Adjustment of EOP and gravity field parameters from SLR observations

Mathis Bloßfeld, Horst Müller, Detlef Angermann

ABSTRACT

Satellite Laser Ranging (SLR) provides the potential to estimate consistently station positions, Earth Rotation Parameters (ERPs) and gravity field parameters of low degree and order. Additionally, parameters which are related to the satellites orbit like the Keplerian elements or empirical accelerations could be estimated within one common adjustment. Since there are high correlations of these parameters among each other, the combined adjustment is a big effort. Although SLR provides highly accurate measurements of the first derivative of UT1-UTC, the Length-of-Day (LOD), the correlation between LOD and the ascending node Ω distorts the estimated parameters systematically. The estimated UT1-UTC values show a significant drift relative to the International Earth Rotation and Reference Systems Service (IERS) 08 C04 time series which is not strictly linear over time.

In this study we quantify the systematic effects on the estimated UT1-UTC values using observations of the satellites LAGEOS 1 and LAGEOS 2. Furthermore, we discuss how the high correlations could be reduced by firstly using longer arc lengths or secondly including observations to more than one satellite in the solution. The gained values of UT1-UTC are validated w.r.t. the IERS 08 C04 time series. Within the solution, gravity field parameters of degree and order two are estimated. For validation, the estimated C_{20} coefficients are compared to a time series of the Center for Space Research (CSR).

1 Correlation factors of LOD, Ω and C_{20}

Satellite Laser Ranging (SLR) is the primary technique to estimate consistently station positions, ERPs and orbit parameters of the satellites together with the spherical harmonics of low degree and order of the Earth gravity field. The big effort of the consistent estimation are the high correlations of the satellite-dependent parameters like Keplerian elements or empirical accelerations, the first derivative of UT1-UTC, called length of day (LOD), and the gravity field parameter C_{20}. The relationship between the ERPs and the orbital elements are given in equation (1) [Rothacher et al., 1999].

$$(UT1 \,\dot{-}\, UTC) = -LOD = -(\dot{\Omega} + \cos i \cdot \dot{u})\rho^{-1} \tag{1}$$

The rate of change of the argument of latitude $\dot{u}$ of a satellite is calculated by $\dot{u} = \dot{\omega} + \dot{M}$ with $\dot{\omega}$ being the rate of change of the argument of perigee and $\dot{M}$ being the rate of change of the mean anomaly. ρ is the ratio of universal time to sidereal time ($\rho \approx 1.0027379$). The secular rate of change of these quantities is caused inter alia by (i) the even zonal spherical harmonics C_{nm} with $n, m = 2,4, \dots$, (ii) by the sine term of the cross-track empirical acceleration or (iii) by relativistic effects like the *Geodetic Precession* or the *Lense-Thirring Effect*. The secular rate of Ω due to C_{20} is calculated with [Beutler, 2005]

$$\dot{\Omega}\Big|_{secular} = \frac{3}{2}\sqrt{\frac{GM}{a_e^3}}\left(\frac{a_e}{a}\right)^{\frac{7}{2}}\frac{C_{20}}{(1-e^2)^2} cosi \tag{2}$$

Satellite-dependent variables are the semi-major axis a and the eccentricity e which define the geometry of the orbital ellipse and the inclination i which is the angle between the orbital plane and the equatorial plane. Variables of the Earth gravity field are the semi-major axis a_e and the gravitational constant multiplied by the mass of the Earth GM (the mass m of the satellite is negligible) and the spherical harmonic coefficient C_{20} which is related to the Earth oblateness.

In order to quantify the correlations, several solutions are calculated. One type of the solutions contains only observations to a single satellite (LAGEOS 1 or LAGEOS 2), whereas the other type contains observations to both satellites (multi-satellite solution). The arc length of both solution types is varied between 7 days and 28 days. The mean values of the orbit fits of the 7-day arc solutions are below 5 mm whereas the mean values of the 28-day arcs are around 1 cm. For calculating the root mean square (RMS) values, only observations to official core stations of the International Laser Ranging Service (ILRS) are considered. If the arc length of the solution is increased from 7 days to 28 days, the correlations between Ω and C_{20} are strongly decreased. Fig. 1 shows the mean correlation factors for the different solution types. On the left side of Fig. 1, the factors for single-satellite and multi-satellite solutions with an arc length of 7 days are displayed whereas the right side shows the values for single- and multi-satellite solutions with an arc length of 28 days. The mean correlation factors of the single-satellite solutions are around ±1.0, the mean correlation factors for the multi-satellite solutions are between -0.6 and 0.15, respectively. The mean inclinations of the two LAGEOS satellites ($i_{LA\,1} \approx 110°$, $i_{LA\,2} \approx 53°$) allow a decrease of the correlation factors. The most uncorrelated solution could be obtained by calculating a multi-satellite solution with observations to both satellites and an arc length of 28 days. The remaining correlation factors are then 0.15 for C_{20} and $\Omega_{LA\,1}$ and 0.3 for C_{20} and $\Omega_{LA\,2}$. These low correlation factors allow to stably estimate both parameters in one common adjustment.

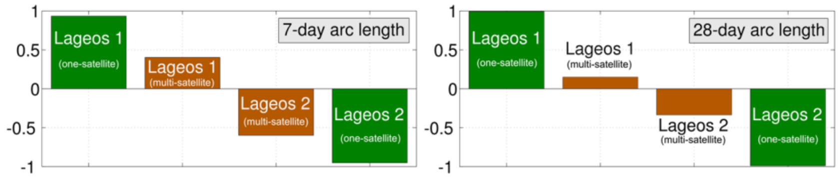


Fig. 1: Mean correlation factors of C_{20} and Ω for single-satellite and multi-satellite solutions.

2 SLR solution (1st iteration)

The DGFI SLR solutions contain various different parameter types. Tab. 1 gives an overview over the different estimated parameters. Every solution contains the station coordinates, the Earth Rotation Parameters (ERP), namely the coordinates of the terrestrial pole (x, y) and UT1-UTC, and the spherical harmonics of degree two. In order to minimize the Observed-Computed residuals, additional parameters like empirical accelerations are set up in the solution in cross-track direction and in along-track direction. The vector of the cross-track acceleration is pointing in perpendicular direction to the orbital plane, the vector of the along-track acceleration points towards the instantaneous flight direction of the satellite (tangential to the orbital ellipse). If the empirical accelerations are not estimated, the orbit fit would get much worse than it is described above.

Tab. 1: Estimated parameters within the DGFI SLR solution.

parameters	**temporal resolution (arc length: 7-day/28-day)**
station coordinates (X,Y,Z)	1 per arc (+ bias if necessary)
pole coordinates (x, y), UT1-UTC	piecewise linear polygon at 0h epochs (8/29 per arc)
spherical harmonics d/o 2 ($C_{20}, C_{21}, S_{21}, C_{22}, S_{22}$)	1 per arc
Keplerian Elements ($a, e, i, \omega, \Omega, M$)	1 per arc (starting element)
factor for solar radiation pressure	3 per arc (start, mid, end of arc)
empirical acceleration (along-track), once-per-revolution	1 per arc (sine-/cosine term)
empirical acceleration (along-track), offset	3 per arc (start, mid, end of arc)
empirical acceleration (cross-track), once-per-revolution	1 per arc (sine-/cosine-term)

The temporal resolution of the empirical accelerations is a very sensitive part in the SLR solution. Although the once-per-revolution cross-track acceleration stabilizes the estimated orbit, high correlations with the rate of change of the ascending node and therefore with LOD falsify the correlated parameters. To describe the impact of the cross-track acceleration W' on the rate of change of Ω, $W' = s \cdot sinu + c \cdot cosu$ is introduced in equation (3) [Beutler, 2005].

$$\dot{\Omega} = \frac{r \cdot sinu}{na^2\sqrt{1-e^2}sini} W' = \frac{r \cdot s}{na^2\sqrt{1-e^2}sini} + \frac{r \cdot c \cdot sinu \cdot cosu - r \cdot s \cdot cos^2 u}{na^2\sqrt{1-e^2}sini} \qquad (3)$$

The variable $n = \sqrt{GM/a^3}$ describes the mean motion of the satellite. The cross-track force W' is acting periodically (once-per-revolution) on the satellite (see Tab. 1). This perturbing acceleration causes a secular rate of change of Ω (1st part on the left side in equation (3)) and a periodical rate of change (2nd part on the left side in equation (3)). Also the temporal resolution of the along-track acceleration causes instabilities of the estimated parameters, but these effects are not discussed in this study. To prevent an impact of the cross-track acceleration on the rate of the node, the sine coefficient s is constrained to zero. The typical order of magnitude is $10^{-12} ms^{-2}$.

2.1 Earth Rotation Parameters

The parameterization of the ERPs (namely the coordinates of the terrestrial pole in x- and y-direction and the rotation angle of the Earth around its rotation axis UT1-UTC) is within all solutions the same. Since SLR is only able to determine LOD, the UT1-UTC values are extrapolated via the estimated LOD values to 0h epochs, which means that a 7-day arc solution contains eight UT1-UTC values representing a piecewise linear polygon. To eliminate the remaining degree of freedom (the offset of the polygon is not defined), the UT1-UTC value at the mid-arc epoch is fixed to its apriori value (IERS 08 C04). Because of the correlations described in the previous equations, errors or non-modeled perturbations of the satellites systematically affect the estimated LOD and the UT1-UTC polygon respectively. Fig. 2 shows the systematic drifts of weekly/4-weekly Δ(UT1-UTC) values which are accumulated over 16.5 years w.r.t. the IERS 08 C04 time series. The spurious drifts of the 7-day single-satellite solutions have an opposite sign and a specific ratio which could be explained with equation (2). Since all satellite independent parameters in equation (2) are nearly the same for both satellites the sign and ratio depend on the ratio of the satellite dependent parameters and thereby mainly on the cosine terms of the inclination. LAGEOS 1 shows a mean drift of 8.23 ms/yr whereas LAGEOS 2 shows a mean drift of -17.57 ms/yr. The ratio is -0.47. The ratio of the cosine terms of the two inclinations is -0.56. Except a small offset, the agreement is quite good. The single-satellite solutions with an arc length of 28-days are not displayed because their values are much more inaccurate than the values for the 7-day arc solution (-38.02 ms/yr for LAGEOS 1 and -26.93 ms/yr for LAGEOS 2) [Rothacher et al., 1999].

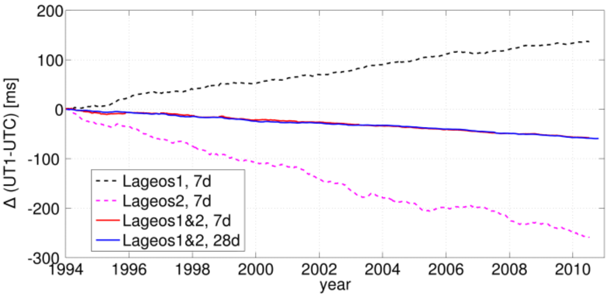


Fig. 2: Accumulated differences of Δ(UT1-UTC) w.r.t. the IERS 08 C04 time series over a time span of 16.5 years.

The mean drifts of the multi-satellite solutions in Fig. 2 are for both arc lengths nearly the same (-3.63/-3.97 ms/yr). These mean drifts are much smaller because the correlations between C_{20} and Ω are reduced significantly (Fig. 1).

2.2 Gravity field parameters

Together with the UT1-UTC values the SLR solutions contain consistently estimated gravity field parameters. The C_{20} coefficients of the two multi-satellite solutions estimated between 1994.0 and 2010.5 are displayed in Fig. 3. All other degree two coefficients are estimated but not discussed here. The 7-day solution and the 28-day solution show a good agreement with the external CSR solution although the CSR solution (monthly mean values; 2002.0 to 2010.5) contains additional observations to Stella, Starlette and Ajisai. The CSR solution is available at ftp.csr.utexas.edu/pub/slr/degree_2 (28.06.11). The C_{20} coefficients show a clear non-linear behavior. In contrast to the estimated parameters the apriori model GGM02S [Tapley et al., 2005] is parameterized as an offset with a rate.

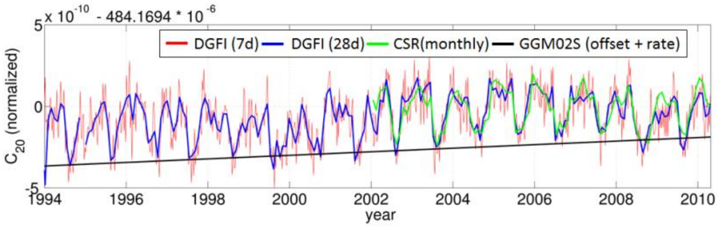
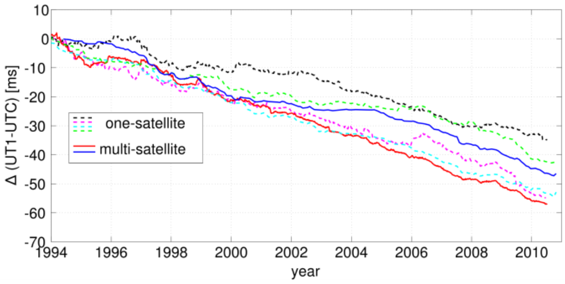


Fig. 3: Estimated normalized C_{20} coefficients of the Earth gravity field. The two DGFI solutions, the CSR solution and the apriori values (gravity field model GGM02S) are shown.

3 SLR solution (2nd iteration)

The high correlations between C_{20} and the rate of change of the ascending node (Fig. 1) cause errors in the estimated gravity field coefficients of the single-satellite solutions. Therefore, a second iteration step is performed. Within this step, the C_{20} coefficients of the multi-satellite solutions of the first iteration step are introduced as new apriori values for the single-satellite solutions. The estimated coefficients are fixed to these apriori values in order to reduce the drift of the estimated UT1-UTC values in the single-satellite solutions which result from the wrong estimated C_{20} coefficients [Rothacher et al., 1999]. The results for the accumulated Δ(UT1-UTC) values are displayed in Fig. 4. The drifts of all single-satellite solutions (LAGEOS 1 and LAGEOS 2, 7-day arc and 28-day arc) are reduced significantly and are now nearly the same as the drifts of the multi-satellite solutions. The mean values for these drifts are now between -2.8 ms/yr and -3.9 ms/yr. Nevertheless there is still a remaining drift in all solution types. This main part of this remaining drift is caused by neglecting the relativistic corrections due to the *Geodetic Precession* and the *Lense-Thirring Effect* [Ciufolini, 2004], which are, at the moment, not



modeled within the SLR solution discussed in this paper. The sum of these two drifts is approximately -3.2 ms/yr and therefore in good agreement with the remaining drifts.

Fig. 4: Accumulated differences of Δ(UT1-UTC) w.r.t. the IERS 08 C04 time series.

4 Conclusions

Within the DGFI SLR solution, station coordinates are consistently estimated together with ERPs and spherical harmonics of the Earth gravity field. This combined adjustment provides the opportunity to study the correlations between the different parameter types. For instance the correlation between C_{20}, Ω and LOD plays a very important role. To reduce the correlation of these three parameters, different solution types were calculated. The multi-satellite solution with an arc length of 28 days shows the smallest correlation factor. In the first of two iterations the observations to LAGEOS 1 and LAGEOS 2 were combined in order to get a stable estimation of the C_{20} coefficients. These coefficients were introduced in the second iteration step as apriori values for the single-satellite solution to reduce the spurious drifts of the estimated UT1-UTC values within these solutions in order to proof that the main drift in the single-satellite solutions is caused by a wrong estimated C_{20} coefficient. At the end a small drift in the accumulated Δ(UT1-UTC) values remains in all solution types. This drift is related to the not modeled relativistic corrections due to the *Geodetic Precession* and the *Lense-Thirring Effect*.

Acknowledgements

The previously presented results are worked out in a project funded by the German Research Foundation DFG within the research group “Earth rotation and global dynamic processes”. The authors want to thank the DFG for making the work in this project possible. The authors also want to thank Univ.-Prof. Dr. phil. nat. Urs Hugentobler from the Technical University of Munich (TUM) for various very fruitful discussions.

References

Beutler, G.: Methods of Celestial Mechanics; Vol. 1: Physical, Mathematical and Numerical Principles & Vol. 2: Application to Planetary System, Geodynamics and Satellite Geodesy, Springer-Verlag, Berlin Heidelberg, 2005

Bloßfeld, M., Müller, H., Seitz, M., Angermann, D.: Benefits of SLR in epoch reference frames, Proceedings of the 17th International Workshop on Laser Ranging, Bad Kötzting, Germany, May 16-20, 2011

Ciufolini, I., Pavlis, E.C.: A confirmation of the general relativistic prediction of the Lense-Thirring effect, Letters to Nature, Vol. 431, 958-960, doi: 10.1038/nature03007, 2004

Rothacher, M., Beutler, G., Herring, T. A., Weber, R.: Estimation of nutation using the Global Positioning System, Journal of Geophysikal Research, 104(B3), 4835–4859, doi: 10.1029/1998JB900078, 1999

Tapley, B., et al.: GGM02 – An improved Earth gravity field model from GRACE, Journal of Geodesy, Vol. 79, Nr. 8, 467-478, DOI: 10.1007/s00190-005-0480-z, 2005

Correspondence

Mathis Bloßfeld, Deutsches Geodätisches Forschungsinstitut, http://www.badw.dgfi.de
Alfons-Goppel-Straße 11, 80539 München

blossfeld@dgfi.badw.de