback calibration" pioneered in TLRS (*Silverberg, 1982*) the measurement of g had to include all the optical delays in the Coude path.

Internal Target and One Ground Target

The single ground target $(i = 1)$ can be used to calibrate the real-time internal target (*Kunimori et al, 1996; Nicolas et al, 1999*) by adopting the surveyed range r_1 and using it rather than its de-composition into coordinates in equation (6). The solution is immediately:

$$g = r_1 - (t_1 - s_1) \tag{9}$$

This equation applies even if the internal target is on the laser/receiver table rather than near the secondary mirror. NRL@SOR uses 2 internal targets, on the optical bench and a Spider Retro "Headring" plus an external target (*Davis et al, 1999*). At Orroral (*Luck, 1992*) we adopted g from the survey results and only used equation (9) to monitor its stability.

RESULTS FROM STROMLO-I (7849)

The history of the Stromlo-I MINICO determinations, from data courtesy Geoscience Australia, is summarized in Figures 5-8, in which the legends are:

DX, DY: Solutions for (x_0, y_0) from equation (6), compared with survey values;

DG: Solutions for g from equation (6), compared with survey values;

DR = $\sqrt{DX^2 + DY^2 + DG^2}$.

Towards the end of the period, Spider Retro was not available so equation (4) was used.

Over the entire period, the single-session RMS in each coordinate including g was typically 1.5 mm, and the RMS of the quadratically combined coordinates was 2.8 mm which thus represents the accuracy with which the system delay q was determined. A good example from 2001 August 15 is given in Table 2, showing how the solutions vary slightly depending upon whether the Spider Retro values are not used (equation (4)), the system delay is eliminated (equation (6)), or Spider Retro measurements are included in the solution for g (equations (4) and (5) together). The RMS $(\hat{\sigma}_0)$ column gives the scatter of the input data about the solutions, whose estimated RMSs are also shown.

Table 2: Solutions from a typical MINICO session, 2001.

Sol'n Equ'n	Error w.r.t. Survey				RMS	RMS of Solutions			
	DX (mm)	DY (mm)	DG (mm)	Dq (ps)	$(\hat{\sigma}_0)$ (mm)	x_0 (mm)	y_0 (mm)	g (mm)	q (ps)
(4)	1.2	-1.9	-	442	1.9	1.5	1.4	-	6.9
(6)	0.0	-1.5	1.9	-	1.2	1.0	0.9	0.7	-
(4),(5)	1.2	-1.9	1.6	442	1.3	1.0	0.9	0.9	4.6

The correlation coefficients between the adjusted parameters are shown in Table 3. They do not vary from session to session. The correlation between g and q is quite high, because they are both treated as constants for all targets, bur they are partially de-correlated because the ground target measurements do not involve the Spider Retro explicitly.

Figures 5 and 6 are time-series graphs of DX,DY and DG,DR respectively (DR>0 always). None of the little patterns seen are considered significant. Figure 7 shows the distribution of the results with time-of-day, the only notable feature being clustering of observations when crews began or finished shifts! Figure 8 plots the planimetric positions of the SRP with respect to its nominal position. Their means, viz DX=0.4 mm, DY=-1.0 mm, therefore represent the inaccuracies in the ground surveys (or unmodelled systematic errors in the ranging system . . .).

Table 3: Correlation coefficients between parameters adjusted in the solution.

	SRP Coordinates		Spider Retro
	x_0	y_0	g
y_0	-.29	1.00	
g	-.28	.09	1.00
$\boldsymbol{q}$	.38	-.12	-.73

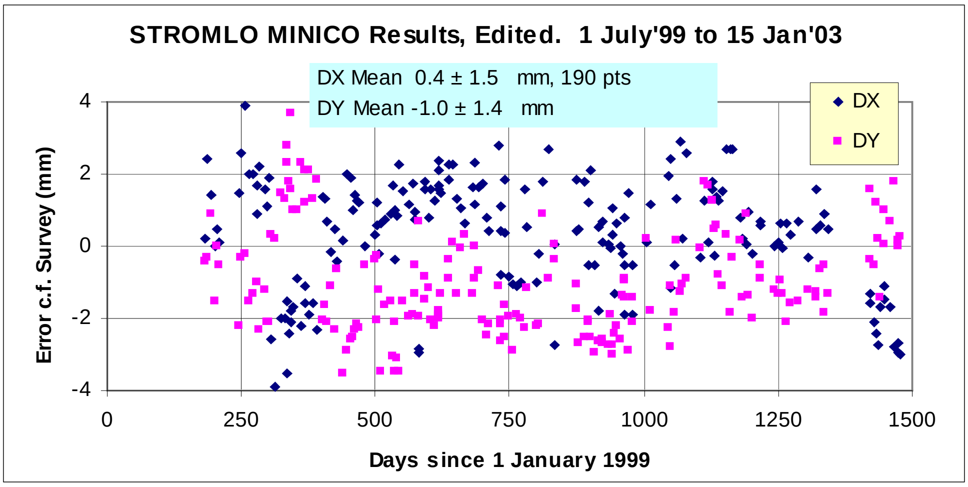


Figure 5: Time-series of MINICO determinations of the East (DX) and North (DY) components of the System Reference Point with respect to their adopted survey values.

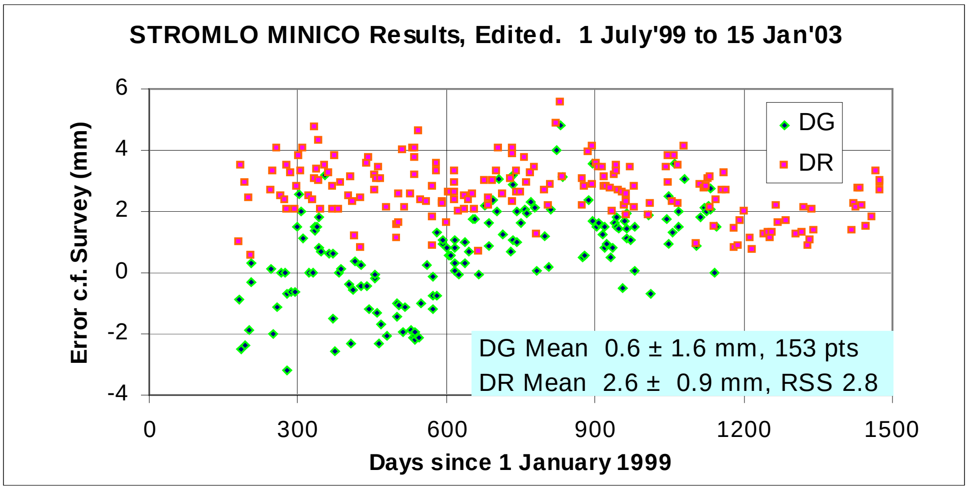


Figure 6: Time-series of MINICO determinations of the Spider Retro distance residuals (DG) with respect to its nominal value, and of the quadratically-combined errors of DX, DY and DG.

Comparison with Standard Single-Target System Calibration

I had planned to present graphs showing how these multi-target system delay determinations improved upon the usual single-target routine calibrations (or degraded them), but of course there is nothing to compare them against, because the value of q resulting from equation (7) depends entirely upon the assumption that the surveyed distance r_1 is absolutely accurate. The only way to check the accuracy of the single r_1 is to repeat the survey (which is recommended by ILRS to be done every two years).

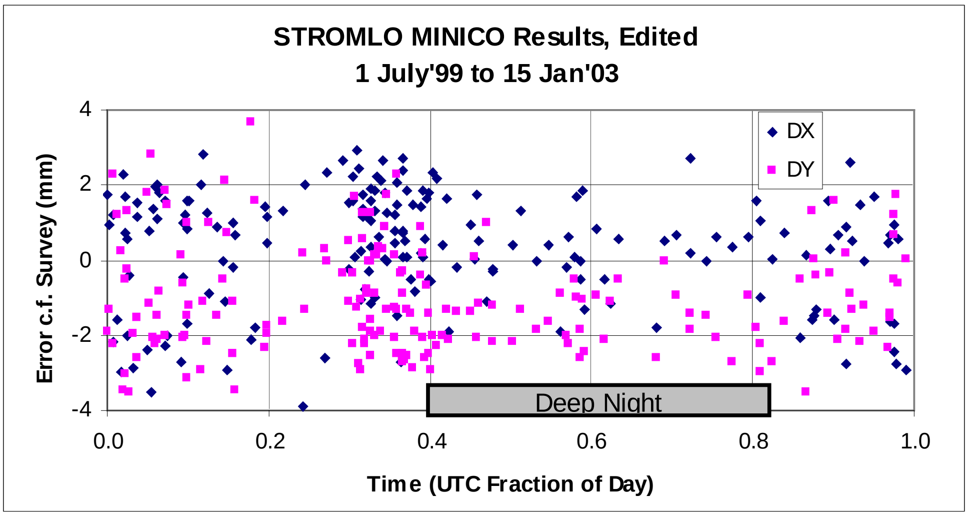


Figure 7: East and North MINICO determinations plotted against time of day (UTC).

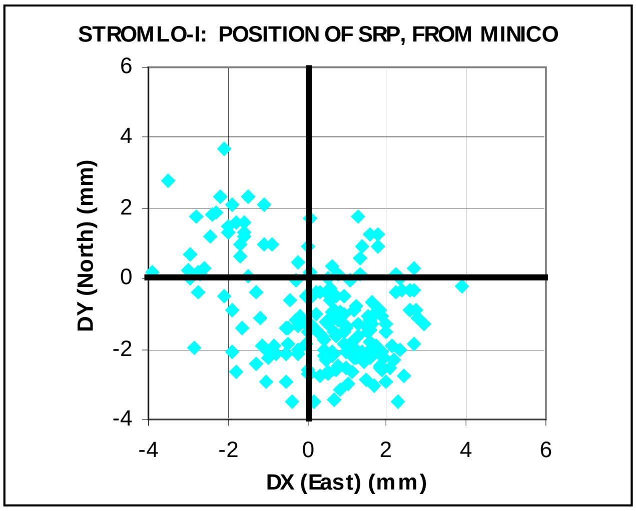
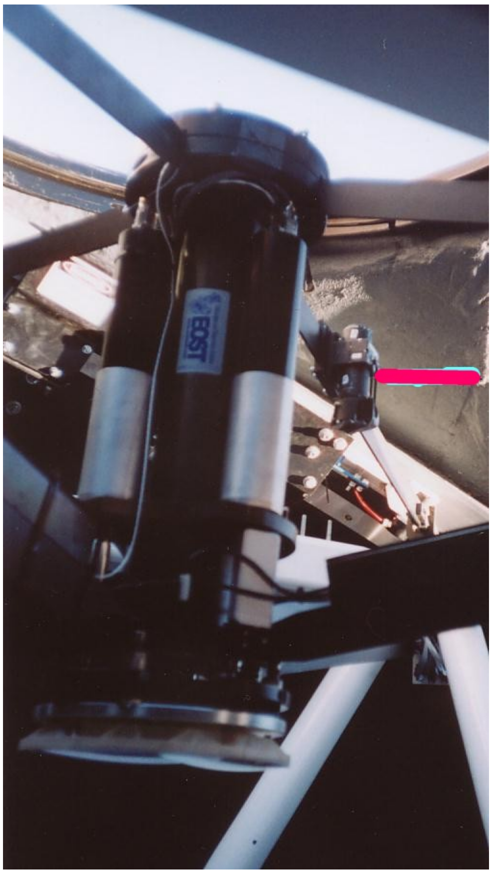


Figure 8: Map of the MINICO-determined positions of the System Reference Point, with respect to nominal.

PROCEDURE AT THE NEW STROMLO STATION (7825)

The four ground targets are available, so equation (4) can certainly be used to get improved values of (x_0, y_0) and hence a revised value of r_1 for use in subsequent single-target calibration. A Spider Retro retained from Orroral has been installed (see Figure 9) and tested in a limited way while ground target ranging at low laser power, but it is currently uncertain whether it can be safely integrated during satellite ranging.

The distance from the Spider Retro will be measured using the methods in this paper, not by scrambling around the telescope with tape measures and other apparatus in a very tight space close to its mirror surfaces. Its position will, however, be measured next time the geodetic collocation survey is performed.



No decision has yet been made on whether the MINICO determinations of the SRP position - and hence of the "survey" values of the ranges to the targets - will be implemented, nor on how often they would be updated. Initially, the MINICO values obtained during Final Acceptance Testing will be checked for any significant differences from the GA 2003 survey values, and either theirs or ours will be adopted. There will follow a period when MINICO will be used to assess the stability of the adopted values, which will decide whether or not to implement a regime of frequent (weekly/monthly) updates.

The first successful MINICO with this system took place on 31 May 2004, yielding survey agreements of $x_0 : -1.0\text{mm} \pm 1.6,\ y_0 : -0.1\text{mm} \pm 1.5\,\text{mm}$ RMS, with g & q RMSs 1.5 mm and 7.2 ps respectively.

Figure 9: Red line points to the Spider Retro to the right of the secondary mirror focusing assembly. The secondary mirror is at the bottom left of the photo.

RESULTS FROM STROMLO-III

The range timing system was substantially revised in mid-September 2004. Results since then have been consistent to better than 0.5 mm (RMS) as shown in Figure 10. The typical RMS yielded by a single MINICO is 0.4 mm in DX and DY, and 1.7 ps in system delay. These are

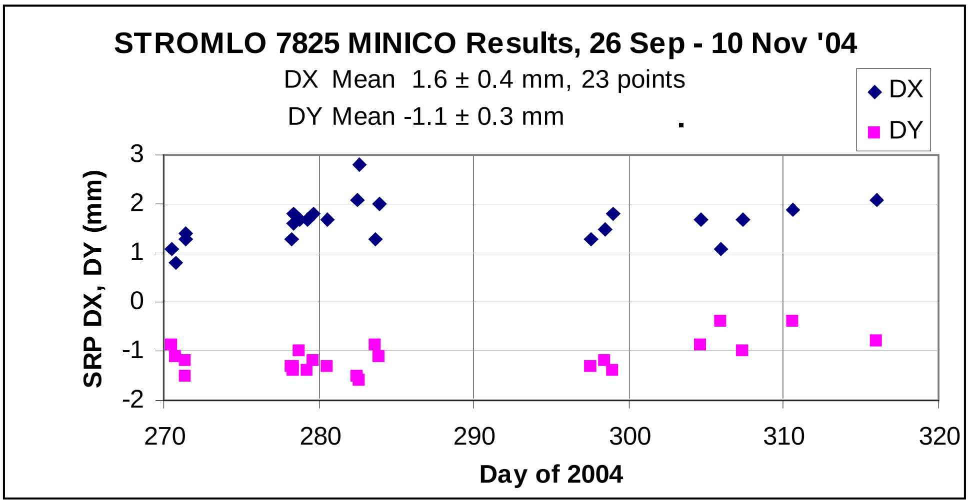


Figure 10: Time-series of MINICO determinations of the East (DX) and North (DY) components of the new System Reference Point with respect to their 2003 survey values.

considerable improvements over the corresponding values from the old system, given in Table 2, solution (4). To date it has not been possible to gather sufficient statistics on Spider Retro.

The 95% confidence interval of the DX mean is (roughly) $\pm 2x0.4/\sqrt{23} = \pm 0.17\text{mm}$, so its difference of 1.6 mm from the survey value is statistically very significant, and similarly for the DY mean of -1.1 mm. It is uncertain whether these are due to survey inaccuracies, to the behaviour of the range timing system over small time intervals (cf. *Benham, Gibbs & Smith, 2004*), or something else. But they are not really large enough to worry about yet.

CONCLUSIONS

This paper addressed the geometric aspects of system delay estimation. Other aspects as well as the geometric, directed towards the accuracy of measurements as they are affected by such things as detector processes and timing system vagaries, were discussed at the ILRS Colloquium on SLR-System Calibration Issues (*Schreiber, 2001*) but are not treated here, nor are historically interesting aspects such as a mysterious background effect from behind the calibration board at Herstmonceux (*Appleby and Matthews, 1990*).

The MINICO method effectively provides a convenient, on-site method for determining the accuracy of the system delay so vital for satellite laser ranging. Such an assessment is not possible by one-target systems except through repeated, costly re-surveys. For Stromlo-1, an upper-bound (1-σ) was 2.8mm. It also provides monitoring of the stability of the horizontal coordinates of the SRP, or looked at the other way, of the actual distances from the SRP to each of the ground targets.

The ground target configuration at Stromlo is not conducive to monitoring the height differences - targets at extremely high and low elevations would be needed - so height variations must be measured by other means, such as laser ranging to satellites . . . (no, it's not a circular argument).

Use of a Spider Retroreflector enables real-time measurement of the system delay, provided that detector and timing system responses do not introduce their own biases. In fact, the system delay is eliminated from the calculations. The MINICO method provides much the best way of estimating the range from the SRP to the Spider Retro.

Results emerging in the final stages of commissioning the new Stromlo SLR, using this method with four ground targets, indicate that the SRP coordinates can be determined with a precision of 0.4 mm RMS, and the system delay with an accuracy of 1.8 ps (0.3 mm).

"The man who has just one ground target only thinks that he knows the system delay. The man who has five can be very confident."

ACKNOWLEDGEMENTS

The staff of Orroral Observatory persevered with use of Spider Retro and proved its usefulness.

Jim Steed and Geoscience Australia kindly provided the Stromlo-1 data from their archives (much of which I had compiled anyway), and performed the local surveys with unprecedented precision and highly verifiable accuracy.

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