

# DESIGN OF LASER RETRO-REFLECTOR ARRAY AND LASER RANGING EXPERIMENT FOR SHENZHOU-4 SATELLITE

Yang Fumin, Chen Wanzhen, Zhang Zhongping, Chen Juping, Wang Yuanming

Shanghai Astronomical Observatory, Chinese Academy of Sciences; [yangfm@shao.ac.cn](mailto:yangfm@shao.ac.cn)

## 1. Introduction

The China's fourth unmanned spacecraft "Shenzhou IV" was launched on December 30, 2002. One module of the spacecraft so called the manned module was returned to the Earth on January 6, 2003. The other part, the orbital module, was remained in the orbit and carried on some scientific experiment. The orbital altitude of the module was 350 km. One of the instruments on board was a microwave altimeter for sea level measurement. There were a laser retro-reflector array (LRA) and a GPS receiver onboard for precise orbit determination. The LRA was designed and manufactured by the Shanghai Observatory. The laser ranging experiment for the Shenzhou-4 satellite among the Chinese SLR stations was carried out during Jan. 7- March 27, 2003.

## 2. Design of the LRA for Shenzhou-4

The photo and the mechanical drawing of the LRA are shown in Figure 1 and 2. The angle between the normals of the central reflector and the side ones is 50 degrees. The diameter of the single corner-cube is 30mm. The diameter of the LRA is 200mm, and the height is 67mm. The divergence of the corner-cube is about 12-16 arcsec.



Diameter: 20cm  
 Corner-cubes: 9  
 Material: Fused quartz  
 Weight: 850g

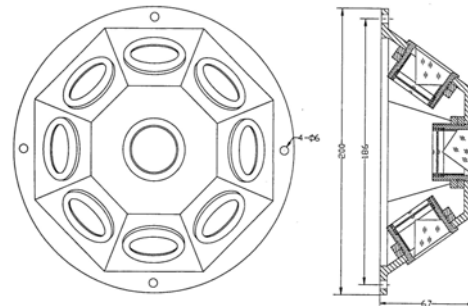


Fig. 1. Configuration of LRA

Fig.2. Mechanical drawing of LRA

## 3. Calculation of Effective Reflection Area of Shenzhou-4

### 3.1. Calculation for the incidence angle of laser beam with respect to the retro-reflector that has an inclination angle with the direction of the satellite-earth's center

The relation between the incidence angle and the relative effective area is given by:

$$\eta = \frac{2}{\pi} \cdot \left( \sin^{-1} \mu - \sqrt{2} \cdot \mu \cdot \operatorname{tg} i_r \right) \cdot \cos i_0$$

Where,  $\mu = (1 - 2 \operatorname{tg}^2 i_r)^{1/2}$ ,  $i_r = \sin^{-1} \left( \frac{\sin i_0}{n} \right)$

$\eta$  is relative effective geometric area,

$i_0$  is incidence angle of laser beam,

$i_r$  is refraction angle of laser beam,

$n$  is index of refraction for retro-reflector, usually the retro-reflector is made of fused quartz ( $n=1.445$ ).

While  $i_0 = 0$ , then  $\eta = 1$

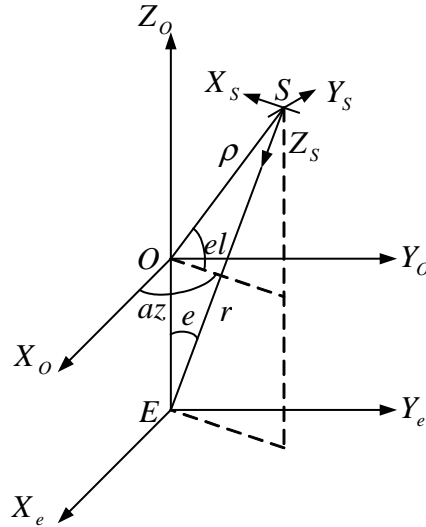


Fig.3. Three coordinate systems

If the normal of the retro-reflector deviates from the direction of the earth's center with a special angle, it is necessary to find out a reference plane to measure the orientation angle of the normal. The orbital plane of the satellite is adopted as the reference plane, and the normal direction is defined as follows: the normal of the reflector deviates from the satellite-the earth's center direction both in the orbital plane with an angle of  $\alpha$ , and in the perpendicular plane to the orbital one with an angle of  $\beta$ . We suppose the deviation angle is invariable as the satellite moves. There are three coordinate systems as follows (Fig. 3):

- (1) Satellite coordinate system ( $S - X_s Y_s Z_s$ ): the satellites ( $S$ ) as the origin, the satellite's instantaneous motion direction as the  $X_s$  axis, forward is positive. The direction from the satellite to the earth's center as the  $Z_s$  axis, to the earth is positive.  $Y_s$  axis determined by right-hand rule.
- (2) Station coordinate system ( $O - X_o Y_o Z_o$ ): The station ( $O$ ) as the origin, the  $X_o O Y_o$  is a tangent plane of the earth's surface.  $Y_o$  axis directs to the projection of the highest point of satellite apparent orbit for the station on the plane  $X_o O Y_o$ .  $Z_o$  axis point to the zenith of the station.
- (3) Geocentric coordinate system ( $E - X_e Y_e Z_e$ ): Move only the origin of the station coordinate

system to the geocentric, and keep  $X_e, Y_e$  and  $Z_e$  axes parallel to  $X_o, Y_o$  and  $Z_o$  axes respectively.

Suppose a satellite is located at an arbitrary position of space, and the incidence angle of laser beam to the retro-reflector is the angle between the laser beam vector and the normal of retro-reflector. Choosing geocentric coordinate system for calculation, we can convert both the laser beam vector from the station coordinate system and the normal of retro-reflector from the satellite system to the geocentric system. Then we can obtain the incidence angle easily.

Azimuth ( $az$ ) is the angle between  $X_o$  axis and the projection of the laser beam on the plane  $X_oOY_o$ . It should be noted that the azimuth here is different from the one defined in astronomy.

The elevation is the angle between the laser beam and its projection on the plane  $X_oOY_o$ , as shown in Fig. 3.

The unity length vector of the laser beam both in station coordinate system and in geocentric coordinate system is the same:

$$L = \begin{bmatrix} \cos(el) \cos(az) \\ \cos(el) \sin(az) \\ \sin(el) \end{bmatrix},$$

In geocentric system, the unity length vector of the satellite position is

$$S = \begin{bmatrix} \sin(e) \cos(az) \\ \sin(e) \sin(az) \\ \cos(e) \end{bmatrix}.$$

Here,  $e$  is the geocentric angle of satellite  $\angle SEO$ . It can be gotten by

$$e = \arcsin\left(\frac{\rho}{r_s} * \cos(el)\right),$$

Where  $\rho$  is the slant distance from the station to the satellite, and  $r_s$  is geocentric distance of the satellite.

In the satellite coordinate system, the normal vector of retro-reflector is

$$\vec{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \cos \beta \sin \alpha \\ \sin \beta \\ \cos \beta \cos \alpha \end{bmatrix}$$

The transformation from satellite coordinate system to geocentric coordinate system is as follows:

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & 0 & -\sin \theta_2 \\ 0 & 1 & 0 \\ \sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix}$$

Where

$$\theta_1 = \cos^{-1}(\cos(e)/\cos(c)), \theta_2 = \pi - c, c = \sin^{-1}(\sin(e) * \cos(az))$$

In geocentric coordinate system, the unity length vector of the normal of the retro-reflector N is

$$\begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_1 & \sin \theta_1 \\ 0 & -\sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{bmatrix} \cos \theta_2 & 0 & -\sin \theta_2 \\ 0 & 1 & 0 \\ \sin \theta_2 & 0 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$$

The incident angle of laser beam to the reflector  $i$  is given by

$$i = \arccos(\vec{L} \cdot \vec{N})$$

Obviously, the incidence angle is the function of the azimuth, elevation of satellite, and the deflection angle of the normal of retro-reflector  $\alpha$  and  $\beta$ . So, the effective reflection area of retro-reflector is also the function of these factors. The total effective reflection area for the LRA can be obtained by summing up the contributions of all retro-reflectors with different deflection angles on the array.

### 3.2 Calculation result of distribution of effective reflection area for Shenzhou-4 LRA

According to the formula above, we calculated the distribution of the effective reflection area for the Shenzhou-4 LRA. Fig. 4 shows the distribution of the satellite's effective reflection area, supposing  $\theta=50^\circ$  and the orbital altitude ( $h_s$ ) is 330km. The value of contour line is the relative effective reflection area, and suppose the reflection area of single retro-reflector as 100. The outermost large circle stands for the horizon circle, and dashed circles for the contour lines with different elevations. From the outermost to the center, the elevation of dashed circles is  $10^\circ$ ,  $30^\circ$ ,  $50^\circ$  and  $70^\circ$  respectively, and the center stands for the zenith of station. The total effective reflection area is about 123 when the satellite is located at the zenith and the maximum area is

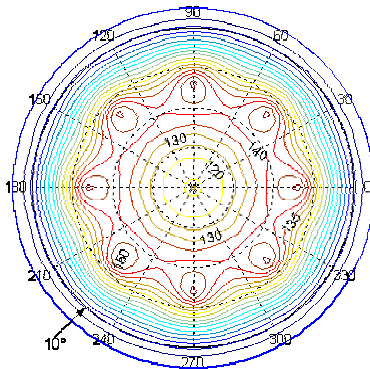


Fig.4  
Calculation result of distribution  
of effective reflection area on  
Shenzhou-4 LRA

about 150 at the elevation of  $32.8^\circ$ . The effective reflection area is greater than 70 at the elevation of  $10^\circ$ . It should be noted that the retro-reflectors of the Shenzhou-4 are without high reflectivity coating. For this kind of fused quartz retro-reflector, they do not always meet the specification of

the total reflection at the incidence angle bigger than  $16.6^\circ$ . It depends on the azimuth of the incident beam. Because the satellite motion is too complicated to calculate the azimuth of the incident beam accurately, the actual total effective reflection area will be less than the calculation result.

#### 4. Optical tests of LRA

##### 4.1 Test of the surface flatness and divergence

The surface flatness and divergence of LRA are measured with a ZYGO Interferometer. The divergence of reflectors are 12-16 arcsec.

##### 4.2 Optical reflectivity measurement

See Fig. 5, the average reflectivity of a single corner-cube is about 92.5%.

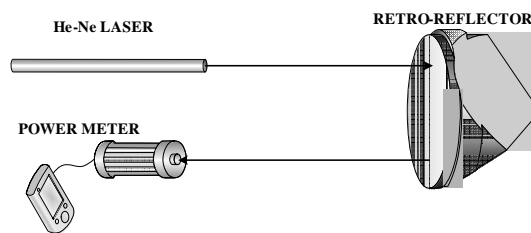


Fig.5 Optical reflectivity measurement

##### 4.3 Relative reflection area measurement

The relative reflection area of a corner-cube was measured as Fig. 6. As the corner-cube had no coating on the back surfaces, the relative reflection area depends on the azimuth angle  $\varphi$  of the incident beam (see Fig. 7). The measurement results shows in Fig. 8.

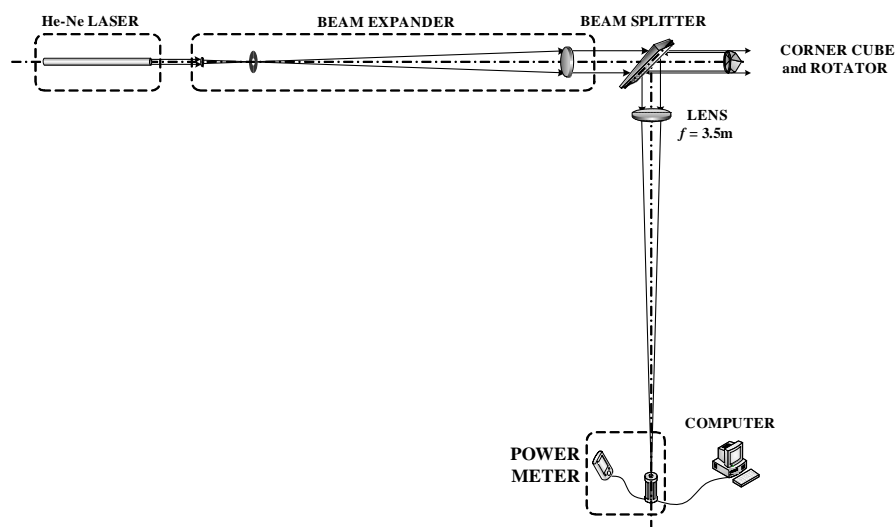


Fig. 6 Relative reflection area measurement

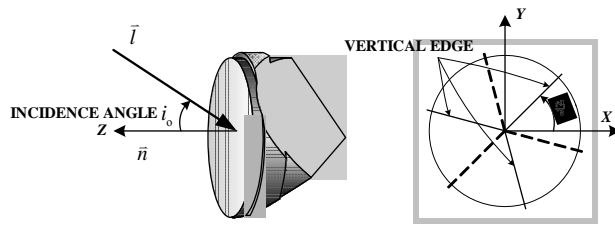


Fig. 7 Azimuth angle of corner cube

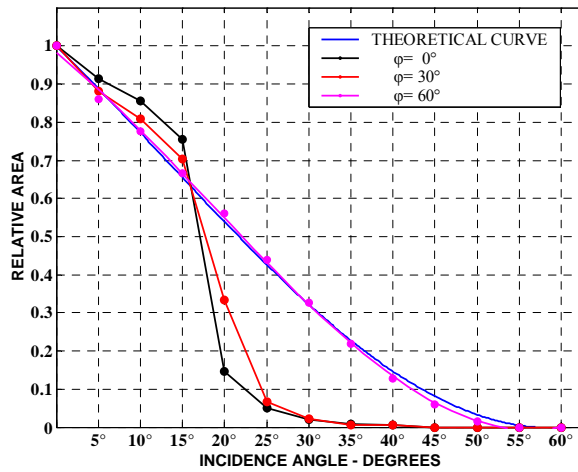


Fig. 8 Measurement result of the relative reflective area via incidence angle of corner cube with different azimuth angles

#### 4.4 Far Field Diffraction Pattern Measurement

The set up of the measurement is shown in Fig. 9 and the results are shown in Fig. 10.

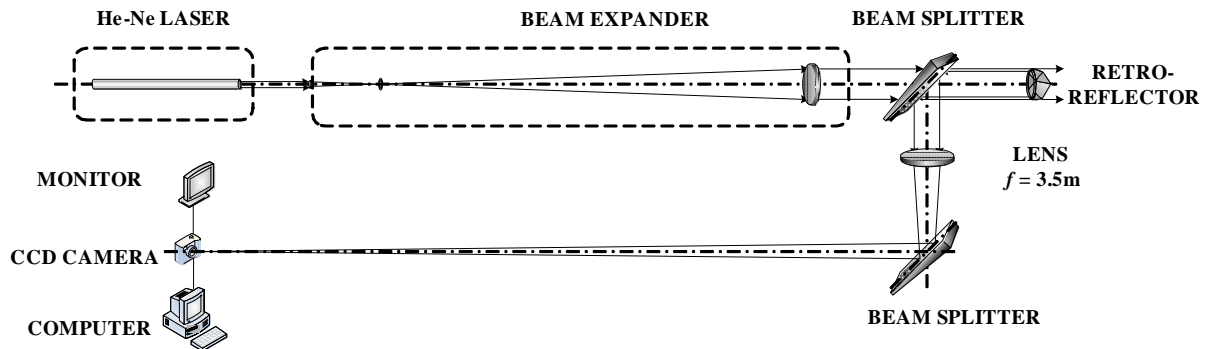


Fig. 9 Far field diffraction pattern measurement



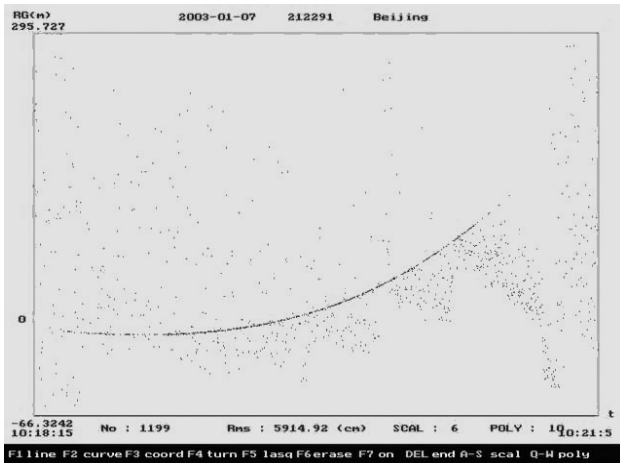


Fig.13 The first pass from Beijing station on Jan.7, 2003

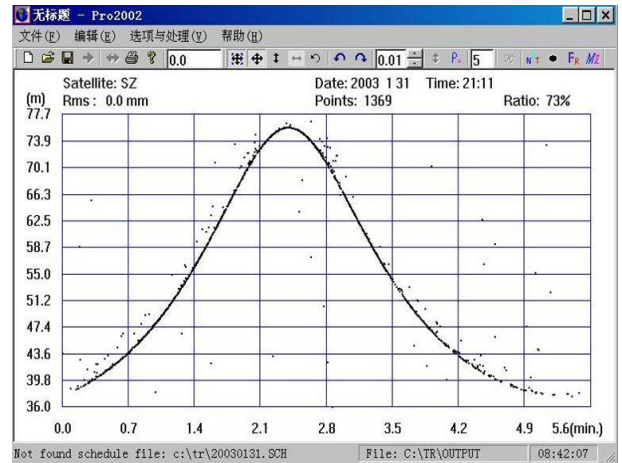


Fig.14 One pass from Changchun station on Jan.31, 2003

## Acknowledgement

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