MOUNT MODEL STABILITY

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Conclusion

The most important factor in mount model stability is to have a stable mount !

Could this have been stable ? :

"The telescope for the GODLAS system (1st SLR, 1964) . . was pointed by a modified Nike-Ajax missile tracking mount controlled by two operators guiding on a sunlit satellite under joystick control. One operator controlled azimuth and the other controlled elevation."

(Degnan, J.J.: "*Thirty Years of SLR*", 1996)

Historical Literature on Mount Modelling

- Hovey, G.R. (1974): Ph.D thesis, Mount Stromlo Observatory
- Wallace, P.T. (1976): Anglo-Australian Telescope, Siding Spring
- Matzke, D.E. (1976): Error Model for X-Y Antenna
- Powell, M.E. (1977): MS Eng. thesis, UTexas at Austin
- **Ricklefs, R.L. (1982): Proc. 4th Laser Ranging Instr. Workshop**
- Luck, J.McK. (1993): Proc. 8th Laser Ranging Instr. Workshop
- Trueblood, M & R.M. Genet (1997): "*Telescope Control*"
- Wallace, P.T. (2004-): TPoint Software web-site
- Meeks, R.L. (2003): Ph.D thesis, Colorado State University/EOST

(And not much else of mathematical significance in my library. What have I missed ?)

Misalignments Generally Modelled

- Encoder zero-point displacements;
- Encoder scales;
- Tilt of the major axis, e.g. the azimuth axis;
- Non-orthogonality of the secondary axis (e.g. the elevation axis) to the major axis;
- Collimation error, i.e. non-perpendicularity of the optical axis to the secondary axis;
- Bending (flexure) in the telescope tube;
- Bending or torsion of the mount, where applicable (e.g. X-axis in alt/alt mount)
- Bearing wobbles and encoder eccentricities.

PLUS:

Empirical Terms –

PROVIDED THAT THEY ARE REPEATABLE EVERY TIME !

Mount Stromlo Mount Model Mount in Terms of Physical Parameters

Residuals (O-C) modelled as linear combination of functions F_j, G_j . Note the same coefficients are used for both _A and _E. There are *m* = 23.

“Computed” predictions are vacuum plus refraction in elevation, per Marini and Murray angle formula.

The Stromlo Mount Model

Term	Description	Azimuth Function (F)	Elevation Function (G)
1	Azimuth encoder offset	1	-
2	Elevation encoder offset	-	1
3	Azimuth axis tilt about North		
4	Azimuth axis tilt about East		
5	Collimation (optical axis misalign)		-
6	Non-orthogonality of Az & El axes		-
7	Azimuth bearing ellipticity (sin)		-
8	Azimuth bearing ellipticity (cos)		-
9	Elevation bearing ellipticity (sin)	-	
10	Elevation bearing ellipticity (cos)	-	
11	Telescope tube flexure	-	
12	Azimuth encoder scale error		-
13	Elevation encoder scale error	-	
14	Bi-periodic in azimuth (empirical)		-
15	Bi-periodic in azimuth (empirical)		-
16	Elevation encoder stiction (sin)	-	
17	Elevation encoder stiction (cos)	-	
18	Elevation bearing stiction (sin)	-	
19	Elevation bearing stiction (cos)	-	
20	Scaled bi-periodic in azimuth (sin)		-
21	Scaled bi-periodic in azimuth (cos)		-
22	Bi-periodic in elevation (sin)	-	
23	Bi-periodic in elevation (cos)	-	
(24)	Observing clock error (not used)		

sin 2 sec A

sin cos tan cos sin cos / 2

STATISTICS

n stars successfully observed;

j;

Dahlquist and Bjorck, 1974) by:

where

so

Part of **Solution from Star Cal of 23 March 2004**

Note large RMSs of parameters 2, 9, 10, 13

Number 29. **Sigma-Hat 1.29 arcsec** **Condition Number 0.6577D+06**

Term	Description		Delta_Parameter (arcsec)	Sigma (arcsec)
1	(Az) Az encoder offset:	1	4686.38	2.31
2	(El) El encoder offset:	1	-507.56	**194.71**
3	(Both) Az tilt about N:	cosA.tanE	15.17	0.29
4	(Both) Az tilt about E:	sinA.tanE	32.98	0.45
5	(Az) Collimation error:	secE	-125.20	3.09
6	(Az) Non-orthogonality:	tanE	-1.11	2.37
7	(Az) Az bearing ellipt:	sinA	-26.59	0.56
8	(Az) Az bearing ellipt:	cosA	-15.24	0.46
9	(El) El bearing ellipt:	sinE	116.61	**79.27**
10	(El) El bearing ellipt:	cosE	-216.26	**147.41**
11	(El) Tube flexure:	cotE	-18.36	8.94
12	(Az) Az encoder scale:	A/twopi	0.87	0.74
13	(El) El encoder scale:	E/twopi	-1924.44	**1088.51**
14	(Az) Az encoder double-cycl:	sin2A	-0.28	0.65

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Part of **Correlation Matrix of Solution**

Insanely large correlations are highlighted

Term	1	2	3	4	5	6	7	8	9	10	11	12
2	0.00											
3	0.40	0.00										
4	-0.68	0.00	-0.40									
5	**-0.97**	0.00	-0.28	0.64								
6	**-0.90**	0.00	-0.10	0.57	**0.97**							
7	-0.41	0.00	-0.26	0.68	0.38	0.33						
8	0.16	0.00	0.61	-0.27	-0.08	0.00	-0.15					
9	0.00	**0.94**	0.00	0.00	0.00	0.00	0.00	0.00				
10	0.00	**-1.00**	0.00	0.00	0.00	0.00	0.00	0.00	**-0.96**			
11	0.00	**-0.92**	0.00	0.00	0.00	0.00	0.00	0.00	-0.72	**0.88**		
12	0.00	0.00	0.05	0.14	0.02	0.02	-0.14	-0.03	0.00	0.00	0.00	
13	0.00	**-0.99**	0.00	0.00	0.00	0.00	0.00	0.00	**-0.97**	**1.00**	**0.87**	0.00

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How to Decrease the Correlations ?

These have been tried:

- Improve the distribution of stars observed
- Delete offending parameters
- "Normalize to the Mean"
- Use Prior Information

Alternative approaches:

- Surface fitting by Legendre polynomials
- Gram-Schmidt orthogonalization of the model functions

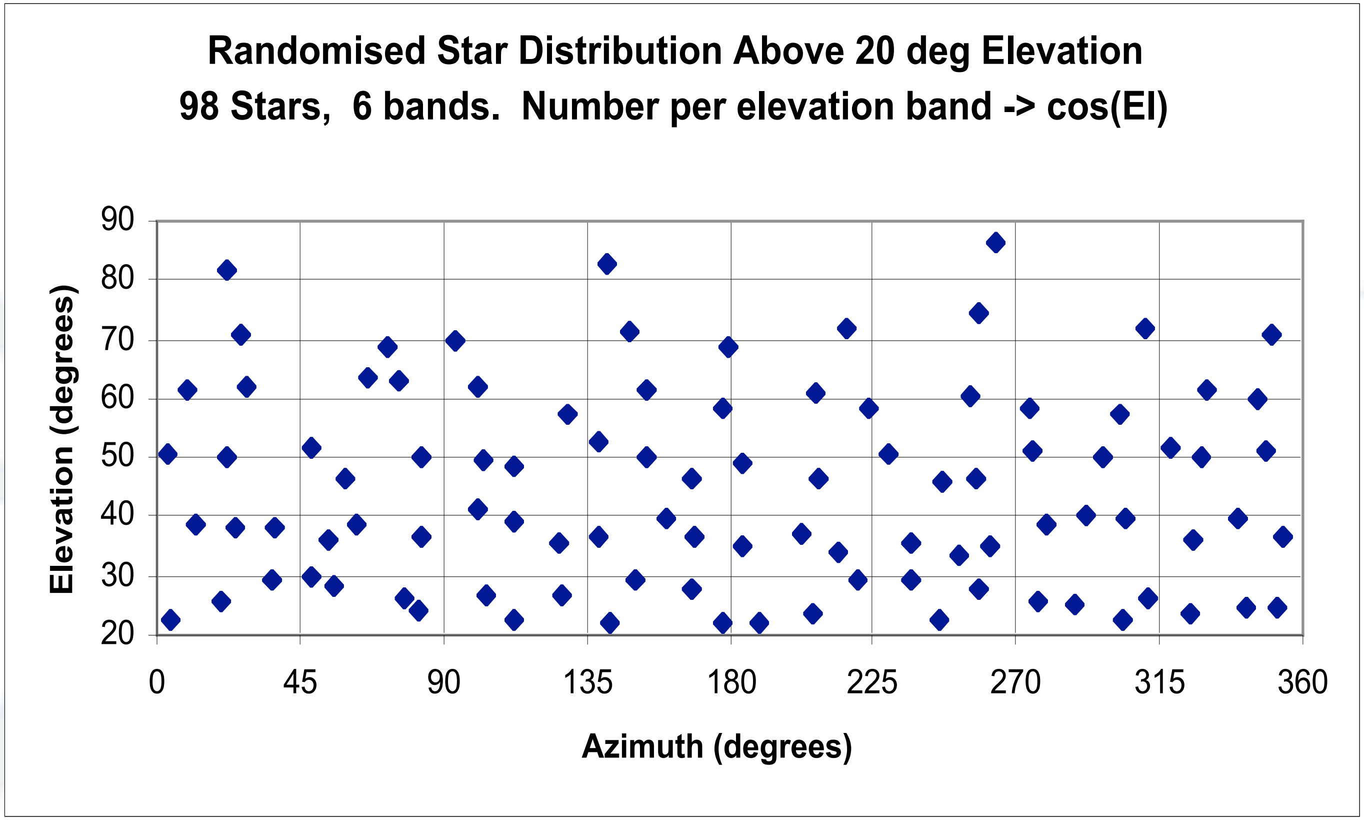
And

- Remove those b*____y INTERPOLATION ERRORS !!

Star Distribution Algorithm

given number of stars wanted and elevation lower limit

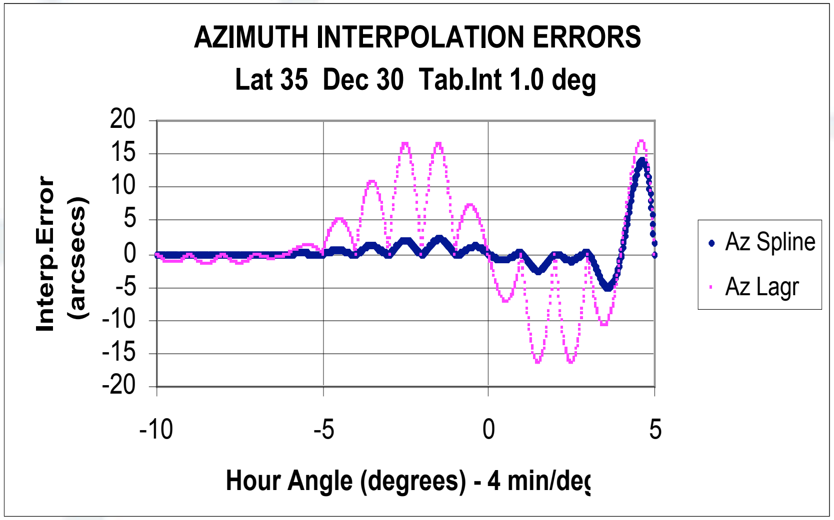
Selected points become centres of catalogue search region

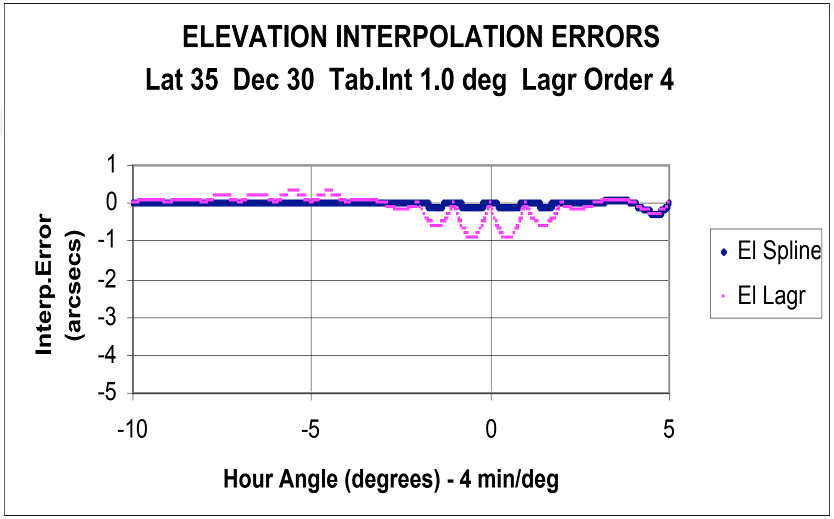


Essential to have Stars near Zenith

BUT

You can get interpolation errors near zenith if you are not careful, which severely corrupt the “Computed” values, hence the Observation Residuals, hence your solution !!!





Parameter Deletion

- The large standard errors of parameter solutions (slide 8) and large correlation coefficients (slide 9) are clues that the observations are over-fitted, i.e. too many parameters.
- And that some model functions are too similar, e.g. tan*E* and sec*E.*
- *Hence the Normal Matrix is SINGULAR.*
- The Condition Number _ (slide 7) measures the degree of singularity. Really, _ = 657,700 is absurd !

SO:

- Some terms must be removed from the solution.

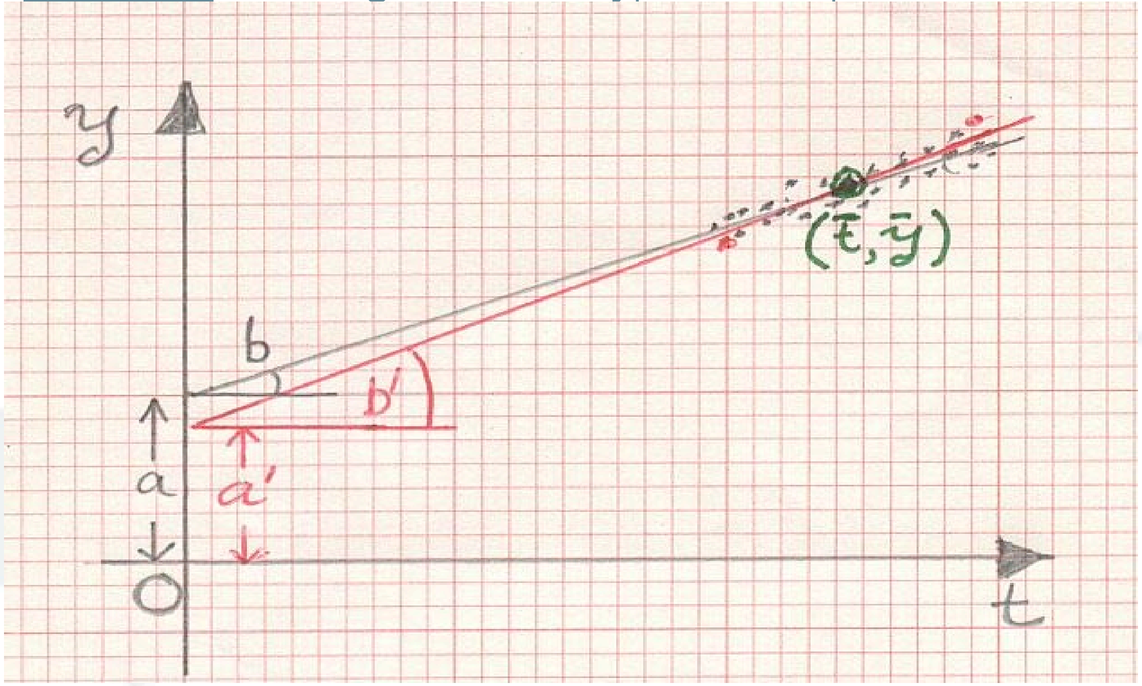
BUT:

- Unwise, willy-nilly deletion of terms may seriously degrade the post-fit RMS of residuals, unnecessarily.

“Normalizing to the Mean”

There must be a better phrase for this !!

Analogy: Straight line fit $y_i = a + b.t_i$



Much Better: $y_i = a + b.(t_i\text{-tbar})$

Removes “Lever Effect” to the origin, hence reduces correlation between a and b to zero.

Normalizing the Mount Model Terms

In analogy to

Effects of "Normalizing to the Mean" and Parameter Deletion

- ***"Normalizing"*** the functions in full 23-parameter solution reduces the condition number **from 658,000 to 68,000**, i.e. from absurd to huge. Post-fit residual RMS remain unchanged at **1.29** arcsecs.
- It also reduces the correlation coefficients between terms which are affected only by the "lever effect" (slide 14), leaving those which are truly correlated much more identifiable.
- For example, correlation between collimation error and azimuth encoder offset reduces from -0.97 to 0.27, whereas Elevation encoder scale with El.bearing remain unchanged at -0.97, 1.00.
- Judicious ***Deletion*** of 5 parameters then reduces the condition number to **32.4**, i.e. from huge to manageable, while increasing residual RMS merely **from 1".29 to 1".32**.

Use of Prior Information
(Bayesian Inference)

- Local Tie survey by GA (*Dawson et al, poster this Workshop*) estimated (amongst many other things) Tilt of azimuth axis from vertical (terms 3 & 4) and Non-Orthogonality betweeen azimuth and elevation axes (term 6), and their standard errors (30”, 30”, 10”).
- Adding these values as weighted constraints, and weighting the observations appropriate to 1”.5, reduced the condition number **from 32.4 to 18.0**, but increased Residual RMS to 1”.5.
- As an experiment, these values were applied with tight constraints (standard errors 1”.5 each) and the observations weighted for standard errors 1”.32 (slide 16) . The condition number reduced to **13.3** while the Residual RMS increased to **1”.7**.

CONCLUSION

The local tie survey results are reasonably consistent with the observations, and correlations between terms decrease accordingly. But there is not much value in including them.

Legendre

HEDONALOPS

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Gram-Schmidt

Acknowledgements

- Chris Moore and Adrian Loeff of EOS for data and development of some of the terms.
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CONCLUSIONS

- Sub-arcsecond **absolute** accuracy in telescope pointing is becoming a real possibility. Stromlo can realistically claim < 1".5 today. (What is APOLLO getting ?)
- Eliminate any trace and last vestige of Interpolation Errors, because they will surely ruin your solutions as well as your observations.
- Do not use too many terms in your Mount Model, else it will become numerically unstable and therefore useless.
- The technique of "Normalizing to the Mean" improves solution stability and enhances identification of offending terms.
- Adding prior information from local tie surveys as weighted constraints improves the stability slightly.
- Sincere apologies for not yet having data for the ultimate test, which is to see how accurately you point next time. But satellite acquisition at Stromlo now seems to need NO handpaddle corrections!

Maybe it's just the predictions….

CONCLUSIONS (continued)

Above all, most importantly:

Get a stable mount.

Or the horse will have bolted