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Relativistic corrections in the European Laser Timing (ELT) experiment

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Introduction

The European Laser Timing (ELT) experiment, which is part of the ESA mission ACES (Atomic Clock Ensemble in Space), aims at enabling picosecond time transfer between ground based clocks and the ultra-stable time scale of the ACES module aboard the International Space Station. To this end, both a classical two-way and an additional one-way optical link shall be established, both of which are based on timing via ultra-short laser pulses. For maximum timing precision, the space based ELT hardware will be equipped with a novel single photon avalanche diode (SPAD), which needs to be gated to reduce the signal-to-noise ratio to an acceptable level. To synchronize pulse transmission dates with the gating of the ELT detector, space and ground clocks shall be referred to a common time scale like UTC.

Hence, the ELT data center is required to compute the relativistic drift of the ACES clock with respect to UTC, as the payload has no access to a sufficiently accurate approximation of UTC, for example through GNSS.

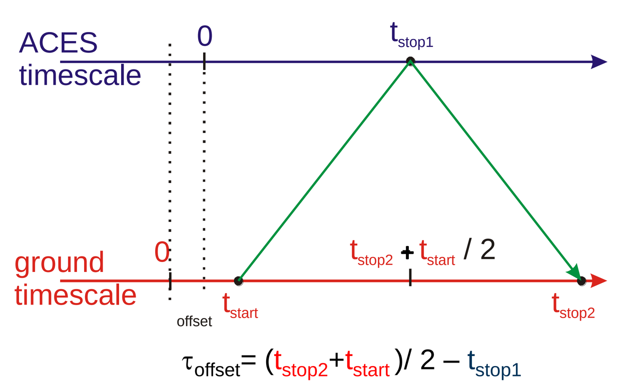


Fig.1: Principle of the ELT experiment

Relativistic effects due to moving clocks and potentials

$$\int_{\text{path}} dt = \int_{\text{path}} d\tau \left[1 - \frac{(V-\Phi_0)}{c^2} + \frac{v^2}{2c^2}\right]$$ [1]

with

- v : Satellite velocity in ECI
- c : Speed of light
- ϕ_0 : Potential of the geoid
- V : Gravitational potential of earth + tidal potentials
- dτ : Coordinate time (TT)

Gravitational potential

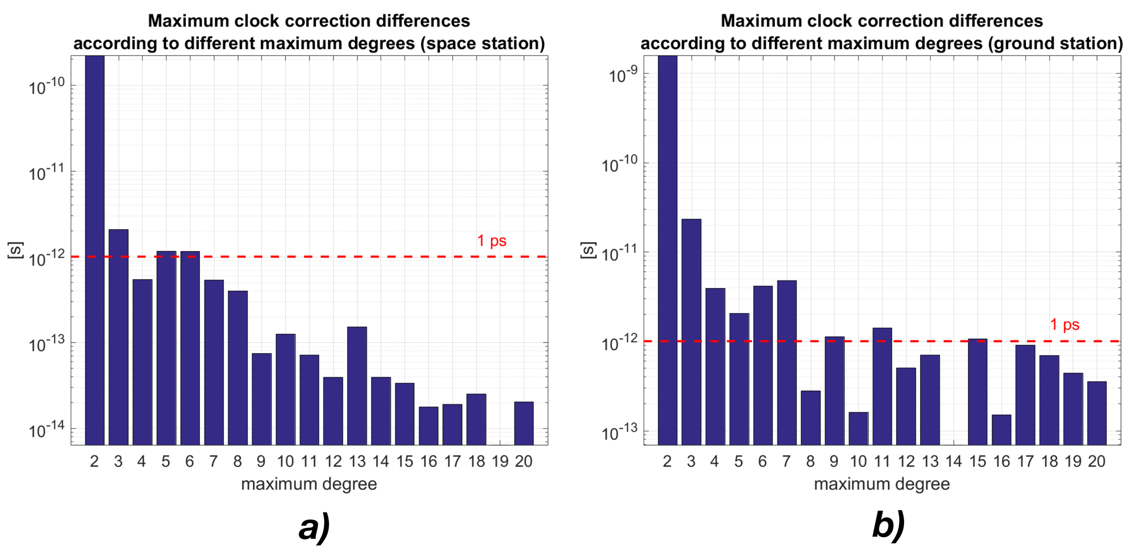


Fig.2: Maximum clock correction differences according to different maximum degrees; a) space, b) ground station.

Figure 2 illustrates the increase of accuracy for the clock corrections using a different maximum degree for the calculation of the gravitational potential. To ensure a time transfer with ps accuracy for space and ground station clocks it is necessary to calculate the gravitational potential at least until degree 15.

Sagnac delay

The Sagnac delay, which is not a real relativistic effect but often added to them, refers to pulse delay corrections. With the Sagnac delay the movement of the receiver station during the signal time of flight is considered. It is only necessary for a one-way link as it cancels for two-way. The effect can be separated into a first (Fig.7) and a second order (Fig.8) delay, which are defined as follows:

First order delay

$$\frac{\mathbf{D}_{AB} \cdot \mathbf{v}_B(t_A)}{c^2}$$ [2]

Second order delay

$$\frac{D_{AB}}{2c^3}\left(\mathbf{v}_B^2 + \frac{(\mathbf{D}_{AB} \cdot \mathbf{v}_B)^2}{D_{AB}^2} + \mathbf{D}_{AB} \cdot \mathbf{a}_B\right)$$ [2]

with

- $\mathbf{D}_{AB}$: Coordinate distance between receiving and emitting station in ECI
- D_{AB} : $|\mathbf{D}_{AB}|$
- $\mathbf{v}_B$: Velocity of receiver station in ECI
- $\mathbf{a}_B$: Acceleration of receiver station in ECI
- c : Speed of light

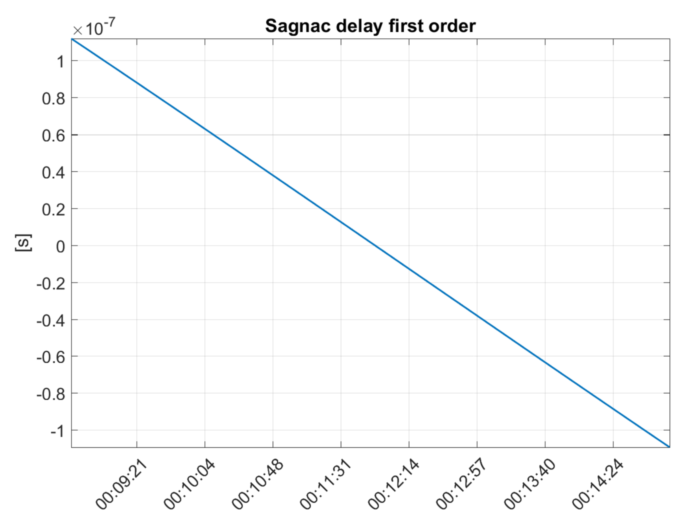


Fig.7: Sagnac delay first order.

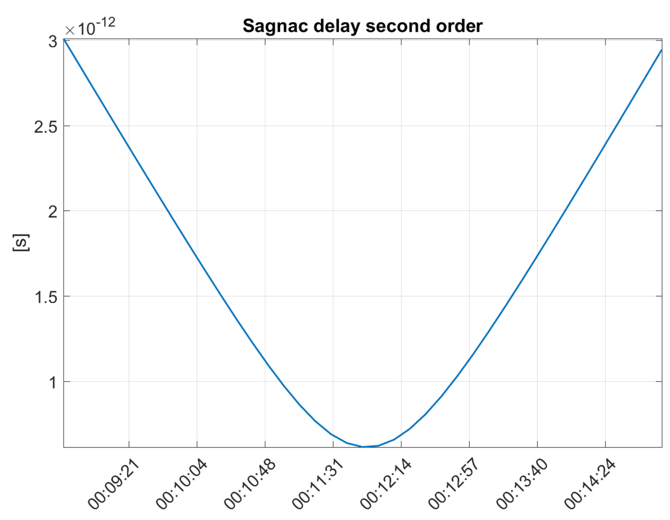


Fig.8: Sagnac delay second order.

Tidal potentials

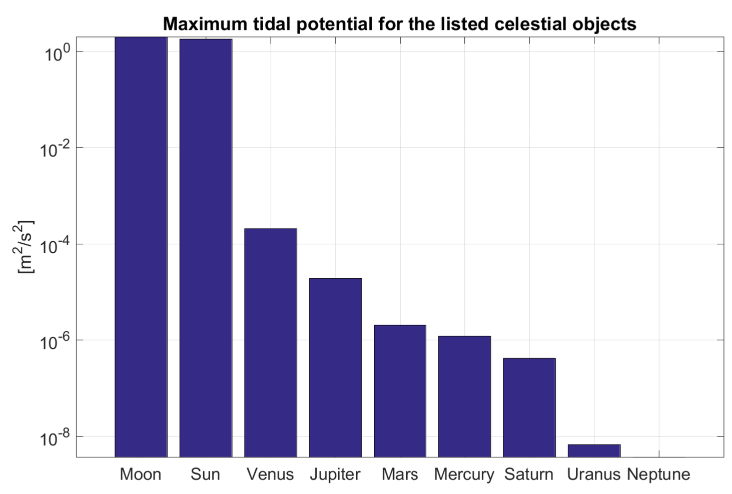


Fig.3: Maximum tidal potentials for the listed celestial objects.

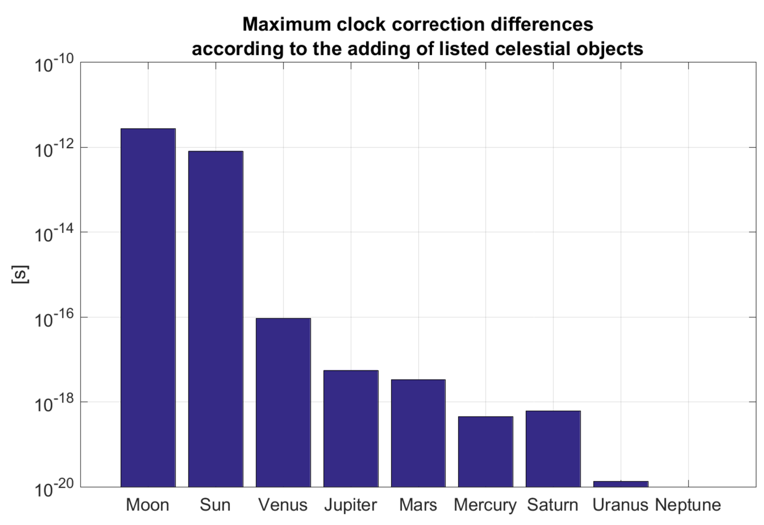


Fig.4: Maximum clock correction differences according to the adding of celestial objects.

Figure 4 illustrates the increase of accuracy for the clock correction calculation in consideration of the tidal potential of each listed celestial object (Fig. 3). Hence, for a picosecond time transfer the effects of Moon and Sun have to be incorporated, whereas the effects of the planets can be neglected.

Shapiro delay

The Shapiro delay (Fig.9) refers to relativistic pulse delay corrections and describes the curvature of the signal way. It is necessary for a one- and two-way link. The effect is defined as follows:

$$\frac{2GM_E}{c^3} \ln\left(\frac{r_A + r_B + D_{AB}}{r_A + r_B - D_{AB}}\right)$$ [2]

with

- D_{AB} : Absolute value of coordinate distance between receiving and emitting station in ECI
- r_A : Absolute value of coordinates of station A in ECI
- r_B : Absolute value of coordinates of station B in ECI
- GM_E : Geocentric gravitational constant
- c : Speed of light

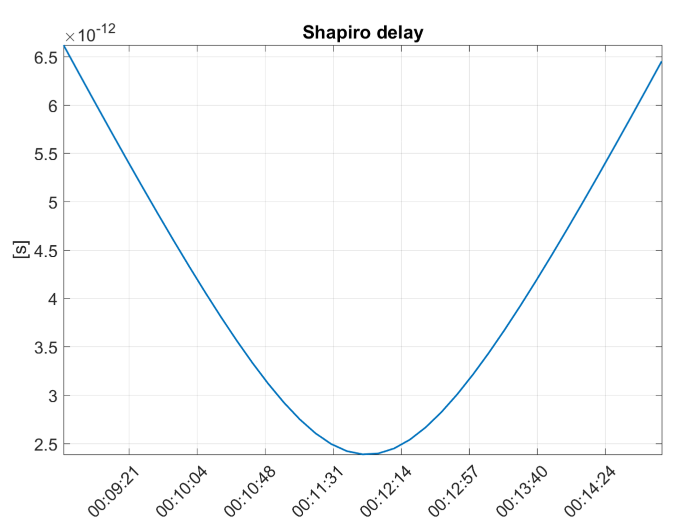
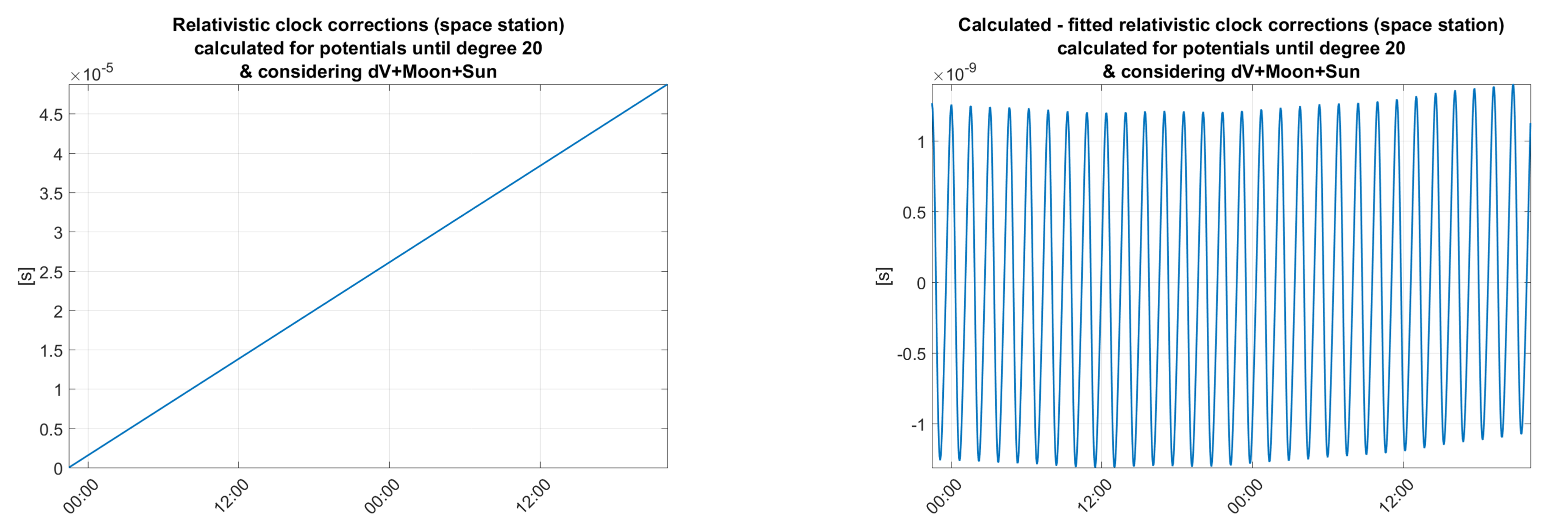


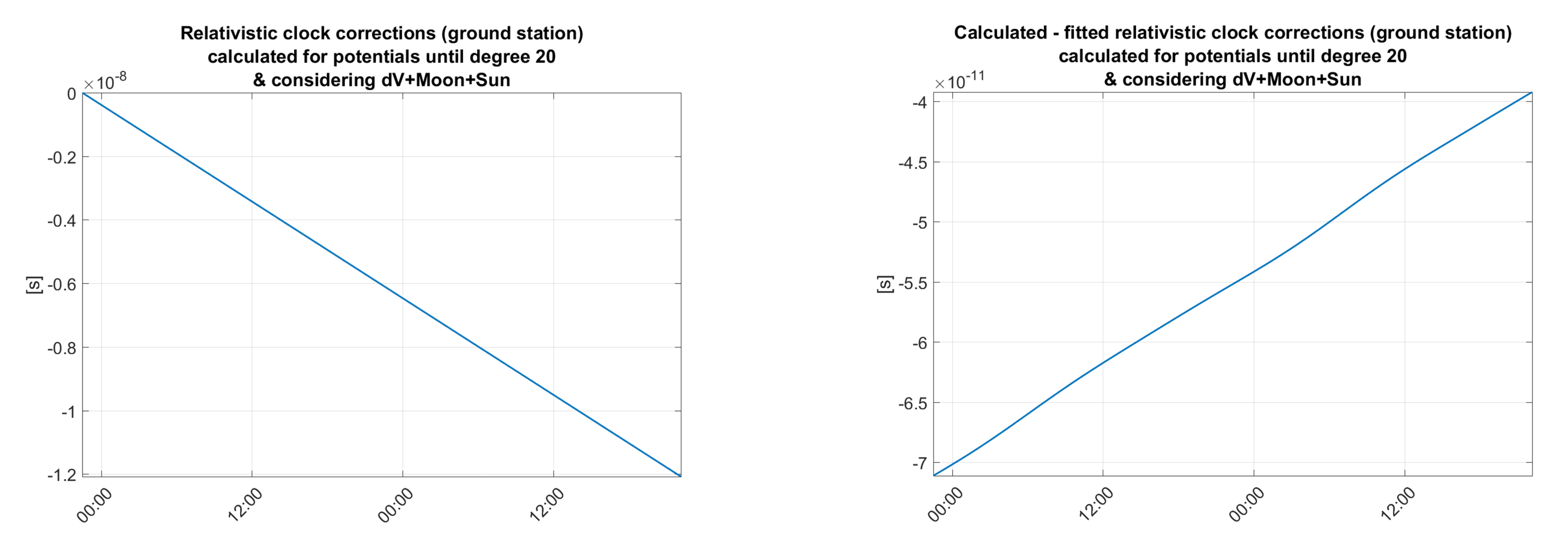
Fig.9: Shapiro delay.

Relativistic clock corrections



a) b)

Fig.5: Clock correction for the space station; a) shows the calculated clock correction, b) shows the difference between the calculated and the linear fitted clock correction.



a) b)

Fig.6: Clock correction for the ground station; a) shows the calculated clock correction, b) shows the difference between the calculated and the linear fitted clock correction.

To offer SLR stations a less intricate calculation of the space and ground station clock offsets to UTC through the relativistic effects of the clocks movements and the potentials (Fig. 5a) and 6a)), a linear coefficient as well as the corresponding offset coefficient will be given to the SLR stations.

Fig. 5b) and 6b) illustrate the difference between the calculated and the linear fitted clock correction. With a maximum difference of about 1,5 ns for the space station and about 70 ps for the ground station clock, we are well below 10 ns, which is the UTC calculation accuracy of the SLR stations.

Summary

Relativistic clock corrections need to be considered due to:

- clocks movements and potentials
- the pulse delay

The corrections due to clocks movements and potentials will be given to SLR stations as a linear coefficient and an offset coefficient. Relating to the simulation the corrections for the pulse delay are for the Shapiro delay around 6,5 ps, the Sagnac delay of first order 110 ns and second order 3 ps. To the proper pulse delay all relativistic delays as well as required non relativistic corrections for instance due to the troposphere and geometry will be added up. Hence, the SLR stations can calculate the offset of the ACES time scale and determine their own offset to UTC.

Literature

[1] Ashby, N. (2003): Relativity in the Global Positioning System, Living Rev. Relativity 6.

[2] Blanchet, L.; Salomon, C.; Teyssandier, P.; Wolf, P. (2000): Relativistic Theory for Time and Frequency Transfer to Order 1/c3, Astron. Astrophys. 370, 320.

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