SELF-MIXING OPTICAL DOPPLER RADIAL VELOCITY MEASUREMENTS: A PROSPECTIVE STUDY

Jean-Louis Oneto[1], Odile Calame[2]
[1] Observatoire de la Côte d'Azur / MEO / GEMINI / CNRS
oneto@obs-azur.fr / Fax: (+33)[0]4.93.40.53.33
[2] Observatoire de la Côte d'Azur / GMC / GEMINI / CNRS
Odile.Calame@obs-azur.fr / Fax: (+33)[0]4.93.40.53.33

Abstract
Optical Doppler measurements of satellite radial velocity would give complementary data to actual ranging measurements. This could help to further constraint the orbitography models. An analytical study of Doppler Effect is presented. A tentative experimental setup using continuous laser with self-mixing autodyne interferometric detection seems very promising.

Introduction

Speed measurements using Doppler were used at the dawn of artificial satellite orbitography, in radiofrequency domain. This was abandoned in favor of laser ranging techniques. While it would be useless to turn back to such methods, the conjoint use of Doppler and laser distance measurements should help to better constraint the orbitography solutions. The former Doppler used downlink techniques with stability provided by on-board atomic clocks. The radiofrequency spectrum gave the advantage of the excellent selectivity of the receivers. The use of two-way Doppler in optical spectral domain takes advantage of existing retro-reflector arrays.

Computation of the satellite motion

For illustrating our purpose, we consider the very simplified case of a satellite orbiting around the Earth, in a Keplerian motion, with a laser station on the terrestrial surface.

The used parameters are defined as following:

R : Earth radius
e : Orbit eccentricity
a, b : Semi-major and semi-minor axes
i : Orbit inclination
Ω : Right ascension of the ascending node of the orbit on the equator:
ω : Argument of perigee
L : Longitude of the laser station
φ : Latitude of the laser station
T_e : Sidereal period of the Earth
w_e : Sidereal angular speed of the Earth ($2\pi / T_e$)
w_s : Angular speed of satellite on its orbit
t : Time

Let denote by T, L and S the position of center of Earth, Laser station and Satellite respectively (Figure 1). The corresponding vectors are derived from them.

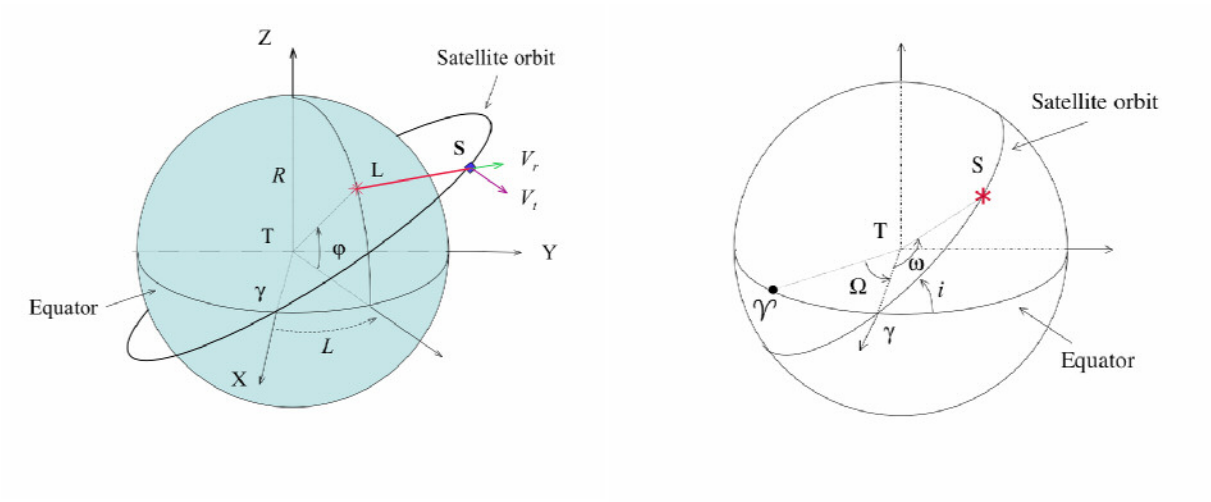

Figure 1: Notation summary

The geocentric coordinates of laser station in the equatorial frame are:

$$\mathbf{TL}_{eq} = \begin{pmatrix} R \cdot \cos(L + w_e \cdot t) \cdot \cos\varphi \\ R \cdot \sin(L + w_e \cdot t) \cdot \cos\varphi \\ R \cdot \sin\varphi \end{pmatrix}$$

The geocentric coordinates of satellite in orbit frame are:

$$\mathbf{TS}_o = \begin{pmatrix} -a \cdot (e - \cos(w_s \cdot t)) \\ b \cdot \sin(w_s \cdot t) \\ 0 \end{pmatrix}$$

By successive rotations of the oriented angles ω, $-i$ and Ω, with respect to the corresponding Z, X and Z axes, we obtain the geocentric coordinates of satellite in the equatorial frame:

$$\mathbf{TS}_{eq} = Rot_Z(\Omega) \cdot Rot_Z(-i) \cdot Rot_Z(\omega) \cdot \mathbf{TS}_o$$

(where $Rot_X(u)$ designates the rotation around the X axis of an u angle) .

The development of the matrix products gives:

$$\mathbf{TS}_{eq} = \begin{pmatrix} \cos\Omega \cdot \cos\omega - \cos i \cdot \sin\Omega \cdot \sin\omega & -\cos\Omega \cdot \sin\omega - \cos i \cdot \sin\Omega \cdot \cos\omega & \sin i \cdot \sin\Omega \\ \sin\Omega \cdot \cos\omega + \cos i \cdot \cos\Omega \cdot \sin\omega & -\sin\Omega \cdot \sin\omega + \cos i \cdot \cos\Omega \cdot \cos\omega & -\sin i \cdot \cos\Omega \\ \sin i \cdot \sin\omega & \sin i \cdot \cos\omega & \cos i \end{pmatrix} \cdot \mathbf{TS}_o$$

We define a local frame by a system with the first axis X in the direction of $\mathbf{LS}$, the two others being directly perpendicular. We calculate the two components of the speed vector of $\mathbf{LS}$ along the X axis ($\dot{\mathbf{LS}}_r = V_r$) and in the normal plane ($\dot{\mathbf{LS}}_t = V_t$). For that, we have to compute the derivatives of the precedent vector coordinates with respect to the time and apply the adequate rotations.

The following numerical applications and graphs use the Lageos1 elements and the Grasse SLR position as example. The dashed lines denote that the satellite is below the local horizon.

The Figure 2 illustrates the variations of these two components of the speed of satellite, in a topocentric frame.

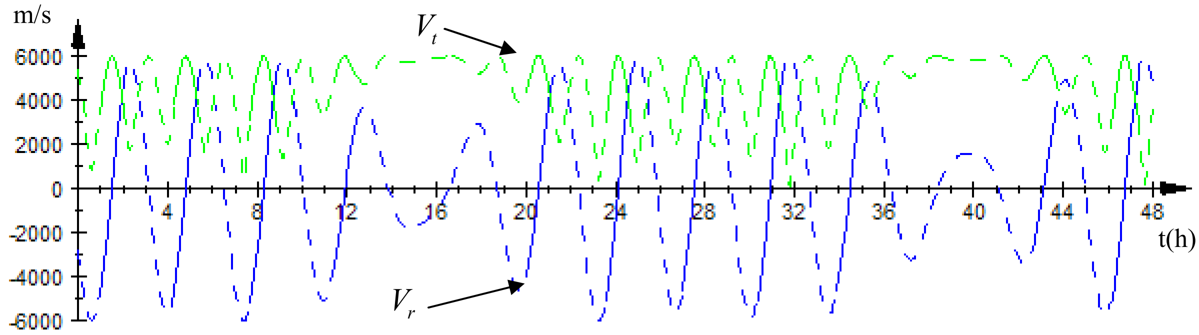

Figure 2: Speed vector components

Doppler Effect

Let define as c the speed of light and λ the wavelength. The corresponding frequency ν is then equal to c/λ. From a relativistic point of view, the Doppler effect in frequency has two components, radial ($\Delta\nu_r$) and transverse ($\Delta\nu_t$).

Let define: $\beta_r = V_r / c$ and $\beta_t = V_t / c$

$$\Delta\nu_r = \nu \cdot \left(\sqrt{\frac{1-\beta_r}{1+\beta_r}} - 1 \right)$$

$$\Delta\nu_t = \nu \cdot \left(\sqrt{1-\beta_t^2} - 1 \right)$$

It is to be noted that β_r and β_t are very small quantities with respect to unity, so that the developments in series give:

$$\Delta\nu_r = \nu \cdot \left(-\beta_r + \tfrac{1}{2}\beta_r^2 + O(\beta_r^3) \right)$$

$$\Delta\nu_t = \nu \cdot \left(-\tfrac{1}{2}\beta_t^2 + O(\beta_t^4) \right)$$

The classical contribution is:

$$\Delta\nu_c = \nu \cdot \left(-\frac{\beta_r}{1+\beta_r} \right)$$

$$\Delta\nu_c = \nu \cdot \left(-\beta_r + \beta_r^2 + O\left(\beta_r^3\right) \right)$$

In absolute value, the relativistic correction to the radial effect is of the same order than the transverse part.
The Figure 3 shows the three corresponding curves.

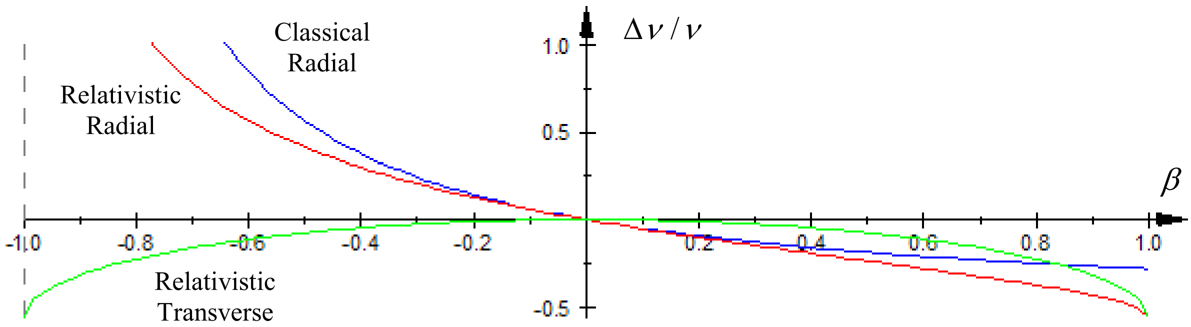

Figure 3: Doppler effects: classical radial and relativistic radial/transverse

The curves, in Figure 4 and Figure 5, show that $\Delta\nu_t$ is by several orders of magnitude smaller than $\Delta\nu_r$.

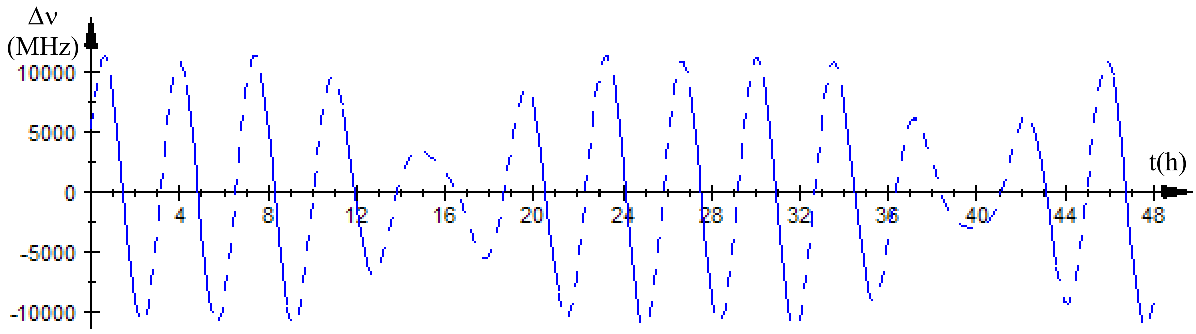

Figure 4: Relativistic Radial Doppler in Frequency

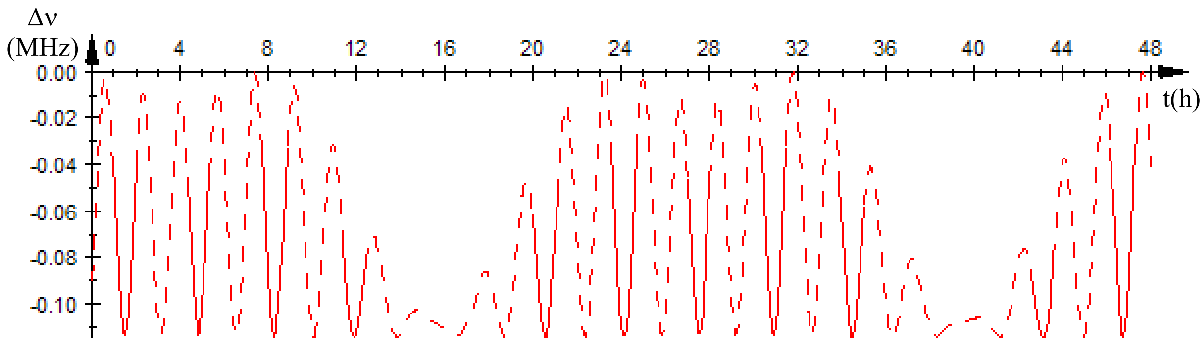

Figure 5: Relativistic Transverse Doppler in Frequency

Nevertheless, the transverse effect cannot be neglected. The physical measurement gives only access to the sum of these two effects, so that in order to retrieve the radial component alone, it is necessary to evaluate the transverse speed and compute the transverse Doppler component. The sensitivity of this effect being very low, even a rather coarse satellite motion determination with respect to the station would be sufficient.

Discussion of detection methods for optical Doppler:

1) Direct spectroscopy:

The radial orbital speed of a satellite (V_r) being of the order of few km/s, $\beta_r = V_r / c$ is of the order of $2 \cdot 10^{-5}$. The associated Doppler effect is of the same order of magnitude, and this leads to $\Delta\lambda \approx 635 \cdot 2 \cdot 10^{-5} \approx 0.01\text{nm}$. In order to determine the speed with enough precision to be of any use, one needs at least a resolution of 1m/s corresponding to a wavelength resolution of $10^{-5}\,\text{nm}$. The achievement of this goal would require a very high resolution spectroscope, very expensive and bulky.

2) Interferometry:

The classical use of interferometry supposes that the path difference must be less than the coherence length. For an ordinary laser, the coherence length is of the order of few meters, and it reaches only a few kilometers, even for an extremely stabilized laser like those used in the Virgo experiment.

In order to obtain interferences between the local reference laser beam and the reflected signal, it would be necessary to have a coherence length greater than twice the satellite distance, i.e. near 1000km even for the lower orbits!

This also applies to the case of the self-mixing Doppler interferometric velocimetry that is often used to check the speed of terrestrial targets.

3) Fixed delay interferometry:

Also known as Fringing Spectroscopy, this method was described by Jian Ge[1] and David Erskine[2], especially for extra-solar planet detection. Even in that case, they have achieved 2m/s resolution and expect sub-m/s results, even without a sophisticated optimization.

In the present case, the source is coherent and easily accessible, the required stability is of the order of the light round trip, i.e. a less than a second, instead of several hours, months or even more. This should leads to a sub-mm/s resolution.

The detection setup (Figure 6) consists of a fixed delay interferometer followed by a low-resolution dispersion spectrometer. In our case (Figure 7), this spectrometer is useless since the laser light is highly monochromatic. This leads to the paradox of a high-resolution spectroscopic device without any dispersive element!

In this design, the delay is chosen to a convenient value: the sensitivity of the device is proportional to the delay, but if must be small enough compared to the coherence length in order to avoid a decrease of the fringe contrast.

The Doppler information is carried by Moiré fringes and must be detected by two-dimensional devices.

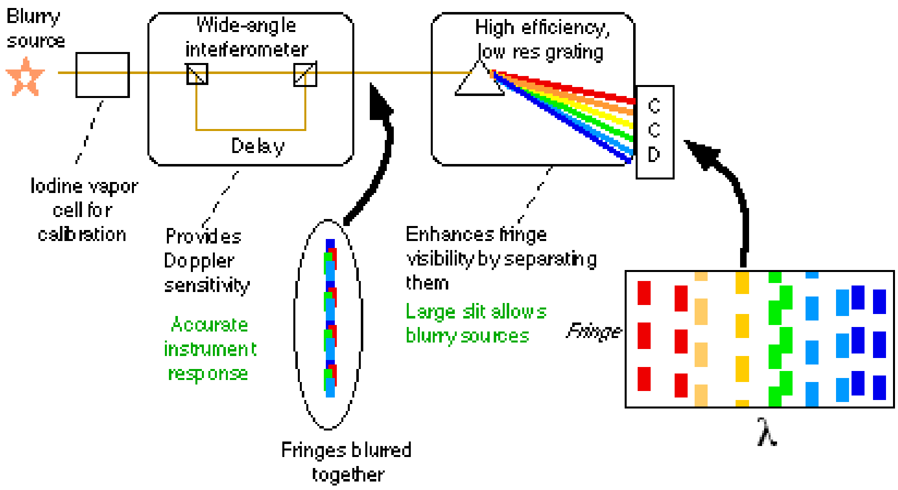

Figure 6: Fixed delay interferometer

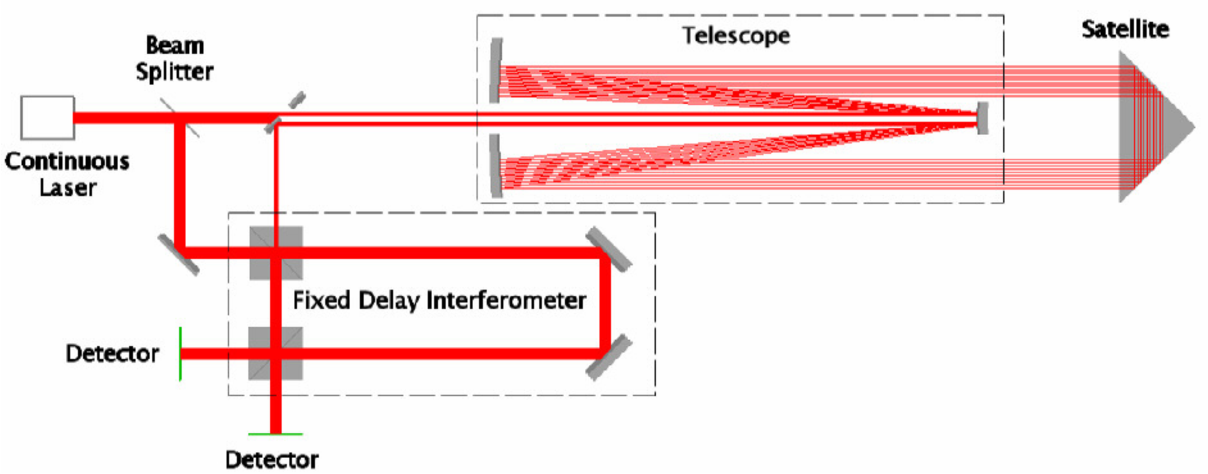

Figure 7: Fixed Delay Interferometer variant for laser

Conclusion

This kind of project may yield a complement of information in conjunction with classical telemetry. Even with distance measurements in the millimeter precision range, direct determinations of speed through Doppler techniques at comparable level, will bring independent and hence significant improvement of satellite dynamics knowledge.

References

[1] Ge, J., Fixed delay interferometry for extra-solar planet detection, ApJ. 571, L165-L168, 2002

[2] Erskine, D., Ge, J., Basri, G., Rushford, M., Macintosh B., and Alcock, C., Doppler Planet Search with a Fringing Spectrometer, published on the Web at: http://www-phys.llnl.gov/H_Div/doppler/