Least square mean effect. Application to the analysis of Satellite Laser Ranging time series

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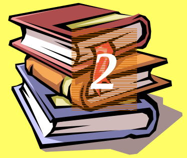


Summary

1- Least square mean effect

- Theoretical considerations
- Numerical examples

2- Alternative models

- Periodic series
- Wavelets

3- New model for SLR data processing

- General considerations
- First results
- Towards global estimations over a long period

4- Prospects

Least square mean effect

Theoretical considerations

Quality of space-geodetic measurements

↓

Representation of studied physical parameters
as time series

Example : terrestrial observing station position time series

Modeling currently used

« Well-known » physical effects = Modeled	+	Other physical effects = Constant estimations

!! We need to get exact and judicious representations !!

Least square mean effect

Theoretical considerations

Vector of physical parameters $\vec{X}$

Time-varying parameters

$$\vec{X}(t)=\vec{X}_0(t)+\delta\vec{X}(t)$$

Modeled effects → $\vec{X}_0(t)$

Studied effects → $\delta\vec{X}(t)$

$\delta\vec{X}(t)$ BUT they are supposed to be constant over $[t_1, t_m]$ $\delta\vec{X}$

-> averages of physical signals over the interval

Measurements modeled as $m(t)\cong f(t,\vec{X})$

Linearization of the model $m(t)\cong f(t,\vec{X}_0(t))+\frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t)).\delta\vec{X}$

$\frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t))$ = f partial derivative matrix at the point $(t,\vec{X}_0(t))$

Physical measurement $m(t)\cong f(t,\vec{X}_0(t))+\frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t)).\delta\vec{X}(t)$

Least square mean effect

Theoretical considerations

Estimation model

$$m(t) \cong f(t,\vec{X}_0(t)) + \frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t)).\delta\vec{X}$$

Linearization of physical measurements

$$m(t) \cong f(t,\vec{X}_0(t)) + \frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t)).\delta\vec{X}(t)$$

Observation equation

$$f(t,\vec{X}_0(t)) + \frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t)).\delta\vec{X} \cong f(t,\vec{X}_0(t)) + \frac{\partial f}{\partial \vec{X}}(t,\vec{X}_0(t)).\delta\vec{X}(t)$$

$$A = \begin{bmatrix} \frac{\partial f}{\partial \vec{X}}(t_1,\vec{X}_0(t_1)) \\ \frac{\partial f}{\partial \vec{X}}(t_2,\vec{X}_0(t_2)) \\ \vdots \\ \frac{\partial f}{\partial \vec{X}}(t_m,\vec{X}_0(t_m)) \end{bmatrix}$$

Matricial relation

$$A.\delta\vec{X} \cong \tilde{A}.\delta\tilde{\vec{X}}$$

$$\tilde{A} = \begin{bmatrix} \frac{\partial f}{\partial \vec{X}}(t_1,\vec{X}_0(t_1)) & 0 & \cdots & 0 \\ 0 & \frac{\partial f}{\partial \vec{X}}(t_2,\vec{X}_0(t_2)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{\partial f}{\partial \vec{X}}(t_m,\vec{X}_0(t_m)) \end{bmatrix}$$

$$\delta\tilde{\vec{X}} = \begin{pmatrix} \delta\vec{X}(t_1) \\ \delta\vec{X}(t_2) \\ \vdots \\ \delta\vec{X}(t_m) \end{pmatrix}$$

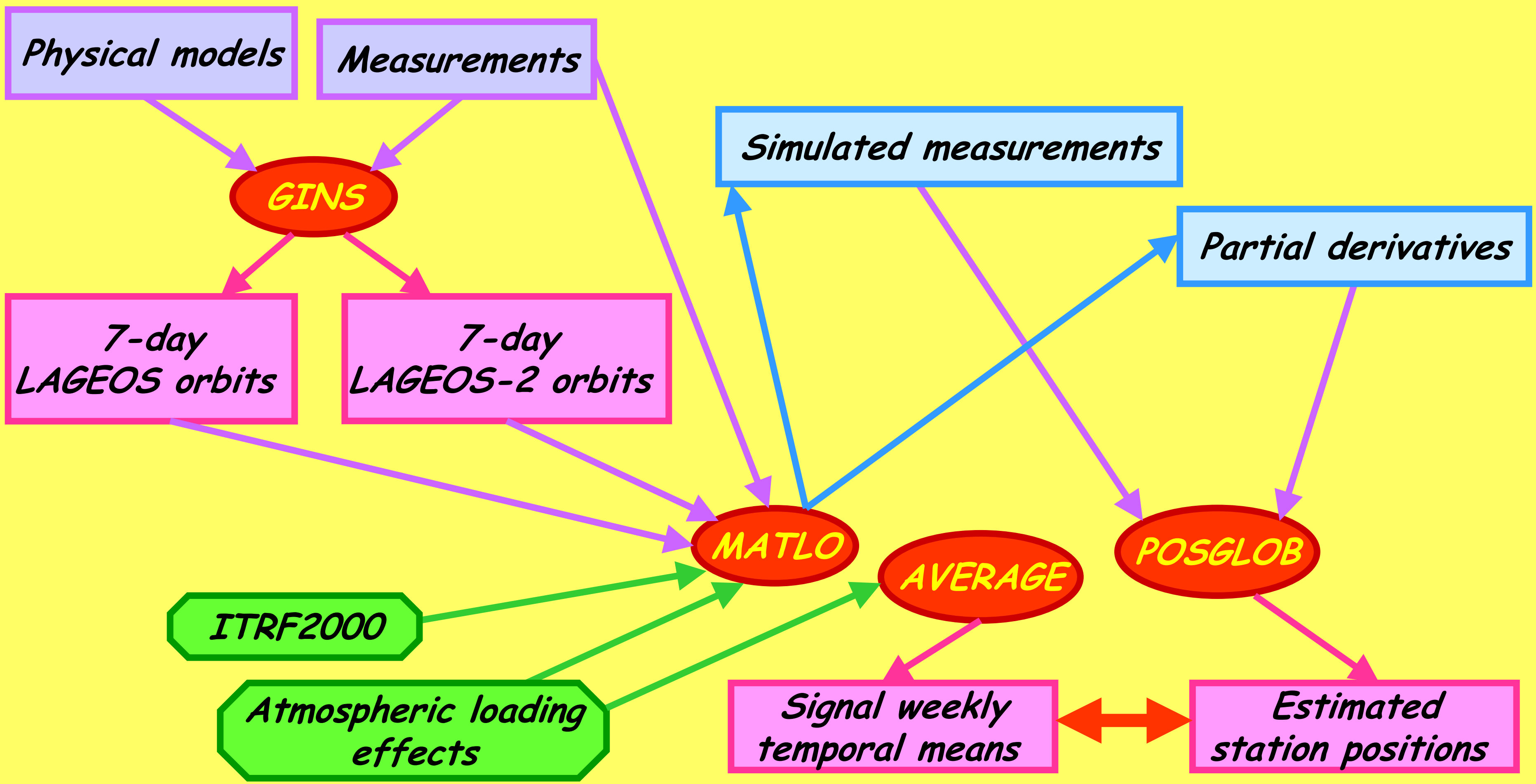
$$\delta\tilde{\vec{X}}_{average} = \begin{pmatrix} \delta\vec{X}_{average} \\ \delta\vec{X}_{average} \\ \vdots \\ \delta\vec{X}_{average} \end{pmatrix}$$

Least square estimation

$$\delta\hat{\vec{X}} \cong \delta\vec{X}_{average} + \left(A^T P A\right)^{-1} A^T P \tilde{A}.(\delta\tilde{\vec{X}} - \delta\tilde{\vec{X}}_{average})$$

Least square mean effect

Numerical examples: method of simulation

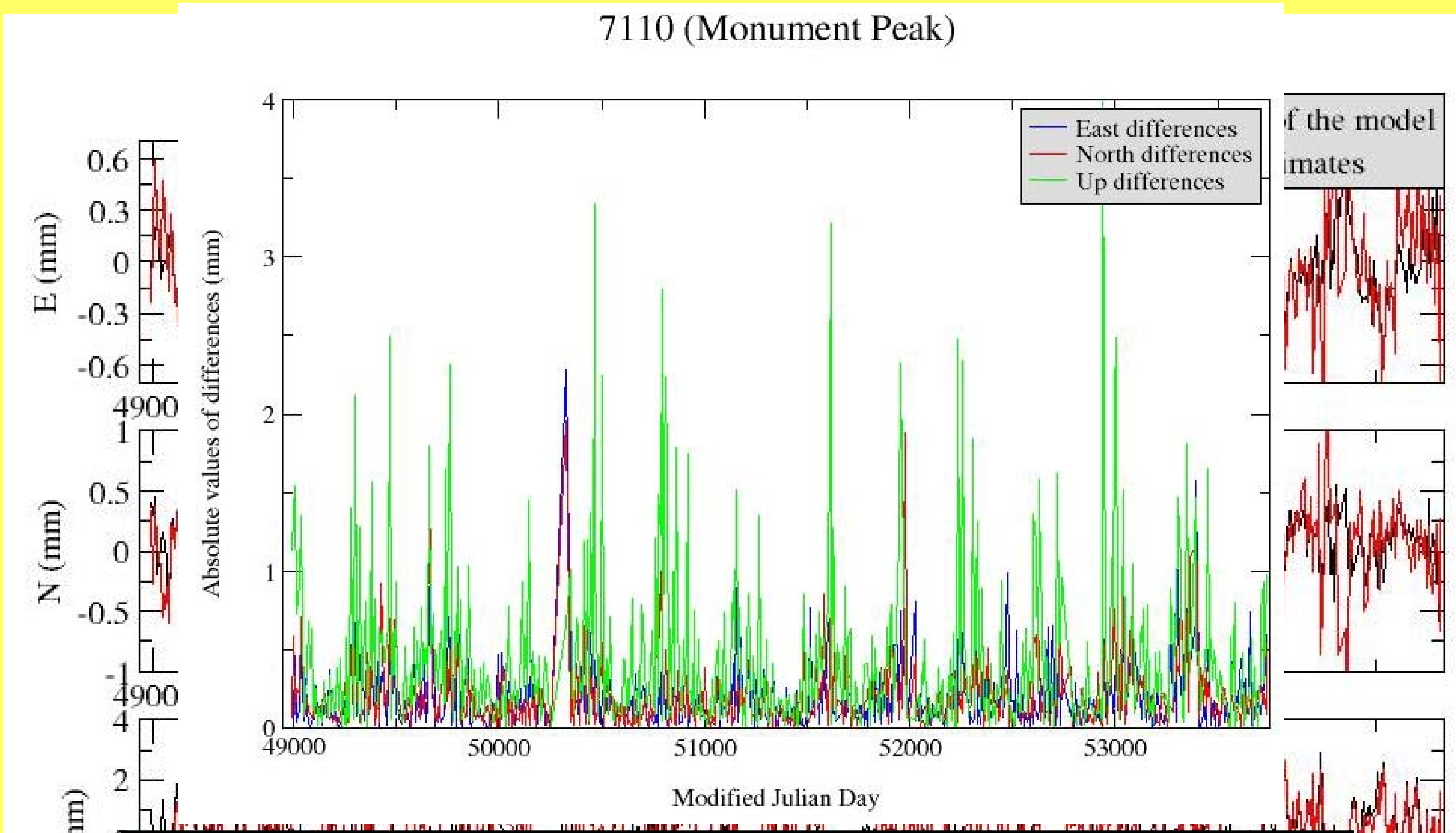


!! Real orbits and real SLR measurement times are used in simulations !!
!! Estimated station position time series contain atmospheric loading signals !!

Atmospheric loading effects are derived from the ECMWF pressure fields http://www.ecmwf.int .

Least square mean effect

Numerical examples: results of simulations



Values (mm)	Minimum	Maximum	Average	RMS
East	$1.57\ 10^{-4}$	2.28	0.19	0.21
North	$3.87\ 10^{-4}$	1.96	0.19	0.22
Up	$3.14\ 10^{-5}$	4.49	0.42	0.51

Alternative models

Periodic series

Model used $\varphi(t) \cong \sum_{i=1}^{n} a_i \cos(\frac{2\pi}{T_i} t) + b_i \sin(\frac{2\pi}{T_i} t)$

$(T_i)_{i=1,n}$ **= characteristic periods of studied signal**

New parameters = sets of coefficients $(a_i)_{i=1,n}$ $(b_i)_{i=1,n}$
for each positioning component

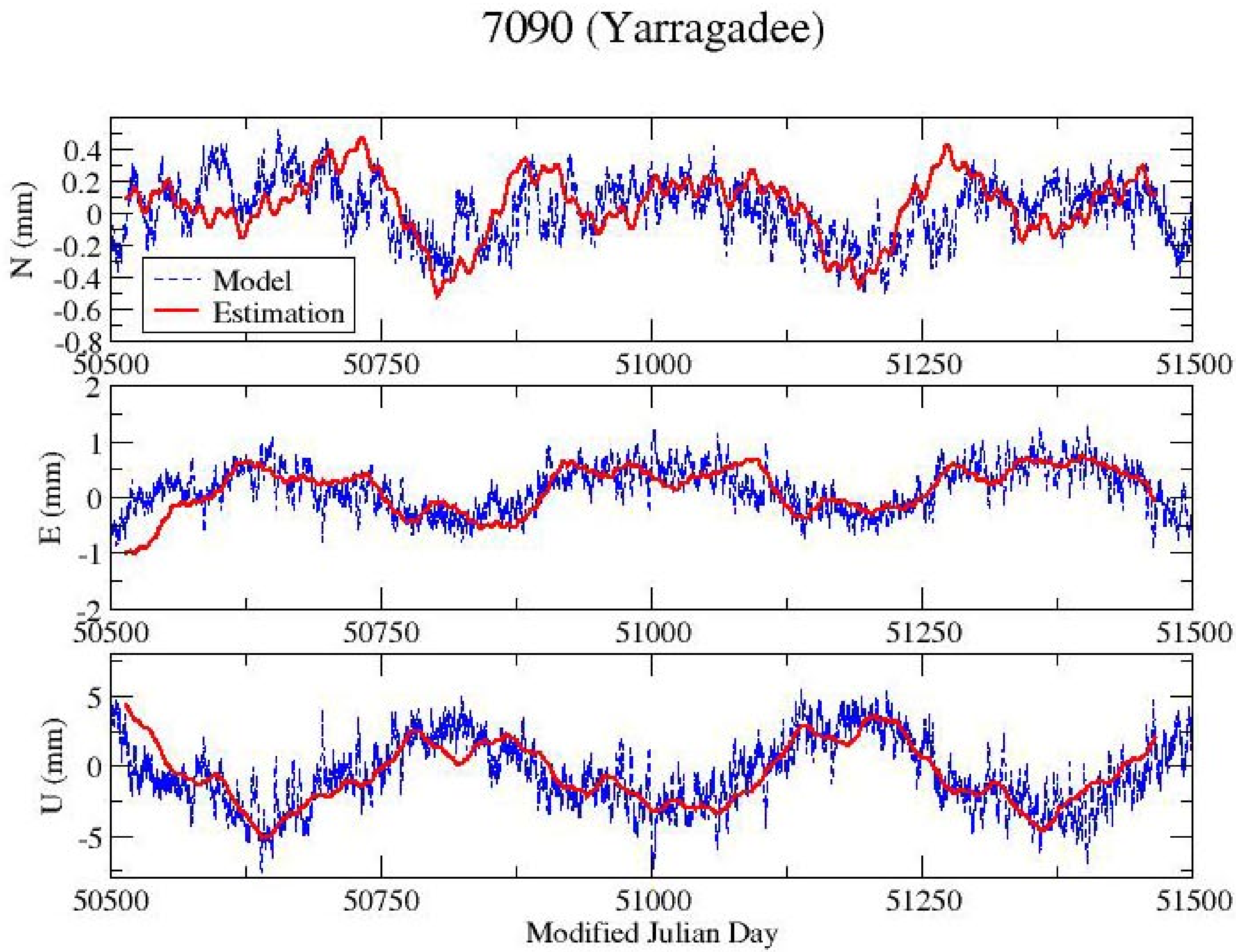
Advantage : no sampling a priori imposed

BUT

- **minimal period allowed imposed by measurements**
- **knowledge of characteristic periods ?!**
- **« discontinuities » of physical signals (earthquakes, seasonality, etc.)**

Alternative models

Periodic series



Alternative models

Wavelets

Model used $$\varphi(t)=\sum_{j=-j_1}^{j_2}\sum_{n=0}^{n_{\max}} a_{j,n}\psi_{j,n}(t)$$

where $$n_{\max}=\begin{cases}2^j-1 \text{ if } j<0\\ 0 \text{ if not}\end{cases}$$ ***and*** $$\psi_{j,n}(t)=\frac{1}{\sqrt{2^j}}\psi\left(\frac{t-2^j n}{2^j}\right)$$

with ψ ***Haar wavelet***

$$\psi(t)=\begin{cases}1 \text{ if } 0\le t<\frac{1}{2}\\ -1 \text{ if } \frac{1}{2}\le t<1\\ 0 \text{ if not}\end{cases}$$

New parameters = sets of coefficients $a_{j,n}$

Advantages : no sampling a priori imposed
discontinuities can be taken into account
representation in time and in frequency

Alternative models

Wavelets

Whatever the model used :

→ For a computation over the global network, we need to guarantee the homogeneity of the involved Terrestrial Reference Frame.

→ We can take the opportunity of this global computation to derive geodynamical signals from global parameters.

New model for SLR data processing

General considerations

This model must allow us to compute together EOPs, station positions in a homogeneous reference frame and weekly Helmert's transformations between weekly TRFs and this reference frame

Classical approach

Observation system : Y=A.δX

Weak or minimum constraints

Y: pseudo measurements or a priori residuals
A: design matrix (partial derivatives)
δX: updates of parameters (mainly EOPs and station positions)

Weekly esti

Helmert's transformation

Solutions :
- Station positions in the a priori reference frame (ITRF2000)
- Coherent EOPs
- Transformation parameters between the weekly TRF and the a priori reference frame

Goal of the new model = to obtain all these parameters in a unique process and directly at the measurement level

New model for SLR data processing

General considerations

New approach

Observation system : $Y=A.\delta X$

$\delta X = \delta X_C + T + D X_0 + R X_0$

$\delta EOP = \delta EOP_C + \varepsilon R_{\{X,Y,Z\}}$

New parameters to be estimated

Theoretical considerations and numerical tests for SLR technique
→ We do not need rotations

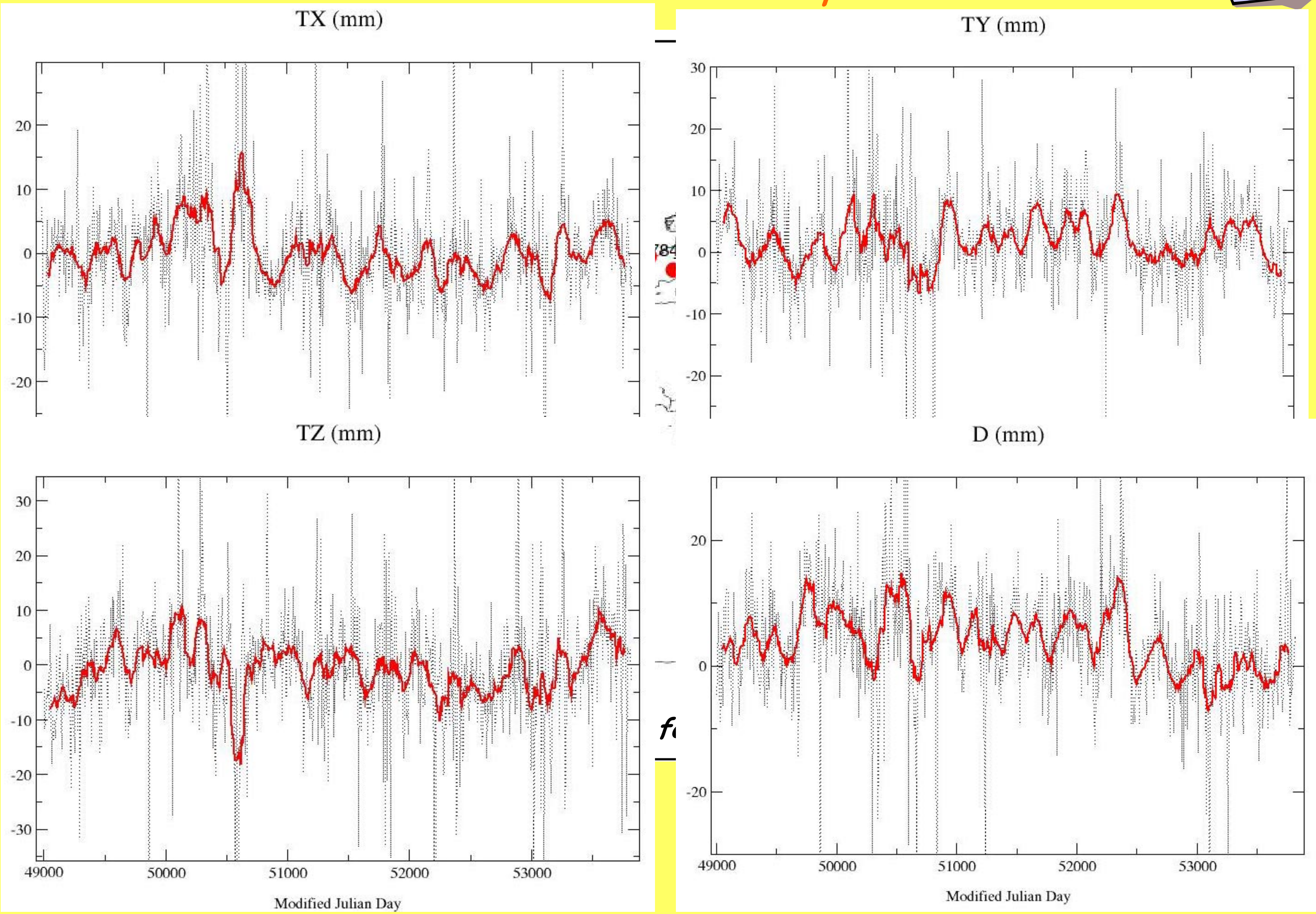
→ Rank deficiency of weekly normal matrices so obtained = 7
→ 7 = 3 (physical orientation not defined)
+ 4 (estimation of the parameters T and D)
= definition of the TRF underlying the estimated δX_C

→ New Observation system : $Y=A'.\delta X'$
with $\delta X'=(\delta EOP_C, \delta X_C, T_X, T_Y, T_Z, D)^T$

The weekly TRF underlying the δXC is defined by minimum constraints with respect to ITRF2000 with a minimum network

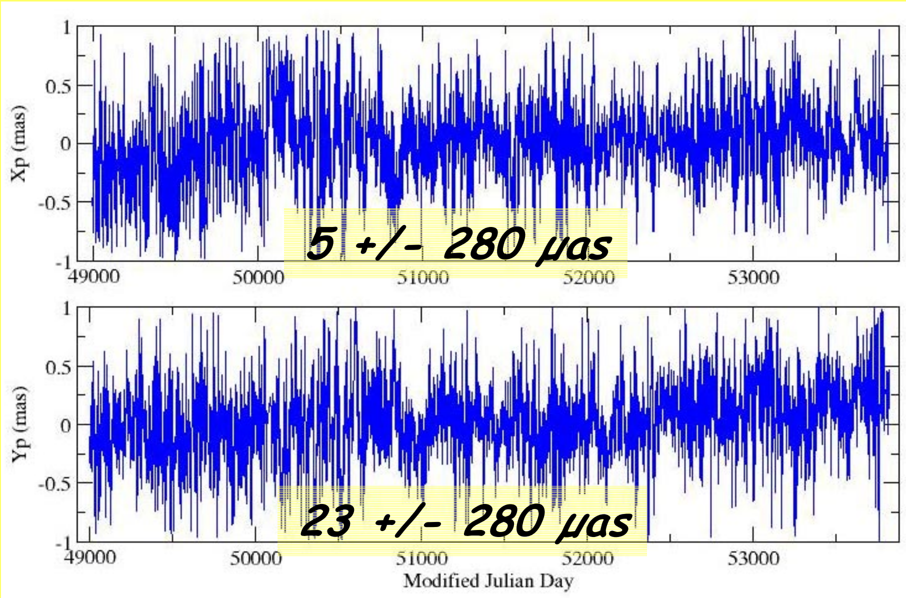
New model for SLR data processing

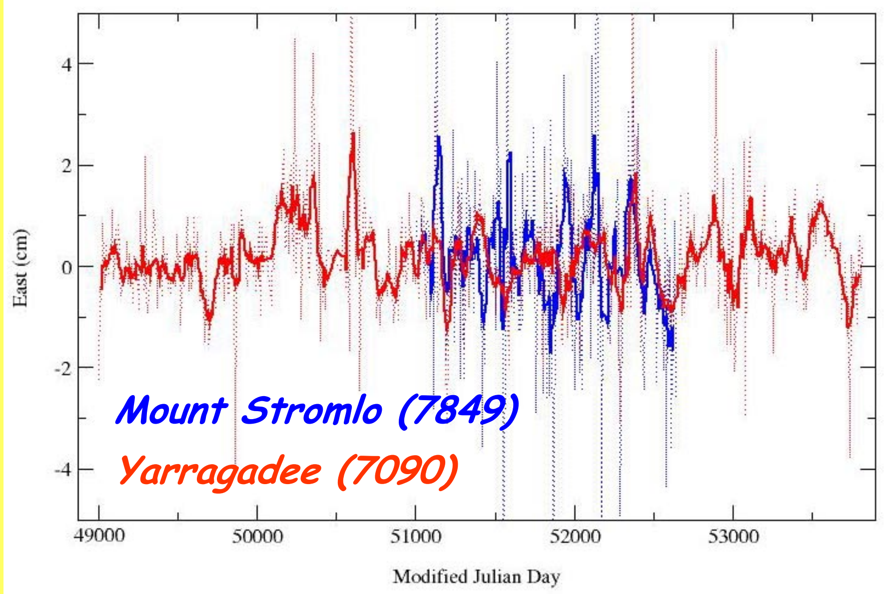
First results : transformation parameters

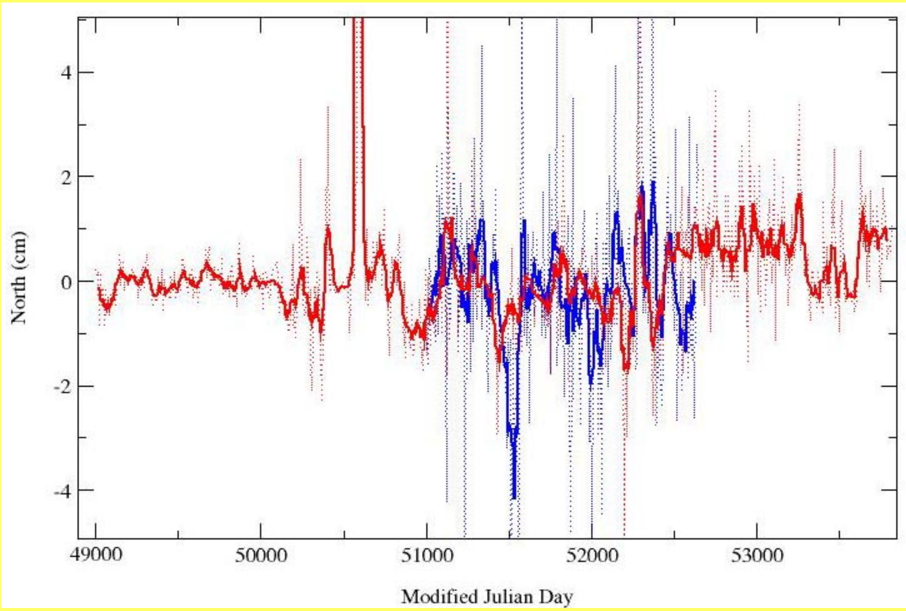


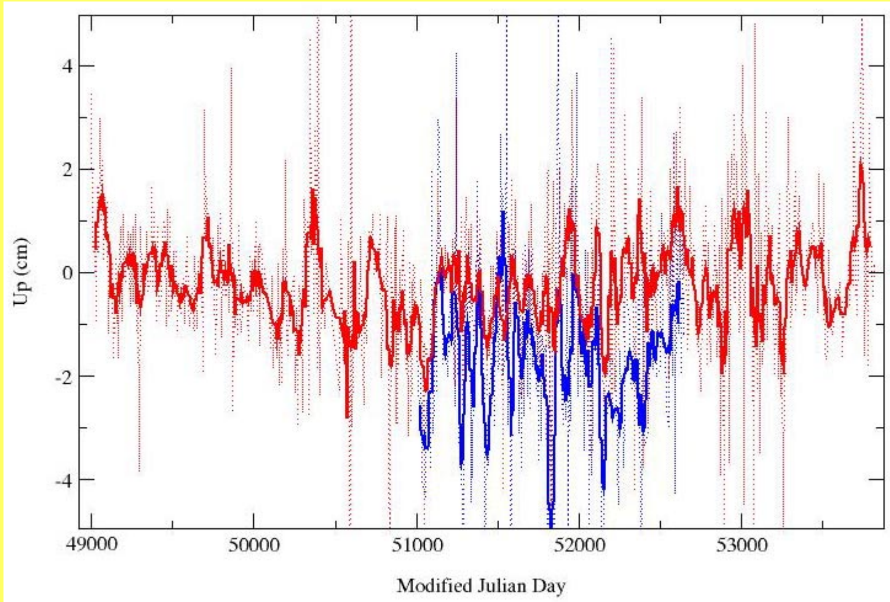
New model for SLR data processing

First results: EOP and station position time series









New model for SLR data processing

Towards global estimations over a long period

How to use this new model to reduce least square mean effect ?

Observation system : $Y=A.\delta X'$ → $\delta X = \underline{\delta X_C} + \underline{T} + \underline{DX_0}$

$\delta EOP = \underline{\delta EOP_C}$

For each parameter δZ, we can use the model

$$\delta Z(t) = \underline{\delta Z_0} + \Sigma_i\, [\underline{az_i\,(t)}\cos(2\pi t/T_i) + \underline{bz_i\,(t)}\cos(2\pi t/T_i)]$$

But

→ Each harmonic estimated for station positions creates an additional Rank deficiency → Generalization of minimum constraints

→ The number of parameters involved is large (several tens of thousands) → Manipulation of large normal systems

New model for SLR data processing

Towards global estimations over a long period

A first experiment …

Computation of the amplitudes of annual signals for the three translations and the scale factor

TX : 2.1 mm
TY : 3.6 mm
TZ : 1.1 mm
D : 0.9 mm

Furthermore, frequency analyses show the disappearance of the annual frequency in the weekly parameters estimated with respect to the annual harmonics.

Prospects

Generalization of the « periodic » model

→ *Global parameters + station positions*
→ *Harmonics linked to the oceanic tides ?*
→ *Diurnal and semi-diurnal signals on EOPs ?*

Coupling of periodic series and wavelets to get a more robust model

Stochastic approaches ?