A NEW APPROACH FOR MISSION DESIGNING OF GEODETIC SATELLITES

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Abstract

Repeat ground track orbits are frequently used in the geodetic applications of an artificial satellite. The usual procedure for designing a mission requiring repetition of the ground track is based on trial and error interactive corrections. We propose a different approach that can be completely automated, and has been implemented as a software tool dubbed SADSaM. In just a few seconds, SADSaM provides the initial conditions of an exactly periodic orbit with the requested repeat ground track cycle, and also provides the orbit stability character, as well as the averaged orbital elements.

Introduction

In the geodetic applications of an artificial satellite, associated measurements are frequently sampled along the ground track of the satellite's nadir point. The points where the ground track of a satellite intersects itself on the surface of the earth are called crossover points. Crossover points provide relevant measures in satellite geodesy, for instance in the calibration of a gravity field model[1]. Techniques for the determination of crossover locations have been introduced in several studies. The relevant bibliography can be found in[2].

The ideal situation occurs when the satellite repeat its ground track on the surface of the Earth, and repeat ground track (RGT) configurations are therefore preferred. The procedure of mission design starts from the experiment requirements, which constrain the orbital parameters to a subset of limited values. Then a first order of J_2 design is done as a rough estimate of the nominal solution. Further refinements of the orbital elements —usually in the presence of a medium degree zonal model, but sometimes including drag— will provide the nominal orbit.

The refinement procedure aerospace engineers normally use is based on trial and error interactive corrections that converge to a good nominal set of orbital elements; "good" meaning that the satellite does not drift substantially from the RGT. A fine "tuning" of the eccentricity in a manual iterative sequence does this refinement.

On the contrary, it has been recently shown that periodic solutions exist for a zonal model of the artificial satellite when the problem is formulated in a synodic frame, i.e. a rotating frame attached to the planet[3]. These periodic orbits repeat exactly their ground track on the surface of the planet and, hence, are ideal candidates as nominal orbits for RGT missions.

In this communication we describe SADSaM, a software tool for computing RGT orbits. It is based on the continuation of families of periodic orbits, and is totally automated. By simply introducing the RGT cycle between the nodal periods and nodal days as input, SADSaM provides the initial conditions of an exactly RGT orbit either sun synchronous or at the required inclination in just a few seconds —even for a high degree (zonal) gravitational model. Besides SADSaM provides the stability character of the RGT orbit and its averaged orbital elements.

We illustrate the usefulness of this tool computing a nominal orbit for ENVISAT, a satellite that is actually tracked by the laser station of ROA. We find that the orbital parameters provided by SADSaM are very close to the real mission parameters.

Frozen orbits

The experiment constrains of an earth observation mission –normally related to technical limitations of the sensors and geographic or geodesic aspects related to the experiment– limit on the acceptable range of the orbit elements as well as the repeat ground track cycle. For these kinds of satellite tasks, mission designers try to minimize the altitude variation of the satellite over the surface of the Earth searching for orbits with a small constant value of the eccentricity and with a frozen argument of perigee. When one only considers the J_2 effect, the Lagrange equations for the secular motion of the orbital elements show that there are no secular variations in semimajor axis, inclination and eccentricity. But there is a constant regression of the line of nodes, and, except at the critical inclination $\sin i = 2/\sqrt{5}$, a constant motion of the perigee given by:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{3n\left(4-5\sin^2 i\right)}{4\left(1-e^2\right)^2} J_2 \left(\frac{\alpha}{a}\right)^2$$

where a is the semimajor axis, e the eccentricity, i the inclination, ω the argument of the perigee, n the mean motion, α the equatorial radius of the Earth, and t is the time. But when considering also the J_3 effect, one finds

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{3n\left(4-5\sin^2 i\right)}{4\left(1-e^2\right)^2} \left[J_2 \left(\frac{\alpha}{a}\right)^2 + \frac{1}{2} J_3 \left(\frac{\alpha}{a}\right)^3 \sin\omega \, \frac{\sin^2 i - e\left(1-\sin^2 i\right)}{e\left(1-e^2\right)^2 \sin i} \right]$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\frac{3n\left(4-5\sin^2 i\right)}{4\left(1-e^2\right)^2} \frac{1}{2} J_3 \left(\frac{\alpha}{a}\right)^3 \cos\omega \sin i$$

The equilibrium solutions $(\mathrm{d}e/\mathrm{d}t) = (\mathrm{d}\omega/\mathrm{d}t) = 0$ of the reduced system above are usually called "frozen orbits", and, besides the critical inclination case mentioned above, one can find frozen orbits whit low eccentricity and argument of the perigee $\pm\pi/2$ at any inclination. As appreciated in Fig. 1, one important property of frozen orbits is that close to the values e_0, ω_0, that "freeze" the orbit, the orbit perigee oscillates.

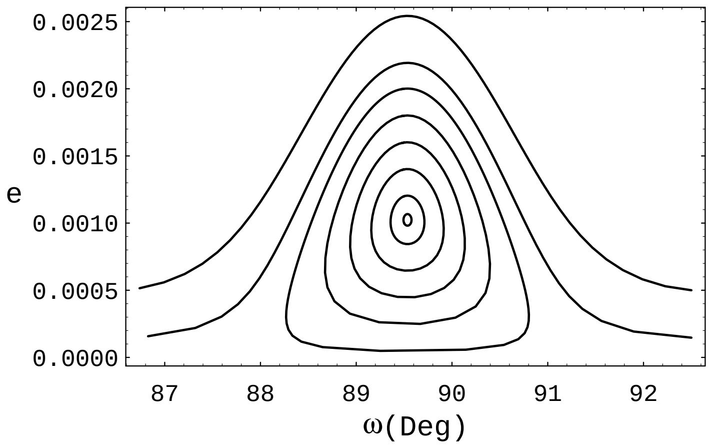

Figure 1. Oscillation of the perigee in the vicinity of a Spot-type frozen orbit (a = 7200.548 km, *i* = 98.723°*)*

Periodic Orbits

A more general definition of frozen orbits is that they are relative equilibriums of an averaged form of the zonal problem. With this definition, and regardless of the motion of the node, one can map frozen orbits onto periodic solutions of the (two dimensional) non-averaged zonal problem. When using cylindrical coordinates, the zonal problem is decoupled into the motion in the (ρ, z)-plane and the motion of this plane: the rotating meridian plane of the satellite. Frozen orbits are periodic solutions of the two degrees of freedom problem that represents the motion in the (ρ, z)-plane[4]. Figure 2 shows an example of a frozen orbit in cylindrical coordinates, with averaged orbital elements: a = 10559.26 km, e = 0.34633, i = 116.556°, ω = 270°.

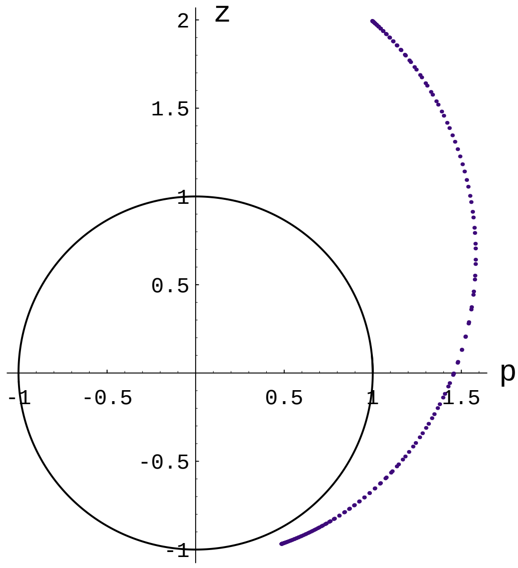

Figure 2. Ellipso™ Borealis-type frozen orbit in the (ρ, z)*-plane*

In some cases the nodes rate of precession is commensurate with the rotation rate of the Earth and the frozen/periodic orbits are exactly periodic solutions in a rotating frame attached to the Earth. That is: orbits that exactly repeat the ground track on the surface of the Earth. As appreciated in Fig. 3 (after Lara, 1999), there exist almost circular, repeat ground track, periodic orbits for all inclinations, and elliptic, repeat ground track, periodic orbits at the critical inclination.

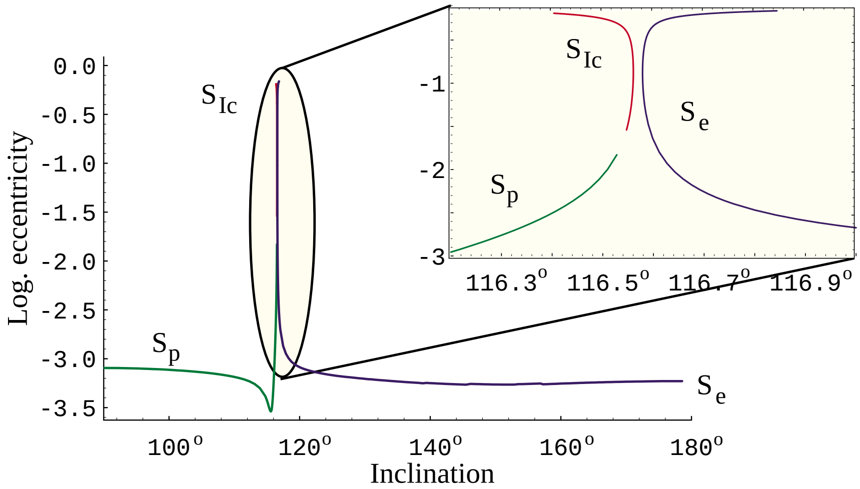

Figure 3. Families of orbits with a repeat ground track cycle of 8 nodal periods (retrograde inclination orbits only)

Therefore, the computation of three-dimensional periodic orbits in a rotating frame attached to the Earth is a new approach for designing satellite missions that require a repeat ground track orbit condition. As described below, this can be done in a totally automated way.

Practical approach: SADSaM

SADSaM, an acronym for a **S**oftware **A**ssistant for **D**esigning **Sa**tellite **M**issions, is a computer application intended for helping mission designers for artificial satellites in their search for repeat ground-track, frozen orbits. Briefly, SADSaM assumes a zonal model for the Earth gravitational potential. Thus, when formulated in the inertial frame the problem is bi-parametric: while the energy E determines the size (and period) of the orbit, the polar component Λ of the angular momentum vector is related to its inclination –except for the critical inclination, where variations of Λ imply variations in the eccentricity for fixed inclination. Therefore, the exploration of the (Λ, E)-plane allows finding either a sun synchronous solution or an orbit at the selected inclination, fulfilling the desired repeat ground-track condition.

The flowchart of Fig. 4 presents the main sequence of computations done by SADSaM. After launching the application, SADSaM asks the user for the number of nodal days and periods after which the ground track of the satellite repeats over itself, and also for the kind of solution desired, either sun synchronous or at fixed inclination or eccentricity. A first approximation to the solution is then computed from a first order of J_2 analytic approximation. It follows the inner loop, where the initial conditions are iteratively refined until finding a frozen orbit. If this frozen orbit does not repeat its ground track, the second loop improves the initial conditions of the orbit that, after differential corrections, will correspond to a repeat ground track orbit. Then, after computing average values of the orbital elements, SADSaM verifies that the orbit parameter –either inclination i, or eccentricity e, or nodes precession rate– corresponds to the value (i_0, e_0, or sun synchronism) specified by the user. If the solution is obtained, then it is written to a file and SADSaM ends. Otherwise, the polar component Λ of the angular momentum of the orbit is varied in the outer loop.

Additional information accepted by the program is the name of the file with the potential model, and the order of the higher zonal harmonic coefficient to take into consideration. By default, SADSaM uses coefficients J_2–J_9 of the WGS84 gravity field.

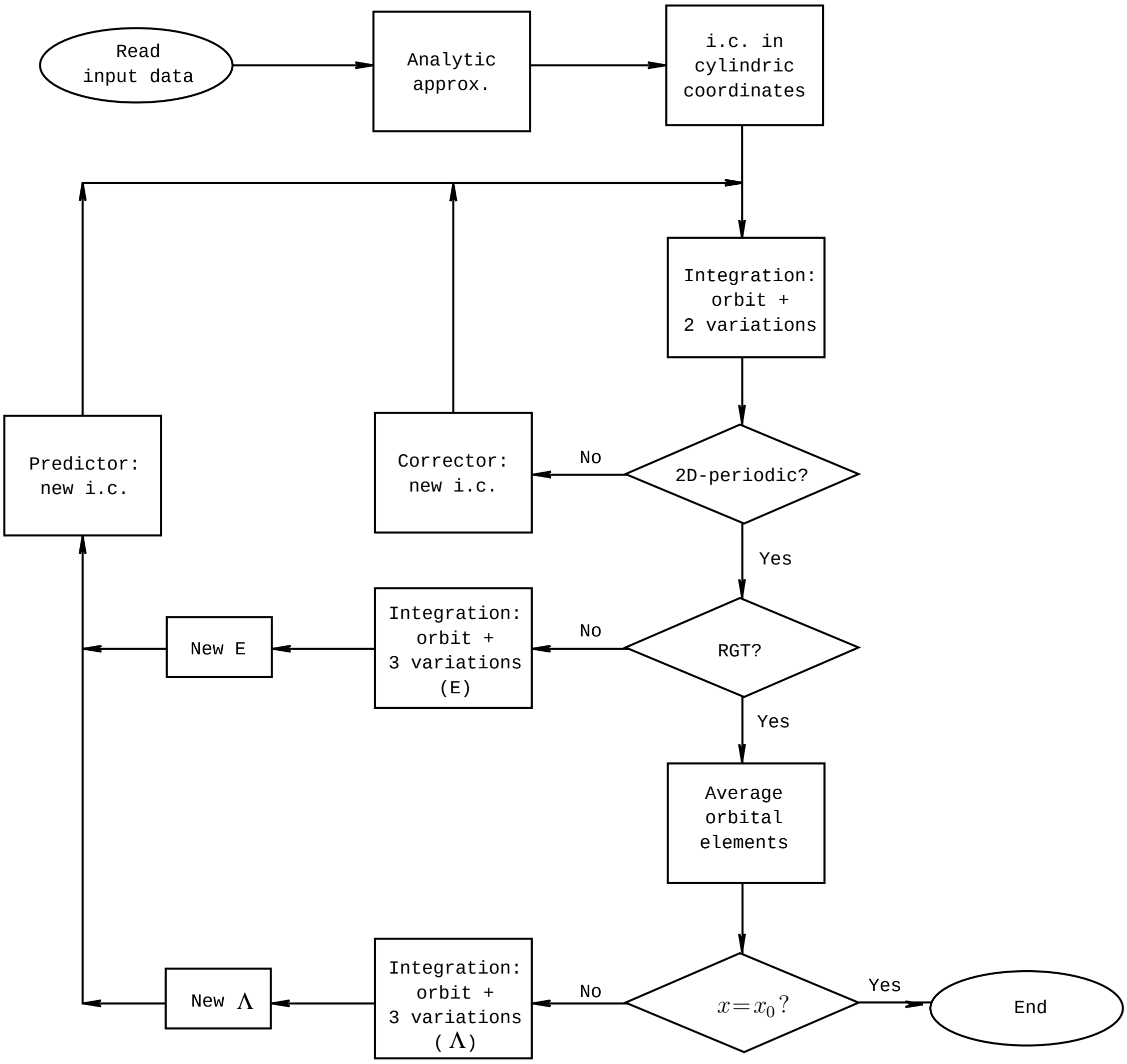

Figure 4. Flowchart of SADSaM

Application to ENVISAT

In order to show the efficiency of SADSaM, we compare the long term propagation of the actual orbital elements of ENVISAT (a = 7159.49 km, e = 0.00115, i = 98.5425°, ω = 91.9°) with those provided by SADSaM for the same mission requirements (sun synchronous; repeat cycle 35 days; length 501 orbits), namely, a = 7159.49 km, e = 0.00114, i = 98.5446°, ω = 90°. Figure 5 present the long-term propagation (2500 days) in each case. A full perturbation model was considered, including the Sun and Moon perturbations, and the GEMT-1 36×36 gravity model. Since the eccentricity of the ENVISAT orbit is very small, the frozen condition is better perceived using the elements $x = e\cos\omega, y = e\sin\omega$. As appreciated in the figure, the results are very similar in both cases but with clear advantage to SADSaM.

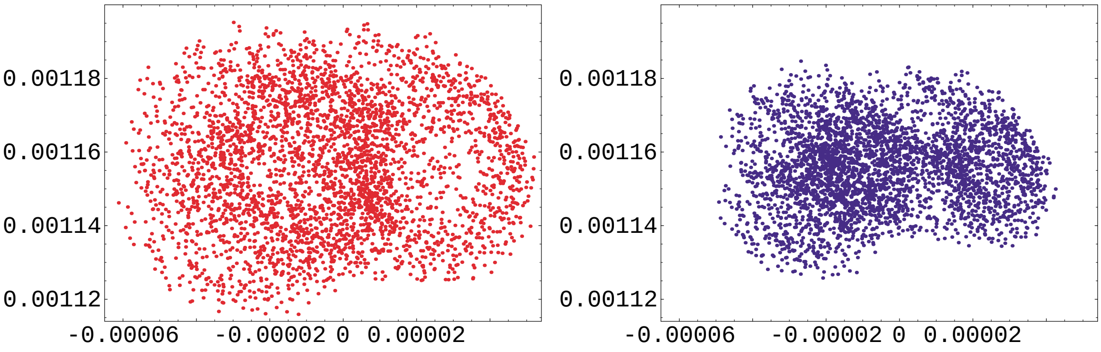

Figure 5. ENVISAT long-term evolution $(e\cos\omega, e\sin\omega)$. *Left: actual. Right: SADSaM.*

Conclusions

Repeat ground track orbit configurations are highly desirable for missions of geodetic satellites. But, when only considering the Earth gravitational force, repeat ground track orbits are found to be three-dimensional periodic orbits in a rotating frame attached to the Earth. The classical approach for mission designing of repeat ground track orbits is based on trial and error manual iterative corrections. On the contrary, the software application described here is a totally automated tool for computing initial conditions of repeat ground track solutions. Our tool has two clear advantages with respect to other software programs: on one side it is not restricted to sun-synchronous solutions; on the other side it automatically determines an accurate value of the eccentricity without need of "a priori" assumptions. Application of this tool to simulate a real mission shows the reliability of the new method proposed.

Acknowledgements

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