

# TEST OF GENERAL RELATIVITY USING LLR DATA AND THE PLANETARY EPHEMERIS PROGRAM (PEP)

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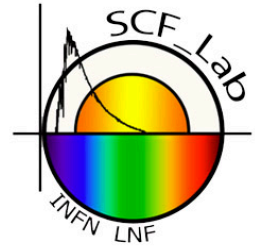
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<sup>2</sup>*Harvard-Smithsonian Center for Astrophysics*

*19<sup>th</sup> International Workshop on Laser Ranging, Annapolis October 27-31, 2014*

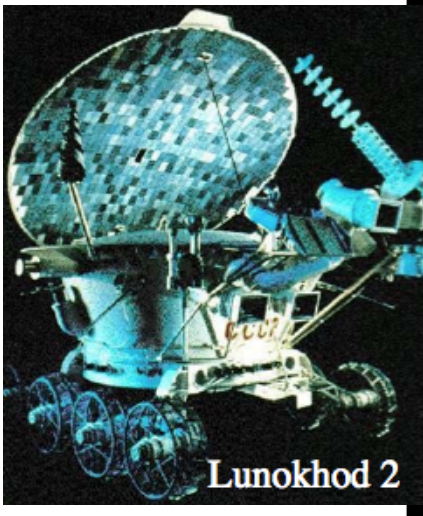
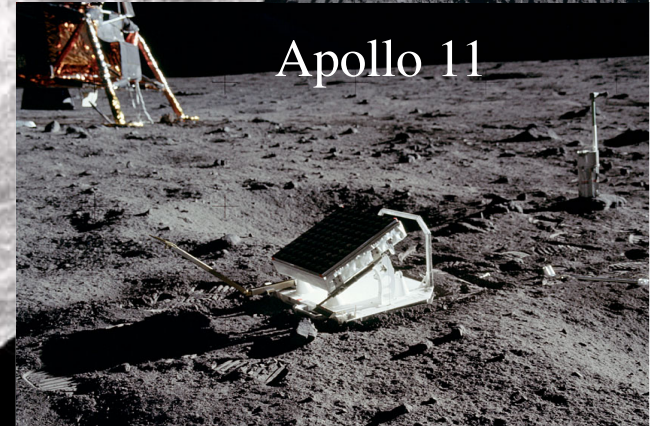
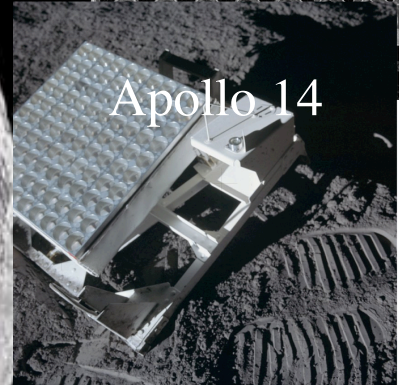
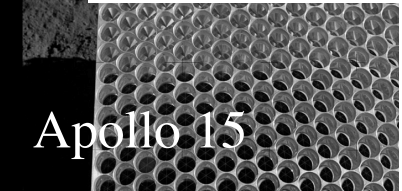
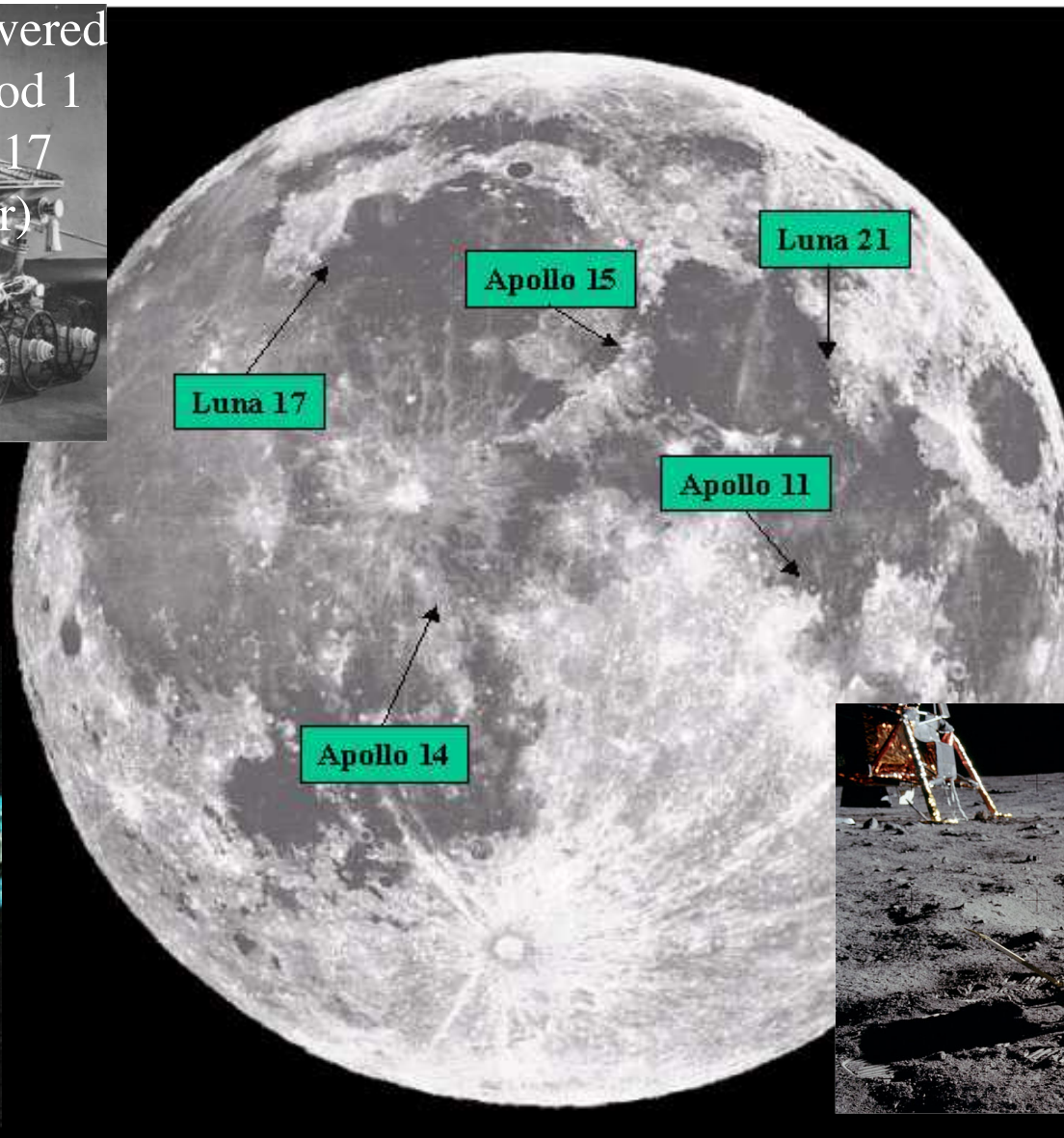
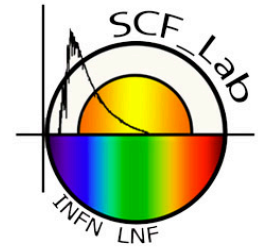
# Outline

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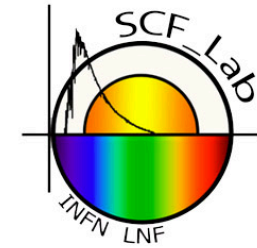


- Introduction
- Test of General Relativity
- Software package
- Data analysis

# CCRs Arrays on the Moon



# Current LLR tests of General Relativity

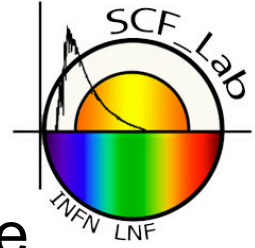


\* J. G. Williams, S. G. Turyshev, and D. H. Boggs, PRL 93, 261101 (2004)

Science measurement / Precision test of violation of General Relativity	Apollo/Lunokhod few cm accuracy*
Parameterized Post-Newtonian (PPN) $\beta$	$ \beta - 1  < 1.1 \times 10^{-4}$
Weak Equivalence Principle (WEP)	$ \Delta a/a  < 1.4 \times 10^{-13}$
Strong Equivalence Principle (SEP)	$ \eta  < 4.4 \times 10^{-4}$
Time Variation of the Gravitational Constant	$ \dot{G}/G  < 9 \times 10^{-13} \text{ yr}^{-1}$
Inverse Square Law (ISL)	$ \alpha  < 3 \times 10^{-11}$
Geodetic Precession	$ k_{gp}  < 6.4 \times 10^{-3}$

# Planetary Ephemeris Program

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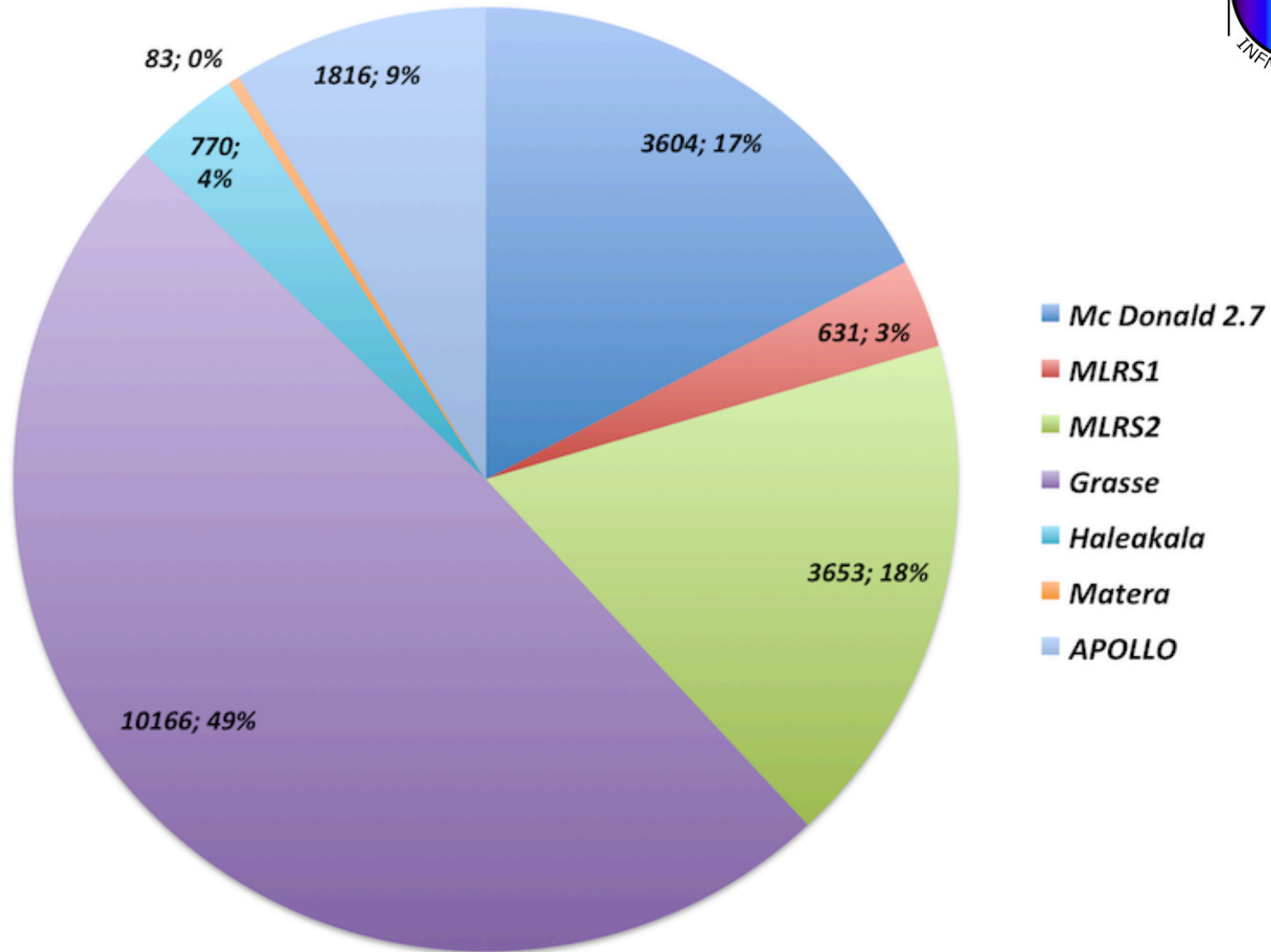
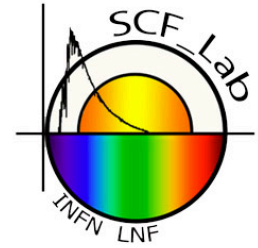
In order to analyze LLR data we used the PEP software, developed by the CfA, by I. Shapiro et al. starting from 1970s.

The model parameter estimates are refined by minimizing the residual differences, in a weighted least-squares sense, between observations (O) and model predictions (C, stands for "Computation"), O-C.

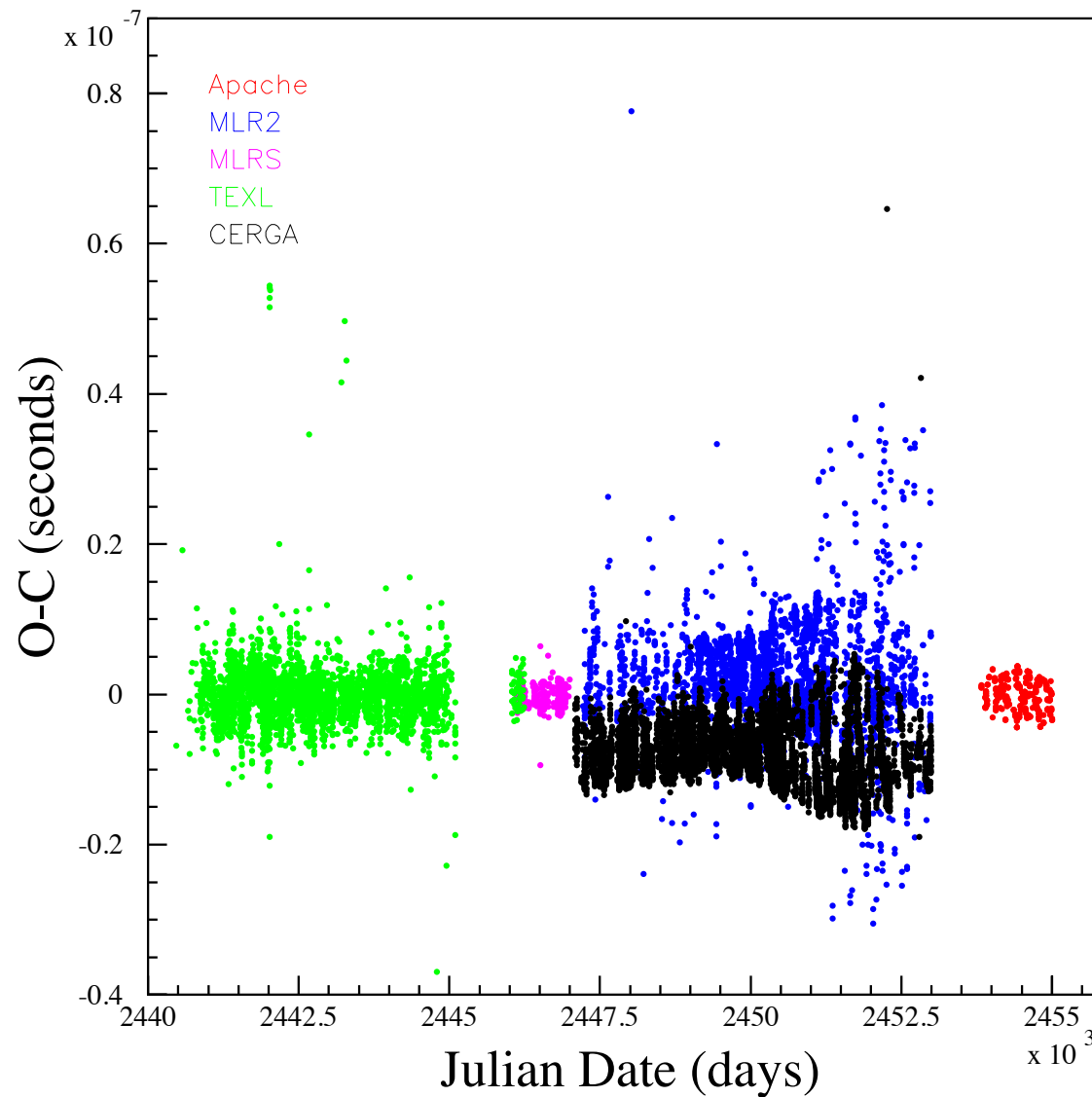
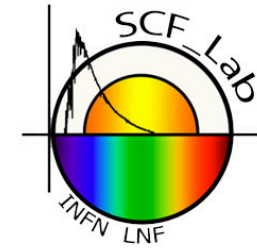
"Observed" is round-trip time of flight. "Computed" is modeled by the PEP software.

# Data Analysis LLR Normal Points

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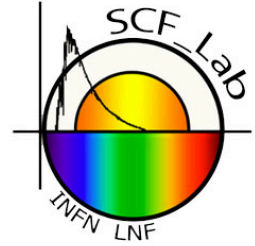


# O-C residual analysis with PEP



# Determination of $K_{GP}$

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$K_{GP}$  is the relative deviation from the value of geodetic precession expected in GR

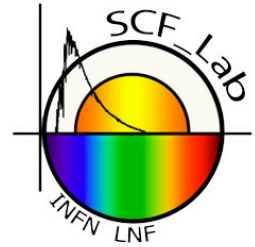
$$K_{GP} = (4.3 \pm 8.6) \times 10^{-3}$$

In this analysis  $\beta=\gamma=1$ ,  $dG/dt=0$ . Nominal errors returned by the fit are significantly smaller than the above estimated values of  $K_{GP}$ .



# Determination of $K_{GP}$

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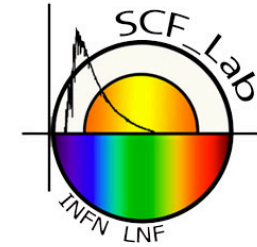


This preliminary measurement must be compared with the best result published by JPL obtained using a completely different software package

$$K_{GP} = (-1.9 \pm 6.4) \times 10^{-3}$$

Our Goals: accuracy on  $K_{GP}$  of few ‰ with current LLR data  
 **$\geq x10$  improvement possible only with MoonLIGHTs**  
PEP simulation of physics reach of new CCRs at lunar poles/  
limbs/equator

# Simulated observations



PEP can make simulated “dummy” observations.

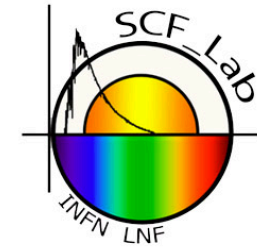
We are simulating new arrays on lunar surface.

We are simulating arrays at the pole of the Moon and we want to see how the PPN parameters change.

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```

# Simulated observations

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	2013	2016	2018	2020	2022	2025	2030
Gdot	1,59E-14	7,73E-15	5,43E-15	3,78E-15	2,74E-15	1,72E-15	1,10E-15
KGP	3,38E-04	2,10E-04	1,55E-04	1,15E-04	1,01E-04	7,83E-05	6,27E-05
beta	6,43E-04	4,16E-04	2,73E-04	2,11E-04	1,88E-04	1,49E-04	1,22E-04

## ARRAYS:

AP11-AP14-AP15-LN1-LN2  
Moon Express 65N 40W  
Astrobotic 50S 35E  
Israel 45N 27.2E

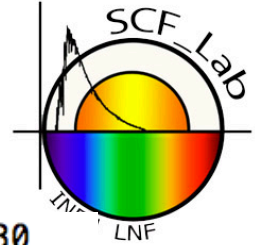
## STATIONS:

APOLLO 3-16 ps  
CERGA 7-33 ps  
MLRS 7-33 ps  
MLRO 7-33 ps

Cadence: 30 days for APOLLO, 20 days for MLRS, 14 days for CERGA, 8 days for MLRO

# Simulated observations

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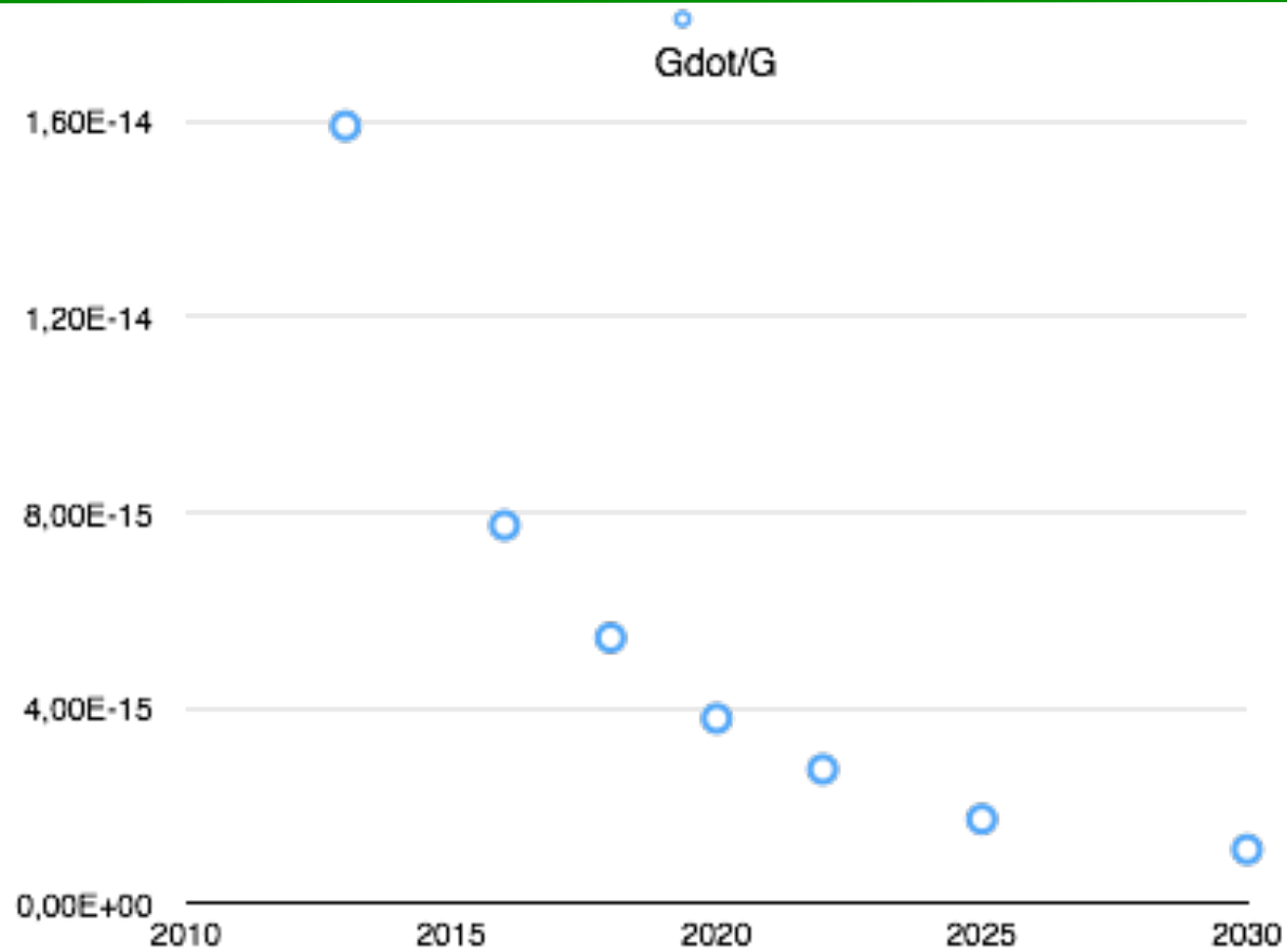
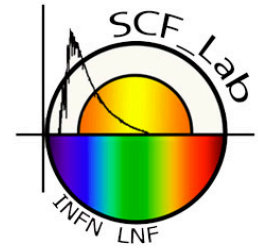
## MNEX 65N:

	2013	2016	2018	2020	2022	2025	2030
Gdot/G	1.586E-14	7.731E-15 7.663E-15	5.432E-15 5.246E-15	3.779E-15 3.593E-15	2.744E-15 2.554E-15	1.722E-15 1.582E-15	1.100E-15 9.927E-16
eta	2.550e-03	1.648e-03 1.536e-03	1.093e-03 9.707e-04	8.216e-04 7.184e-04	7.364e-04 6.264e-04	5.882e-04 4.930e-04	4.900e-04 4.157e-04
beta	6.425E-04	4.156E-04 3.861E-04	2.729E-04 2.417E-04	2.113E-04 1.854E-04	1.881E-04 1.606E-04	1.490E-04 1.255E-04	1.223E-04 1.044E-04

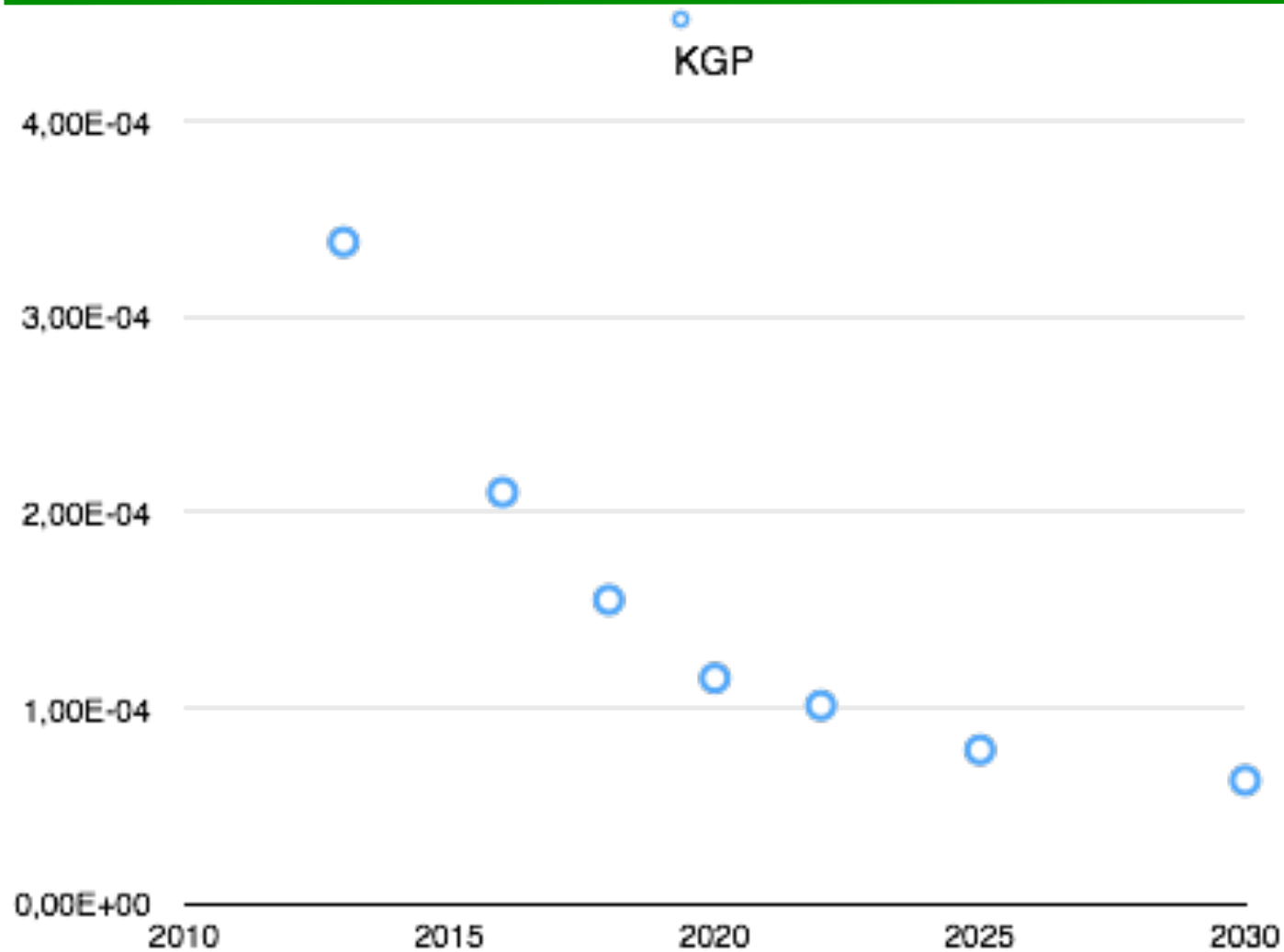
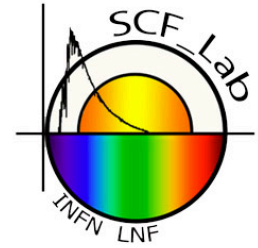
## MNEX 87N:

	2013	2016	2018	2020	2022	2025	2030
Gdot/G	1.586E-14	7.813E-15 7.703E-15	5.663E-15 5.384E-15	4.157E-15 3.711E-15	3.283E-15 2.721E-15	2.126E-15 1.688E-15	1.432E-15 1.061E-15
eta	2.550e-03	1.788e-03 1.561e-03	1.269e-03 1.007e-03	1.003e-03 7.577e-04	9.075e-04 6.655e-04	7.211e-04 5.286e-04	6.098e-04 4.536e-04
beta	6.425E-04	4.515E-04 3.916E-04	3.160E-04 2.495E-04	2.572E-04 1.949E-04	2.317E-04 1.705E-04	1.834E-04 1.349E-04	1.532E-04 1.141E-04

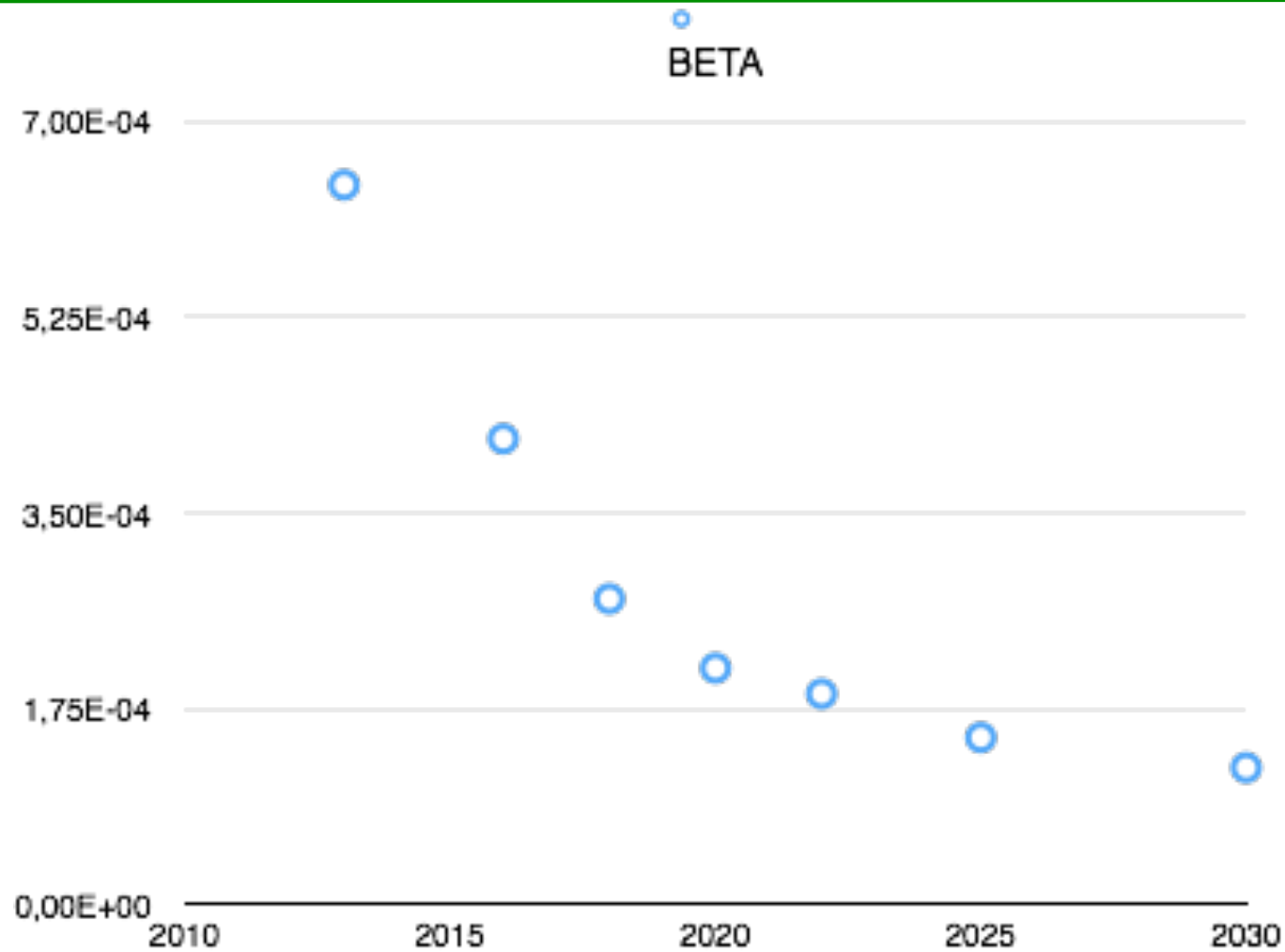
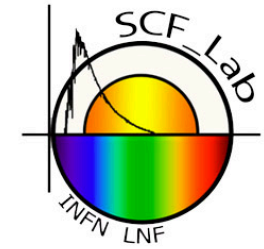
# Simulated observations



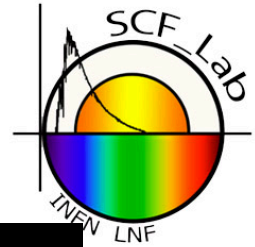
# Simulated observations



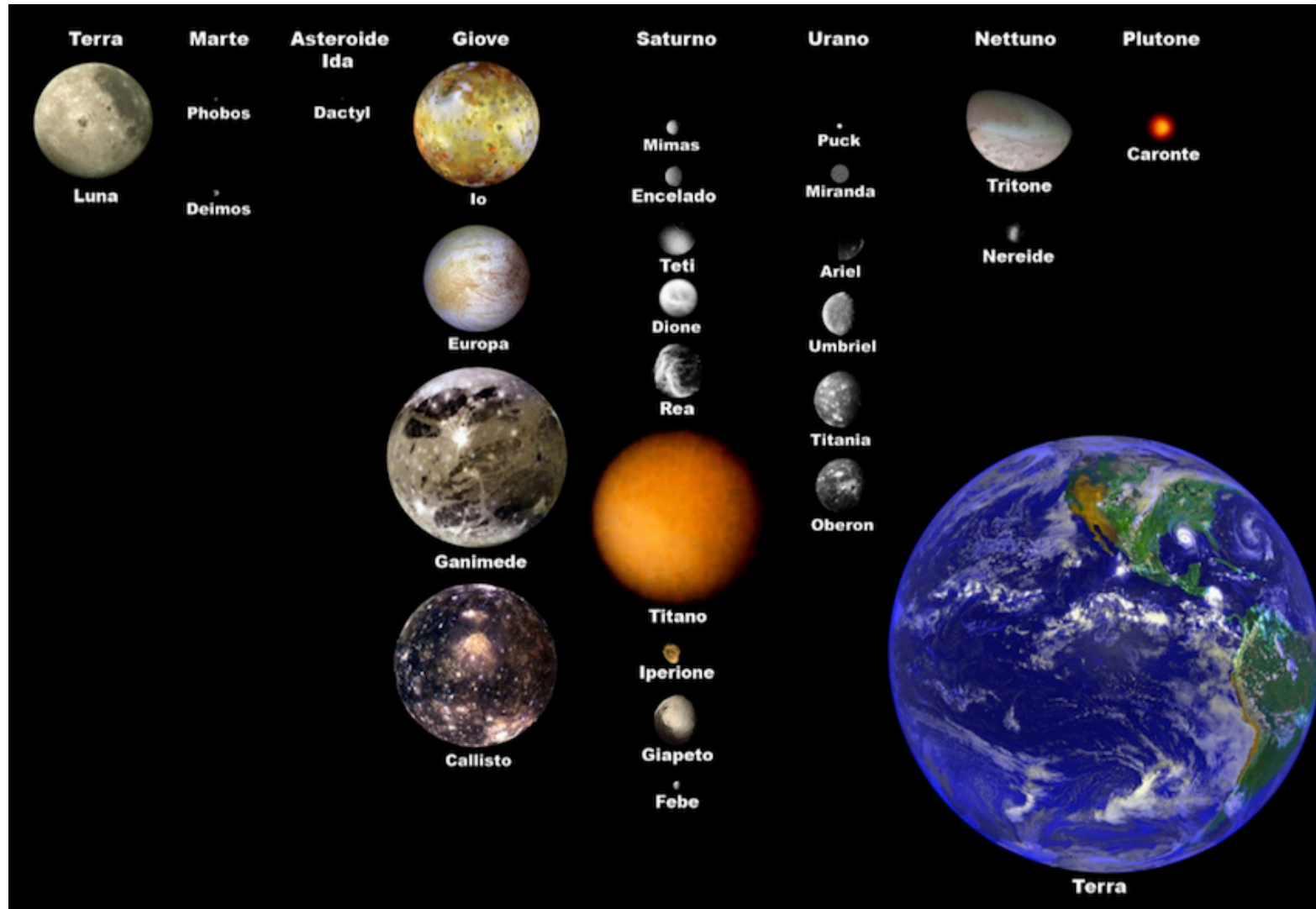
# Simulated observations



# Other Solar System applications



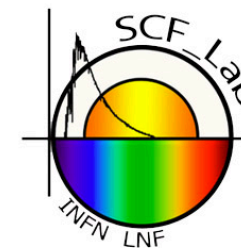
See talk by S. Dell'Agnello





# Conclusion

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- Deepen our knowledge about data and software in order to better estimate (and reduce) the measurement uncertainty on  $K_{GP}$  and on other GR parameters.
- Improve the precision of these kind of General Relativity measurements by using not only LLR data, but also SLR data to Earth satellites and primarily to LAGEOS.
- We have the option to implement the equations of motion of new gravity theories (like SPACE-TIME TORSION and NON-MINIMALLY COUPLED GRAVITY – see talk by S. Dell’Agnello) inside PEP and study not only the secular variation of the geodetic precession , but also periodic signatures of NEW PHYSICS on the geodetic precession and on other PPN parameters

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***THANK YOU  
FOR YOUR ATTENTION***

**ANY COMMENTS/QUESTIONS?**