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Model of the Moon orbital and rotational motion developed by Paris Observatory Lunar Analysis Center (POLAC).

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Abstract. *We report the current development of a numerical solution of the differential equations governing the orbital and rotational motion of the Moon carried out at SYRTE in Paris Observatory by POLAC. This numerical solution will serve two main purposes. The first one is to enable tests of General Relativity (GR) with Lunar Laser Ranging (LLR) data at POLAC. The second one is to improve the existing semi-analytical solutions of Moon motion, namely the ELP ephemeris (Ephéméride Lunaire Parisienne) for the Moon orbital motion and the Lunar libration model of M. Moons for the Moon rotational motion.*

1 – Introduction.

Nowadays, LLR is the most accurate method to measure the Earth-Moon distance. The observable is a normal point based on several round-trip light times between their emission by a LLR station, their reflection by one of the retro-reflectors on the surface of the Moon and their detection back on Earth. Although the current internal precision of each LLR measurement is sub-centimetric, their link to the Earth-Moon distance by the current LLR reduction processing is performed within a few centimeters of accuracy. The very high quality of LLR measurements (over the distance, it represents a fractional accuracy of one part over 10^{11}) led scientists to improve theories of fundamental and gravitational physics. To put this into perspective, the most important periodic term over the Earth-Moon distance is due to the relativistic interaction between the ponctual masses of the Moon and the Earth which reaches an amplitude of 1 m, widely above the sub-centimetric precision. Therefore, LLR can provide a very good test for GR (see [12] for a non-exhaustive list of possible tests). In order to enable GR tests at POLAC, a new numerical solution for the orbital and rotational motion of the Moon has been implemented and will be fit on real LLR data for different theories of gravitation.

In section 2, we introduce the mathematical modeling of the orbital and rotational motion of the Moon and bodies in the Solar System as well as their numerical integration. In section 3, this numerical solution is compared with the INPOP10e Moon's ephemeris. In the last section we present the ongoing work concerning tests of alternative theories of gravitation and the improvement of ELP.

2 – The mathematical model and its numerical integrated solution.

Our mathematical model is based on the dynamical equations governing the orbital and the rotational motion of the planets, the Sun, the Moon, Pluto and 50 of the biggest asteroids including contributions from point-mass interactions, figure effects, tides and Lunar physical librations. Each contribution is described in the following. Our solution consists of calculating numerically the positions and velocities of Solar System bodies as well as the Moon libration angles. These integrations are achieved thanks to ODEX integrator [7] in quadruple precision. The numerical error has been evaluated to 11 μm on the Earth-Moon distance over a timespan of 100 years with a 10^{-20} error tolerance.

2.1 – Point-mass interactions.

The numerical integration is made in the International Celestial Reference Frame (ICRF). The origin of our frame is centered at the Solar System Barycenter (SSB) and has no relative rotation with distant extragalactic sources like quasars. The position and velocity of the SSB are fixed at the origin of the frame at the beginning of the integration, using the invariant mass/energy quantity of the *n*-body metric [3]. Then, using a Parametrized Post Newtonian (PPN) n-body metric we deduce accelerations at the relativistic order $O(c^{-4})$ [10, 14] due to the mutual interactions between all the point-mass bodies.

2.2 – Figure potential.

The Moon, the Sun and the Earth can not be considered only as point-mass bodies because the contribution of their figure potential leads to an additional acceleration, above the centimeter level, on the orbital motion of bodies. From the formulation of Kaula (see [9] Eq. (1.31)), we use spherical harmonics to describe their figure potential. We consider an expansion up to degree 4 in zonal harmonic for the Earth, up to degree 4 in zonal, sectoral and tesseral harmonic for the Moon and degree 2 in zonal harmonic for the Sun. As we are computing accelerations in ICRF frame, we need to orientate the Earth, the Sun and the Moon in this frame. The Earth orientation is forced through a precession/nutation modeling based on the IAU-routines of *Standards Of Fundamental Astronomy (SOFA)* [15]. The orientation of the Sun is fixed relative to ICRF [4]. Finally for the Moon, we integrate its rotational equations of motion, as described in section 2.4.

2.3 – Tides and spin.

For an accurate computation of the orbital motion of the Moon, we need a higher degree of modeling. Indeed, since the Earth and the Moon are close to each other and their rotational motion are fast, we have to take into account distortions raised upon these two extended bodies. These distortions lead to an additional acceleration on the orbital motion of bodies. The distortions are from two kinds. The most important one is caused by tidal effects, because of the presence of other point-mass bodies. The second one is the spin distortion caused by the variation of the angular velocity vector of the extended body. This one is only computed for the Moon, since orientation of Earth is forced as seen in section 2.2.

In both case, to get a more accurate modeling, distortions are evaluated considering anelastic bodies. Since anelastic body does not react immediately to a perturbation, there is a time-delay in its reaction because of friction inside it, leading to dissipation. Therefore, to take into account this dissipation for tides, we introduce a phase lag between the position of a perturber and the direction of the tidal bulge. For the spin velocity vector, dissipation is considered computing the angular velocity vector at time *t* minus the time delay.

Then, in our software, distortions induce variations in second degree harmonics of extended bodies. Subsequently, the impact of extended bodies on the orbital motion of point-mass body is computed with the figure potential formalism described in section 2.2.

2.4 – Lunar librations.

We orientate the Moon with respect to the ICRF with three Euler angles. Therefore, to integrate their evolution, it is necessary to compute their acceleration which is given thanks to Euler's equation of motion. It relates the change in Moon angular velocity vector depending on Moon total moment inertia tensor, as well as external torques acting on the Moon. The Moon total moment inertia tensor is time varying since second degree harmonics of the Moon are time varying as well (see §2.3). On the other hand, external torques acting upon the Moon are from different contributions. First of all, we have implemented torque due to all point-masses, except asteroids, interacting with Moon figure. Secondly, we introduce momentum due to interaction of the figure of the Earth with the figure of the Moon [5]. Finally, we compute the geodetic precession effect upon the Moon orientation, which is given by [13].

3 – Comparison to INPOP10e.

We compare our computation describing the orbital and rotational Moon motion with INPOP10e numerical solution [6, 11]. The aim of this comparison is to validate all the steps of our model implementation. Currently, the two dynamical models are close except three main differences. Firstly, in our model, the orientation of Earth is forced whereas it is integrated in INPOP10e. Secondly, we integrate the position and velocity of the 50 biggest asteroids, while the position and velocity of 300 asteroids are computed in INPOP10e. Thirdly, INPOP10e takes into account a flat ring in order to model the remaining asteroids of the main belt which is not taken into account in our model. Integrations have been computed over 200 years centered on J2000 without any adjustment, using the physical parameters and the initial conditions provided by INPOP10e at J2000.

The left panel of figure 1 shows the differences between our numerical solution and the one of INPOP10e, for the Earth-Moon distance as a function of time. The right panel shows the distribution of these differences around its mean value.

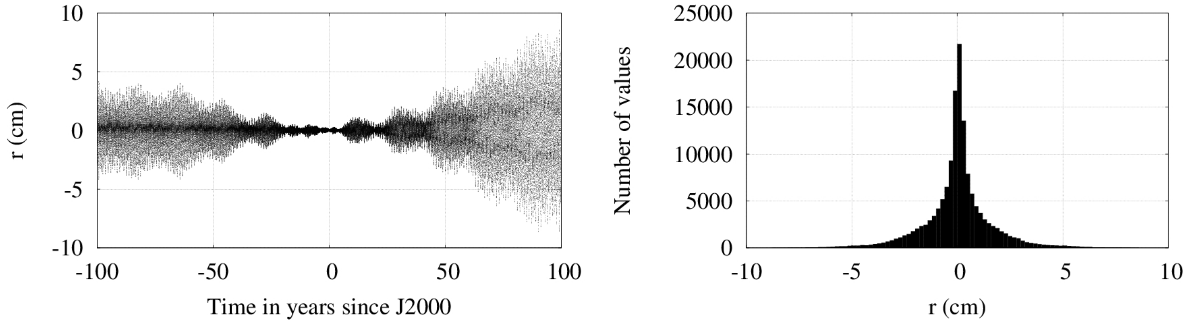

Figure 1. On the *left side plot* is shown differences (in cm) between our solution and INPOP10e for the Earth Moon distance. X axis is TDB time expressed in years from J2000. Distribution of theses differences around the mean value are plotted on the *right side plot*.

Figure 2 represents these differences for the six lunar Keplerian elements and figure 3 shows the differences for the three lunar Euler angles and their time derivatives. All the amplitudes of these differences are entirely compatible with the differences between the two models described above. It validates our current implementation and integration of our dynamical model. The next step in the development of our Moon solution, will be to fit initial conditions and physical parameters with real LLR data.

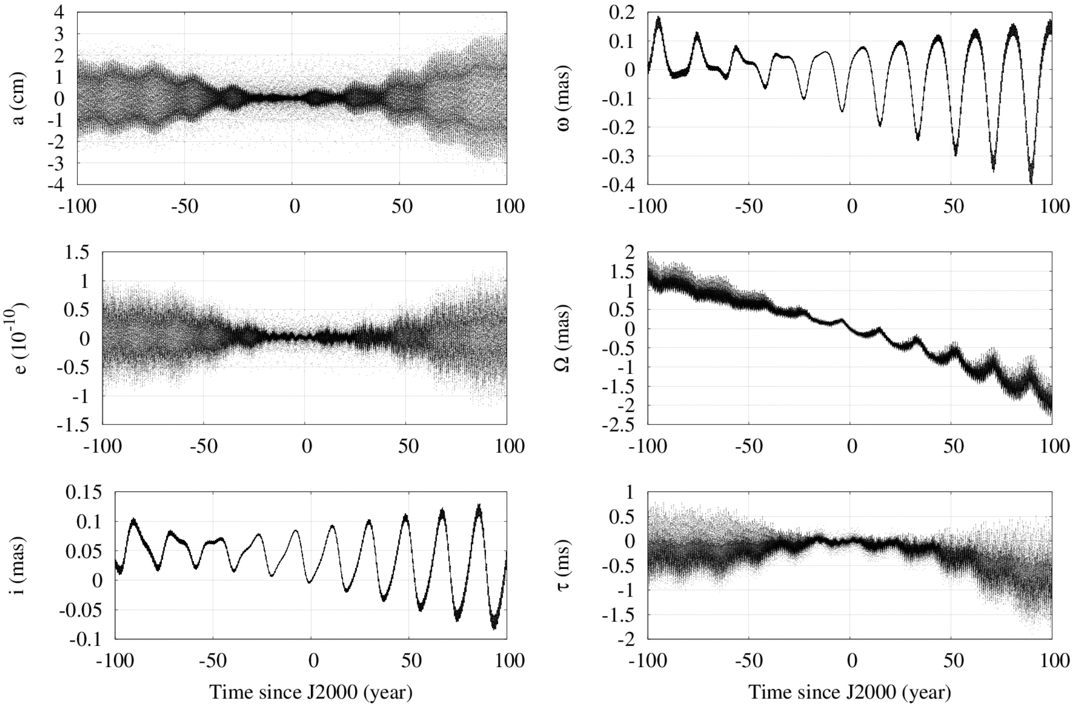

Figure 2. Differences between our solution and INPOP10e for the 6 lunar Keplerian elements. *a* is the semi major axis, *e* is the eccentricity, *i* is the inclination, *ω* is the perigee argument, *Ω* is the longitude of the node and *τ* is the time of passing perigee.

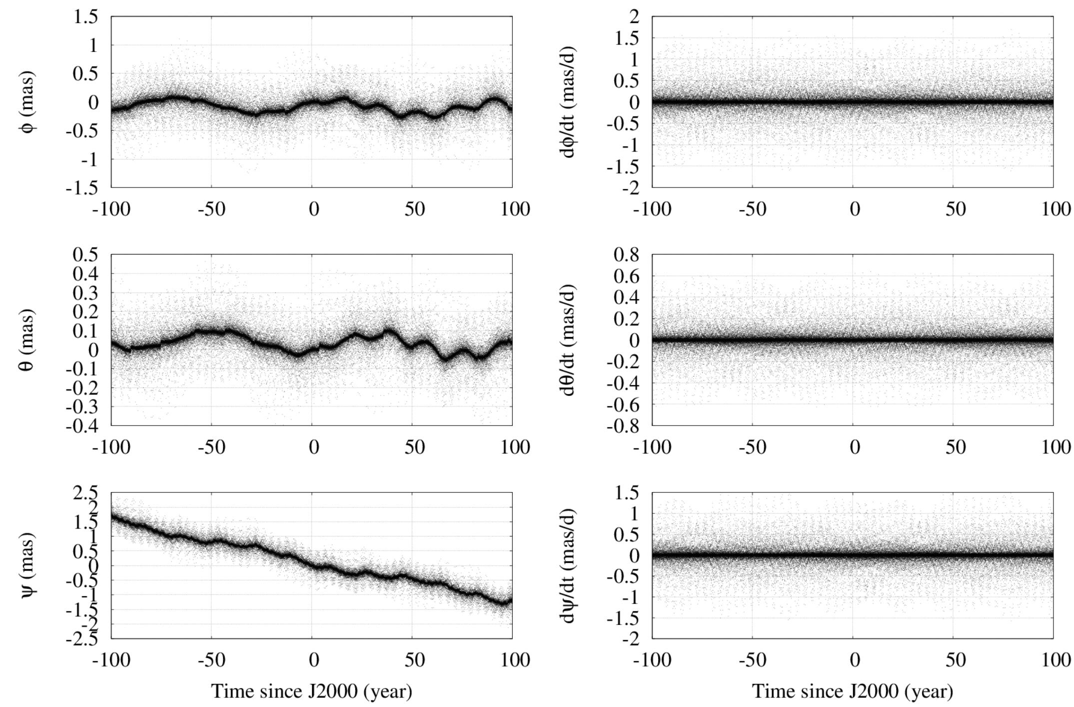

Figure 3. On the *left side plot* is shown differences (in mas) between our solution and INPOP10e for the three Euler angle (*Φ,θ,Ψ*). On the *right side plot* is shown differences (in mas/d) for the three Euler angles time derivatives. X axis is TDB time expressed in years from J2000.

Conclusion : futures applications.

The model described above and its software implementation has been developed for two main future applications. The first one concerns fundamental physics and test of GR. The second one concerns the improvement of ELP semi-analytical theory.

We focus on a new generation of software that simulates the observables from a given space-time generic metric [8]. To do that, computing the equations of motion and time transfer for light travel time is needed. The equations of motion will be directly computed in the software, whereas equations for light propagation will be implemented in the LLR reduction software. We use Post Newtonian (PN) approximation limiting ourselves to first order GR corrections. In the weak field approximation, gravitation is seen as a perturbation to the flat Minkowskian space. Brumberg shows (*cf.* [1] Eq. (2.2.49)) that the equation of motion, as a function of the coordinate time t, can be expressed only with the gravitational part of the metric tensor, and their derivatives with respect to coordinates. This flexible approach allows to perform simulations in any alternative metric theories of gravity. The output of this software will provide templates of anomalous residuals that should show up in real data if the underlying theory of gravity is not GR. Those templates can be used to give a rough estimation of the constraints on additional parameters involved in alternative theory of gravity. They also provide signals that can be searched for LLR data aimed at testing gravitational laws.

ELP is a semi-analytical solution of the dynamical equations governing the motion of Moon gravity center. This solution is developed since the 70's by M. Chapront-Touzé, J. Chapront and G. Francou and are still of great interest (*e.g.* for the study of underlying resonances or to split the different contributions). The latest solution, ELP-MPP02 [2], takes into account all effects with theoretical signal larger than 1 mm over the Earth-Moon vector. The first order of modeling is the “Main Problem” which corresponds to the case where the Earth, the Sun and the Moon are considered as point-masses with the Sun following a Keplerian ellipse around the Earth-Moon barycenter. All other effects are treated as perturbation to the Main Problem *i.e.* planetary perturbations (direct and indirect), Earth figure, GR, tide on Earth and Moon figure. The ELP solution consists of Poisson series for the geocentric lunar distance, longitude and latitude where coefficients are numerical while the trigonometric arguments are kept under literal form. In its purely semi-analytical form this solution does not reach millimetric precision. Indeed after a fitting to INPOP8, the residuals of the differences over 200 years centered at J2000 are 71 cm for the distance, 1.54 mas for the longitude and 0.56 mas for the latitude. These differences are mainly explained by the slow convergence of Poisson series (in particular for planetary effects), the truncation of series during their computation and some implicit physical parameters impossible to fit or update.

With the help of the software described above, we plan to investigate the origin of these differences by comparing effect-by-effect the ELP semi-analytical solution with the numerical solution of exactly the same theoretical model. Then we intend to reduce these differences by improving the not enough accurate Poisson series or, in in the case of too slowly convergent series, replace them by their numerical counterpart numerical.

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