A new general model for the evolution of the Spin vector of the two LAGEOS satellites and LARES and the LARASE research program

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LARASE LASER RANGED SATELLITES EXPERIMENT

Consiglio Nazionale delle Ricerche ISTI

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OUTLINE OF THE TALK

- The LARASE experiment and its goals
- The rotational dynamics and the Spin Model for the LAGEOS and LARES satellites
    - The internal structure of the two LAGEOS satellites
    - Previous Spin models
    - LARASE spin model
    - LARASE spin model versus measurements
    - An example of application of LARASE model to POD
- Conclusions

LARASE EXPERIMENT AND ITS GOALS

- The LAser RAnged Satellites Experiment (LARASE) main goal is to provide accurate measurements for the gravitational interaction in the weak-field and slow-motion limit of General Relativity by means of a very precise laser tracking of geodetic satellites orbiting around the Earth (the two LAGEOS and LARES)
- Beside the quality of the tracking observations, also the quality of the dynamical models implemented in the Precise Orbit Determination (POD) software plays a fundamental role in order to obtain precise and accurate measurements
- The models have to account for the perturbations due to both gravitational and non-gravitational forces in such a way to reduce as better as possible the difference between the observed range, from the tracking, and the computed one, from the models
- In particular, LARASE aims to improve the dynamical models of the current best laser-ranged satellites in order to perform a precise and accurate orbit determination, able to benefit also space geodesy and geophysics

LARASE ACTIVITIES

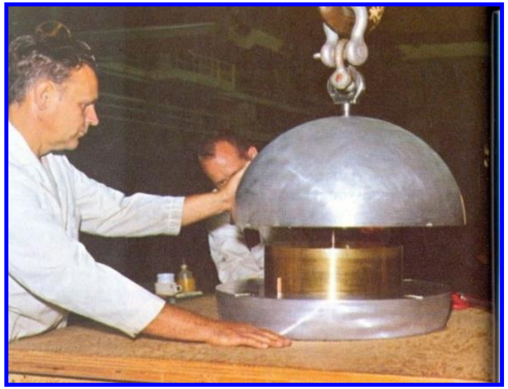
- Develop a complete spin model for LAGEOS and LARES satellites
- Apply LARASE spin model to correctly model thermal effects (the solar Yarkovsky-Schach effect and the Earth's Thermal drag, also known as Earth-Yarkovsky effect or Rubincam effect)
- Impact of the neutral drag on the two LAGEOS satellites and on LARES
- Solid and Ocean tides on the two LAGEOS satellites and on LARES
- Precise Orbit Determination for the two LAGEOS satellites and for LARES
- Precise and accurate measurements of relativistic physics in the field of the Earth

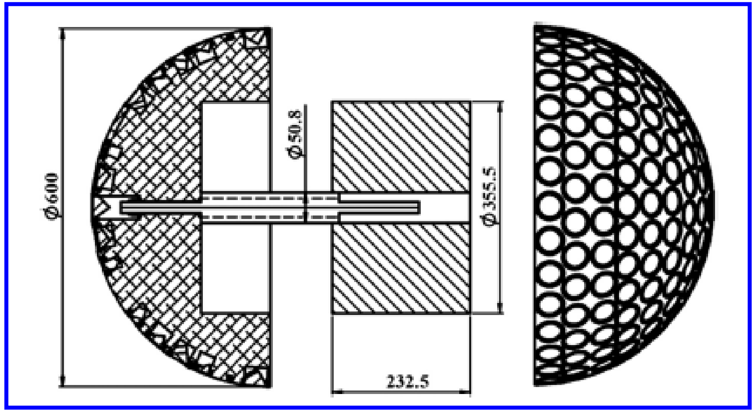
THE LARASE ACTIVITY ON SPIN

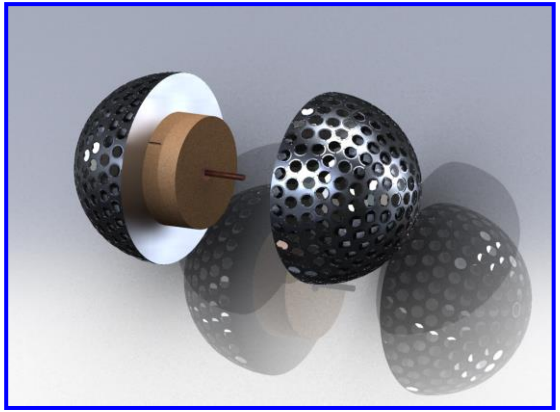
- Gather the documentation with the characteristics of the satellites: the two LAGEOS and LARES
- Build a 3D realistic model of each satellite to calculate its moments of inertia
- Write the equations describing the different interactions
- Write a MATLAB simulation program with a easy graphic interface to model and view the results

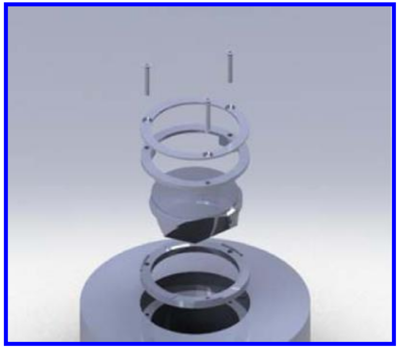
THE INTERNAL STRUCTURE - LAGEOS

- We gathered and critically analyzed the scientific literature and to the official documents
- We were able to solve the problems concerning materials and dimensions
- Therefore we built a realistic model of LAGEOS and LAGEOS II









LAGEOS MASS PROPERTIES

Using our model, and the most likely material properties

Materials used for the construction of the two LAGEOS satellites (Cogo, 1988) and their nominal densities.

Satellite	Material density ρ_n (kg/m^3)		
	Hemispheres	Core	Stud
LAGEOS	AA6061 2700[a]	QQ-B-626 COMP.11 8440[a]	Cu-Be 8230[b]
LAGEOS II	AlMgSiCu UNI 6170 2740[c]	PCuZn39Pb2 UNI 5706 8280[c]	Cu-Be QQ-C-172 8250[c]

we calculated masses and moments of inertia of the satellites in different arrangements

Satellite origin of value	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement				
Computed value in NASA (1975)	409.8	11.516	11.084	11.084
Measured value in NASA (1975)	406.965	–	–	–
Values computed in the present work using nominal density of Table 1	405.93	11.40	10.93	10.93
LAGEOS balance model				
Computed value in NASA (1975)	440.3	13.14	12.71	12.71
Measured value in NASA (1975)	440.0	13.11	12.69	12.71
Value computed in the present work using nominal density of Table 1	437.68	13.09	12.62	12.62
Values computed in the present work using normalized density	440.00	13.16	12.68	12.68
LAGEOS II flight arrangement				
Computed values in Fontana (1990)	–	11.45	11.00	11.00
Measured value in Fontana (1990), Fontana (1989) and Cogo (1988)	405.38	–	–	–
Values computed in the present work using nominal density of Table 1	404.97	11.44	10.99	10.99
LAGEOS II without CCRs				
Computed value in Fontana (1989)	386.59	10.39	9.95	9.95
Measured value in Fontana (1989)	387.20	9.67	9.37	9.15
Values computed in the present work using nominal density of Table 1	386.71	10.41	9.95	9.95
Values computed in the present work using normalized density	387.20	10.42	9.96	9.96

~10^{-3}

LAGEOS SATELLITES MASS PROPERTIES

And in particular we fixed the dynamical parameters of the satellites in flight arrangement

Satellite	Mass (kg)	Moments of inertia (kg m^2)		
	M	I_{xx}	I_{yy}	I_{zz}
LAGEOS flight arrangement	406.97	11.42 ± 0.03	10.96 ± 0.03	10.96 ± 0.03
LAGEOS II flight arrangement	405.38	11.45 ± 0.03	11.00 ± 0.03	11.00 ± 0.03

We believe to have solved definitely the problem concerning these fundamental dynamical parameters

Further details in: Visco, M., Lucchesi, D.M., 2016. *Review and critical analysis of mass and moments of inertia of the LAGEOS and LAGEOS II satellites for the LARASE program*. Advances in Space Research 57, 1928-1938. doi:10.1016/j.asr.2016.02.006

LARES MASS PROPERTIES

- LARES geometry is much more simple than that of LAGEOS
- Currently, we have not yet been able to recover the full documentation of the satellite
- With a method similar to that used for LAGEOS we built a realistic model of LARES computing the mass and moments of inertia (not available in literature)

A rendering of LARES

PREVIOUS SPIN MODELS

The best spin models developed in the past are:

- Bertotti and Iess (JGR 96 B2, 1991)
- Habib et al. (PRD 50, 1994)
- Farinella, Vokrouhlicky and Barlier (JGR 101, 1996); Vokrouhlicky (GRL 23, 1996)
- Andrés, 1997 (PhD Thesis) and LOSSAM

- These studies, with the exception of Habib et al., attack and solve the problem of the evolution of the rotation of a satellite in a terrestrial inertial reference system in the so-called *rapid spin approximation*, and they introduced equations for the external torques that are averaged over characteristic time intervals: the orbital period and the day

- Averaged models, especially LOSSAM, well fit observations, while the model by Habib et al., using a body-fixed reference system and non-averaged torques, fail to properly fit the observations

LARASE SPIN MODEL

Our target was to build a dynamical spin model well fitting the available experimental data and that is able to predict the future trend of the spin evolution in the general case and not only in the fast spin approximation

We have deeply reviewed previous spin models, in particular we:

- first built our own spin model in the *rapid spin approximation*
- adopted *non-averaged torques* in the equations to describe the general problem
- introduced in the equations all known possible torques (like in LOSSAM model)
- solved the equations in a body-fixed reference system in order to better describe the misalignment between the symmetry axis and the spin

LARASE SPIN MODEL – THE EQUATIONS

We wrote the Euler equations in the body frame using the Euler angles with respect to the Earth Centered Inertial (ECI) reference frame

$$\begin{aligned} I_x\,\dot{\omega}^b_{sx} - \omega^b_{sy}\omega^b_{sz}(I_y - I_z) &= M_x \\ I_y\,\dot{\omega}^b_{sy} - \omega^b_{sx}\omega^b_{sz}(I_z - I_x) &= M_y \\ I_z\,\dot{\omega}^b_{sz} - \omega^b_{sx}\omega^b_{sy}(I_x - I_y) &= M_z \end{aligned}$$

$$\begin{aligned} \ddot{\theta} \;=\;& \frac{\cos\psi M_x}{I_x} - \frac{\sin\psi M_y}{I_y} - \dot{\phi}\dot{\psi}\sin\theta\frac{I_z}{I_y} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\frac{I_y - I_z}{I_x} \\ & + \frac{I_x - I_y}{I_x}\left[\dot{\theta}\left(\dot{\psi} + \dot{\phi}\cos\theta\right)\frac{\sin(2\psi)}{2}\frac{\Lambda}{I_y} + \dot{\phi}^2\frac{\sin(2\theta)}{2}\sin^2\psi\frac{\Lambda}{I_y} - \dot{\phi}\dot{\psi}\sin\theta\left(\frac{I_y - I_z}{I_y} - \sin^2\psi\frac{\Lambda}{I_y}\right)\right] \qquad (4) \end{aligned}$$

$$\begin{aligned} \ddot{\phi} \;=\;& \frac{\cos\psi M_y}{I_y\sin\theta} + \frac{\sin\psi M_x}{I_x\sin\theta} + \frac{\dot{\psi}\dot{\theta}}{\sin\theta}\frac{I_z}{I_y} - \dot{\phi}\dot{\theta}\frac{\cos\theta}{\sin\theta}\frac{\Lambda}{I_x} + \\ & \frac{I_x - I_y}{I_y}\left[\frac{\dot{\psi}\dot{\theta}}{\sin\theta}\left(\frac{\Lambda}{I_x}\sin^2\psi - 1\right) - \frac{\Lambda}{I_x}\dot{\phi}\frac{\sin(2\psi)}{2}\left(\cos\theta\dot{\phi} + \dot{\psi}\right) - \dot{\phi}\dot{\theta}\frac{\cos\theta}{\sin\theta}\frac{\Lambda}{I_x}\cos^2\psi\right] \qquad (5) \end{aligned}$$

$$\begin{aligned} \ddot{\psi} \;=\;& \frac{M_z}{I_z} - \frac{\cos(\theta)}{\sin(\theta)}\left(\frac{\cos(\psi)M_y}{I_y} + \frac{\sin(\psi)M_x}{I_x}\right) + \dot{\phi}\dot{\theta}\frac{1}{\sin\theta}\left(\cos^2\theta\frac{I_y - I_z}{I_x} + 1\right) - \dot{\psi}\dot{\theta}\frac{I_z\cos\theta}{I_y\sin\theta} + \\ & (I_x - I_y)\left[\dot{\phi}\dot{\theta}\frac{1}{I_z}\frac{1}{\sin\theta}\left(\sin^2\theta\cos(2\psi) + \cos^2\psi\cos^2\theta\frac{\Lambda}{I_xI_y}\right) - \dot{\theta}^2\frac{\sin(2\psi)}{2I_z} - \dot{\phi}^2\frac{\sin(2\psi)}{2I_z}\left(\cos^2\theta\Lambda\frac{I_z}{I_xI_y} - \sin^2\theta\right)\right. \\ & \left. -\dot{\psi}\dot{\theta}\frac{\cos\theta}{I_y\sin(\theta)}\left(\sin^2\psi\frac{\Lambda}{I_x} - 1\right) + \dot{\phi}\dot{\psi}\cos\theta\sin(2\psi)\frac{\Lambda}{2I_xI_y}\right] \qquad (6) \end{aligned}$$

where $\Lambda = I_x + I_y - I_z$

LARASE SPIN MODEL – THE TORQUES

We considered four main torques acting on the satellites:

- The torque from Earth magnetic field (eddy currents)
- The torque from Earth gravitational field (satellite oblateness)
- The two torques due to the sun radiation pressure over the satellite (distance of C.M. from geometrical center and hemispheres' reflectivity asymmetry)

MAGNETIC TORQUE

- The torque from Earth magnetic field

$$M^E_{mag} = V\sum_{i=1}^{9}\frac{|\mathbf{B}_i|^2}{2|\boldsymbol{\omega}_s|}\{A''_i[1+\cos(2\omega_i t+2\varphi_i)] - D'_i\sin(2\omega_i t+2\varphi_i)\}\,\boldsymbol{\omega}_s +$$

$$V\sum_{i=1}^{9}\frac{\mathbf{B}_i\cdot\boldsymbol{\omega}_s}{2|\boldsymbol{\omega}_s|^2}\{[\alpha'(\omega_i)-A'_i][1+\cos(2\omega_i t+2\varphi_i)] - [D''_i+\alpha''(\omega_i)]\sin(2\omega_i t+2\varphi_i)\}\,(\boldsymbol{\omega}_s\times\mathbf{B}_i) +$$

$$V\sum_{i=1}^{9}\frac{\mathbf{B}_i\cdot\boldsymbol{\omega}_s}{2|\boldsymbol{\omega}_s|}\{-A''_i[1+\cos(2\omega_i t+2\varphi_i)] + D'_i\sin(2\omega_i t+2\varphi_i)\}\,\mathbf{B}_i$$

The torque has one constant and 7 periodic components: at Earth's angular speed $\omega_\oplus$, at twice of the orbital mean motion n and at their combinations

$$B = B_0 + \sum_{i=1}^{8}\mathbf{B}_i\cos(\omega_i t+\varphi_i)$$

$$\begin{aligned}\omega_1 &= \omega_2 = \omega_\oplus - 2n\\ \omega_3 &= \omega_4 = \omega_\oplus + 2n\\ \omega_5 &= \omega_6 = 2n\\ \omega_7 &= \omega_8 = \omega_\oplus\end{aligned}$$

$$A'_i = \frac{\alpha'(\omega_s-\omega_i)+\alpha'(\omega_s+\omega_i)}{2} \qquad D'_i = \frac{\alpha'(\omega_s-\omega_i)-\alpha'(\omega_s+\omega_i)}{2}$$

$$A''_i = \frac{\alpha''(\omega_s-\omega_i)+\alpha''(\omega_s+\omega_i)}{2} \qquad D''_i = \frac{\alpha''(\omega_s-\omega_i)-\alpha''(\omega_s+\omega_i)}{2}$$

The polarizability has to be corrected using a parametric factor (no trivial explanation)

Information from aerodynamics studies showed that due to its metallic content, the satellite would spin down from its initial 90 RPM to nearly no spin in less than one year. (NASA document Phase B 1975 TM X 64915)

GRAVITATIONAL TORQUE

- The torque from Earth gravitational field

$$M^b_{grav} = 3\omega_\oplus^2 \left\{ \hat{s}^b \times \left[I_x (\hat{s}^b \cdot \hat{x}^b)\hat{x}^b + I_y (\hat{s}^b \cdot \hat{y}^b)\hat{y}^b + I_z \,(\hat{s}^b \cdot \hat{z}^b)\,\hat{z}^b \right] \right\} = 3\omega_\oplus^2 \begin{bmatrix} (I_y - I_z)\, s^b_y\, s^b_z \\ (I_z - I_x)\, s^b_x\, s^b_z \\ (I_x - I_y)\, s^b_x\, s^b_y \end{bmatrix}$$

$$\begin{aligned} s^b_x &= \cos(\omega + M_0 + n\cdot t)\,[\cos(\phi-\Omega)\cos(\psi) - \cos(\theta)\sin(\phi-\Omega)\sin(\psi)] + \\ &\quad \sin(\omega + M_0 + n\cdot t)\,\{\sin(\phi-\Omega)\cos(I)\cos(\psi) + [\sin(\theta)\sin(I) + \cos(\theta)\cos(\phi-\Omega)\cos(I)]\sin(\psi)\} \end{aligned}$$

$$\begin{aligned} s^b_y &= -\cos(\omega + M_0 + n\cdot t)\,[\cos(\theta)\sin(\phi-\Omega)\cos(\psi) + \cos(\phi-\Omega)\sin(\psi)] + \\ &\quad \sin(\omega + M_0 + n\cdot t)\,\{[\sin(\theta)\sin(I) + \cos(\theta)\cos(\phi-\Omega)\cos(I)]\cos(\psi) - \sin(\phi-\Omega)\cos(I)\sin(\psi)\} \end{aligned}$$

$$\begin{aligned} s^b_z &= \cos(\omega + M_0 + n\cdot t)\sin(\theta)\sin(\phi-\Omega) + \\ &\quad \sin(\omega + M_0 + n\cdot t)\,[\cos(\theta)\sin(I) - \sin(\theta)\cos(\phi-\Omega)\cos(I)] \end{aligned}$$

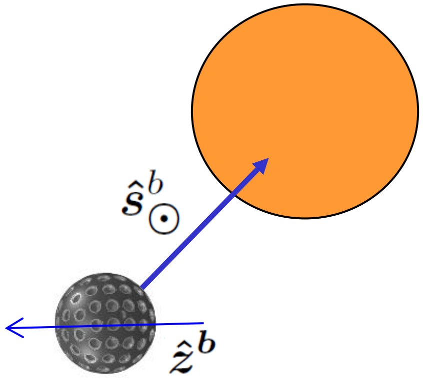
- The gravitational torque depend on asymmetry and is null for a perfectly symmetric satellite ($I_x=I_y=I_z$)
- It is periodic at the orbital period

TORQUES FROM SOLAR RADIATION PRESSURE

- The torque from the offset:

$$M^b_{off} = \nu\pi\rho^2 \frac{\Phi}{c} C_R \left(\boldsymbol{h}^{\boldsymbol{b}} \times \hat{s}^b_\odot\right)$$

- ν – shadow function
- ρ – radius
- Φ – Solar flux
- C_R – Radiation coefficient
- $\boldsymbol{h}^{\boldsymbol{b}}$ – Distance of C.M. from geometrical center

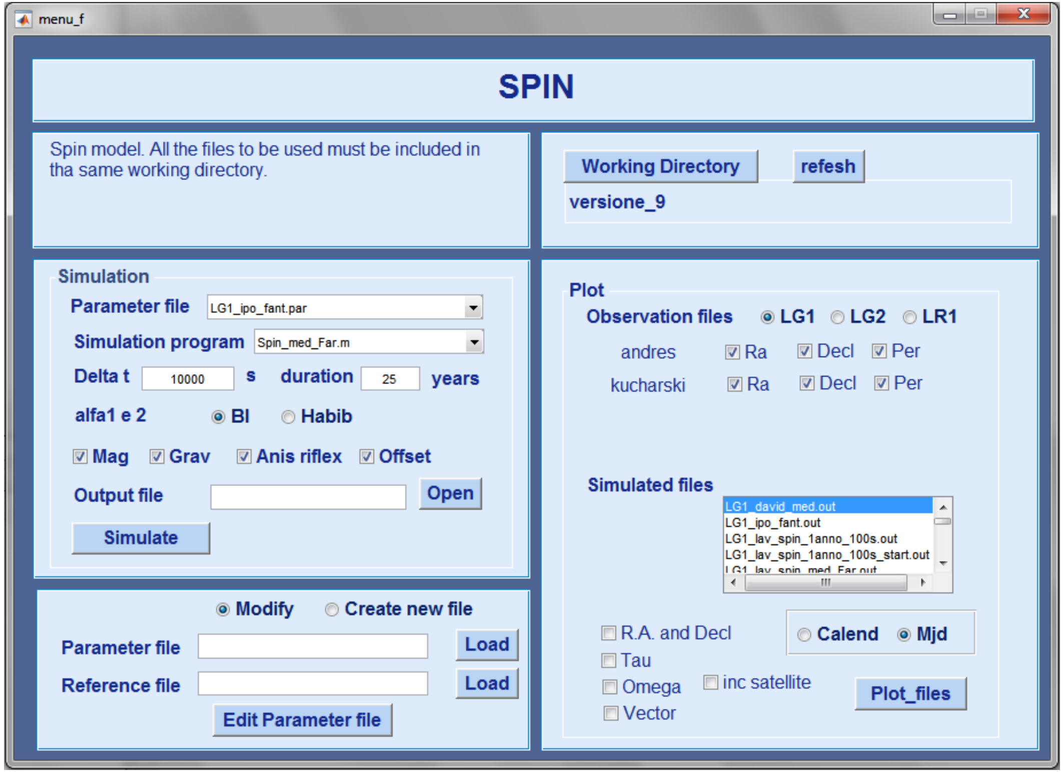


- The torque from the asymmetric reflectivity of north-south surfaces:

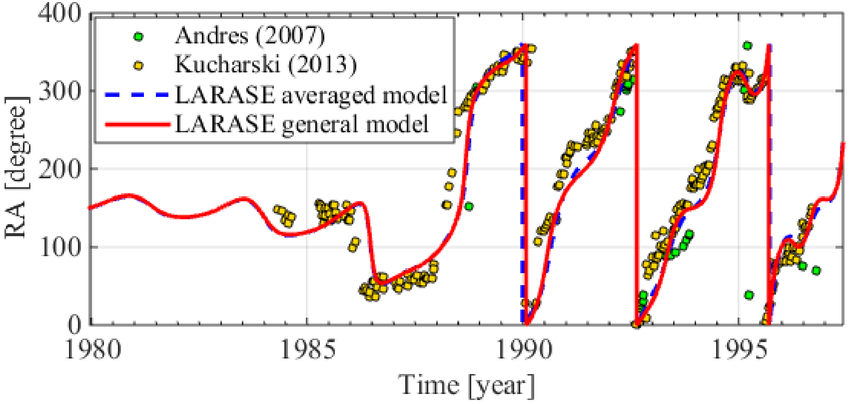
$$M^b_{ar} = \nu \frac{2}{3}\rho^3 \frac{\Phi}{c} \Delta\rho\, C_R \left(\hat{\boldsymbol{z}}^{\boldsymbol{b}} \times \hat{\boldsymbol{s}}^{\boldsymbol{b}}_\odot\right) \left|\hat{\boldsymbol{z}}^{\boldsymbol{b}} \times \hat{\boldsymbol{s}}^{\boldsymbol{b}}_\odot\right|$$

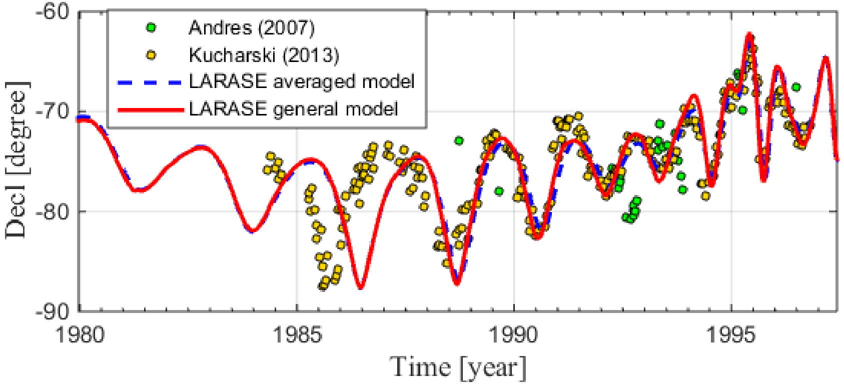
$$\Delta\rho = \frac{C_R^N - C_R^S}{C_R}$$

LARASE SPIN MATLAB PROGRAM

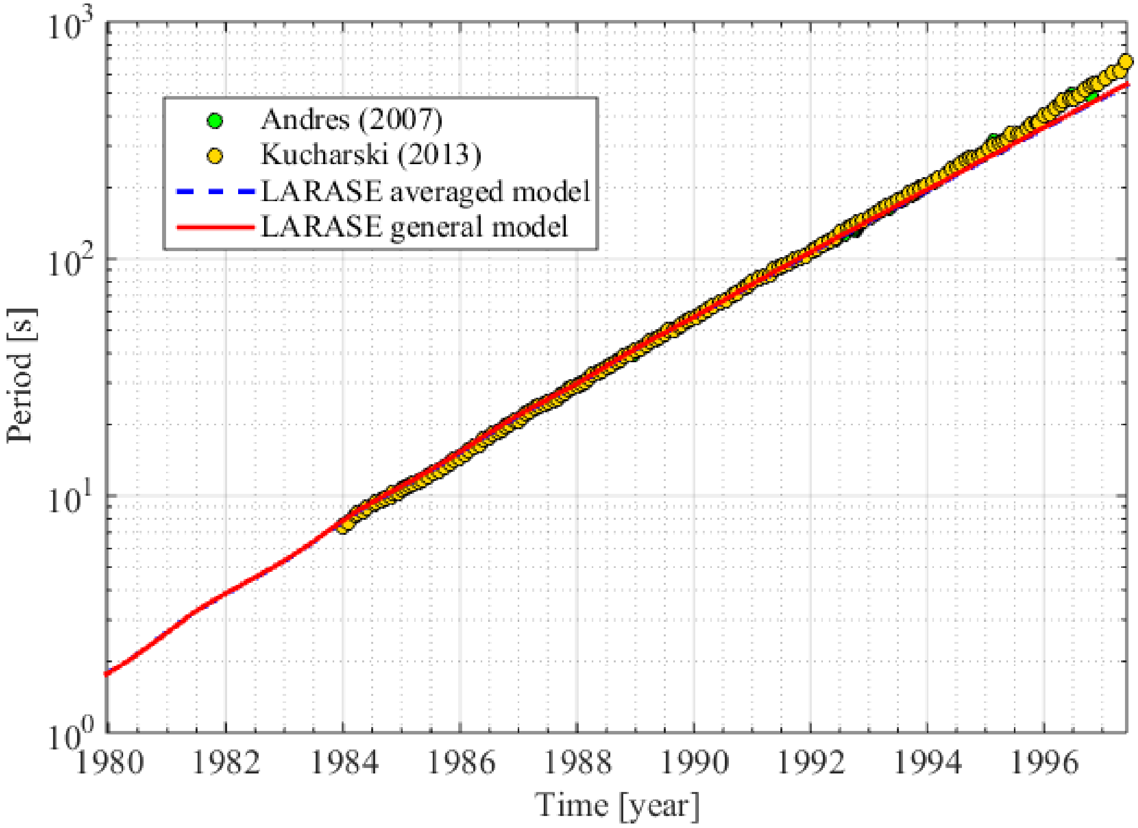


MODEL RESULTS LAGEOS





MODEL RESULTS LAGEOS

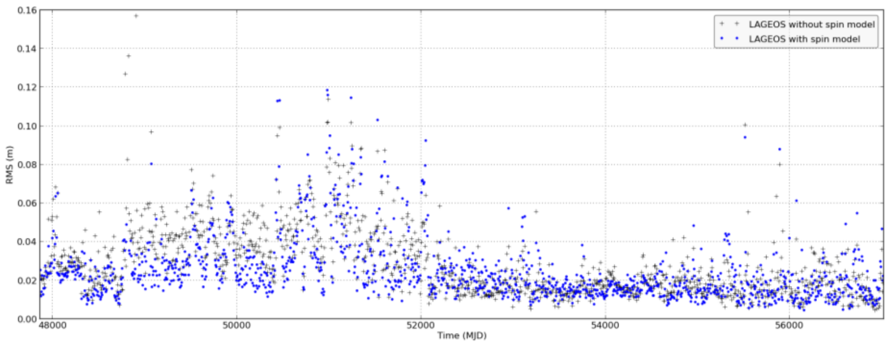


PRELIMINARY MODELING OF THE EARTH-YARKOVSKY EFFECT WITH LARASE SPIN MODEL

- Root Mean Square (RMS) of the range residuals in POD (without empirical accelerations)

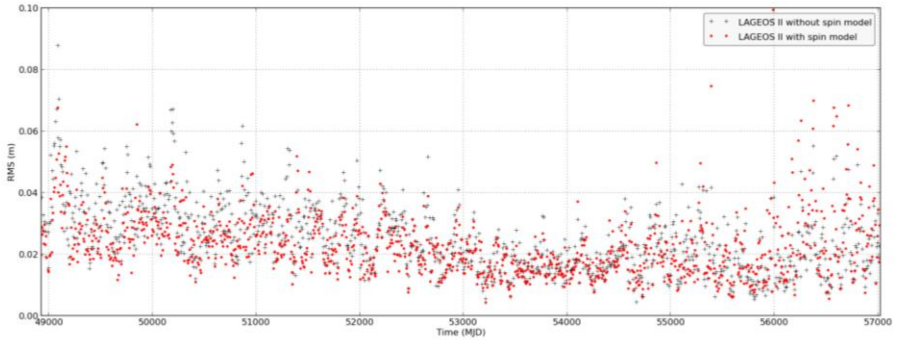
LAGEOS

RMS: 2.8 cm → 2.5 cm



LAGEOS II

RMS: 2.5 cm → 2.2 cm



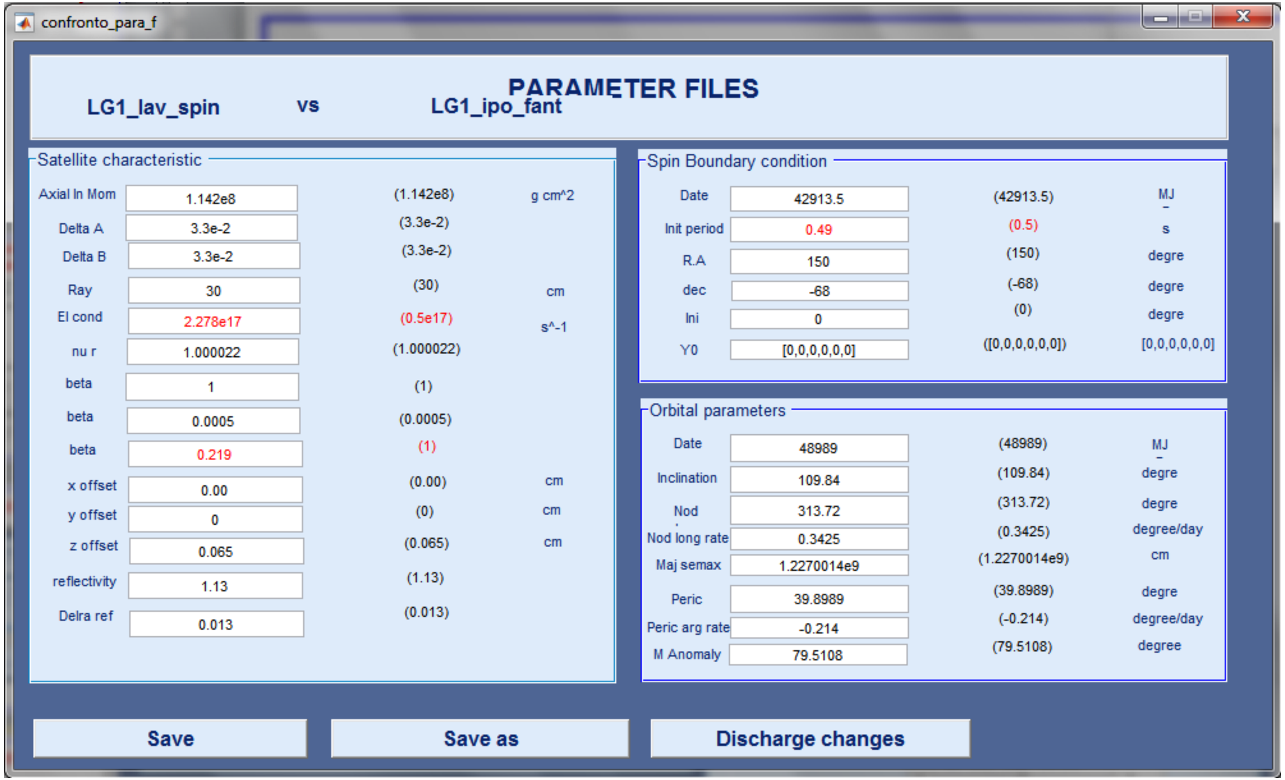
CONCLUSIONS

- We developed a model (LARASE) for the spin appropriate for any experimental configuration
- In principle, this general model could be applied to predict the current behavior of the spin of LAGEOS' satellites, starting from the last available measurements
- We have not yet completed the full investigation of the problem, but we believe that in order to obtain more robust results, some new measurements of the spin are needed (maybe via radar-ranging to the Ge CCRs)
- The knowledge of the spin vector allows a precise calculation of the surface thermal thrust perturbations
- The possibility to directly model the thermal thrust effects could be useful not only for the LARASE program, but in general for a precise orbit determination at the millimetric level in the range residuals with the resulting benefits in the fields of space geodesy and geophysics

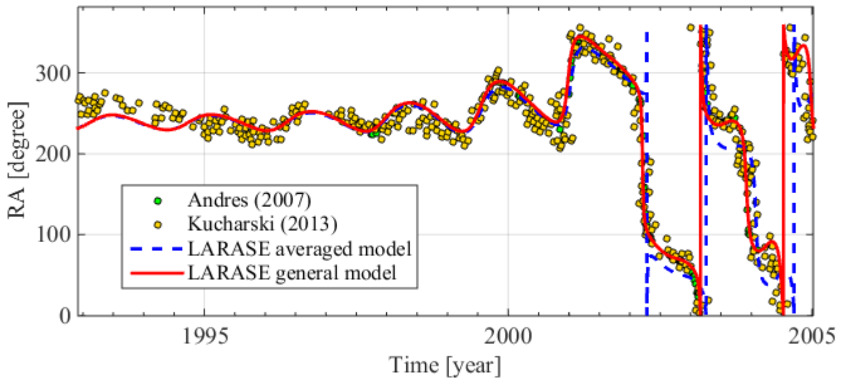
THE END

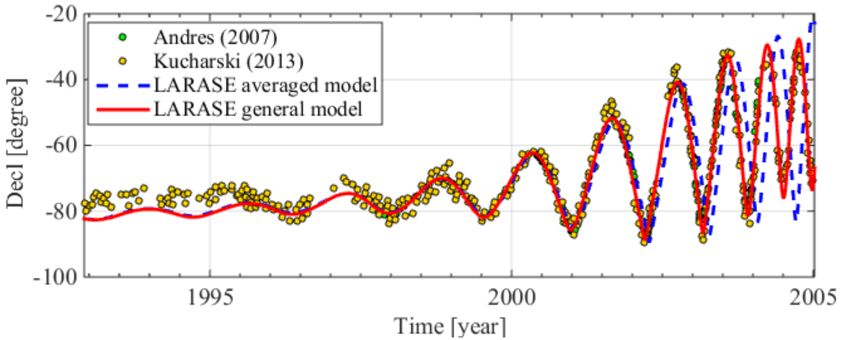
EXTRA

PARAMETERS EDITINGS

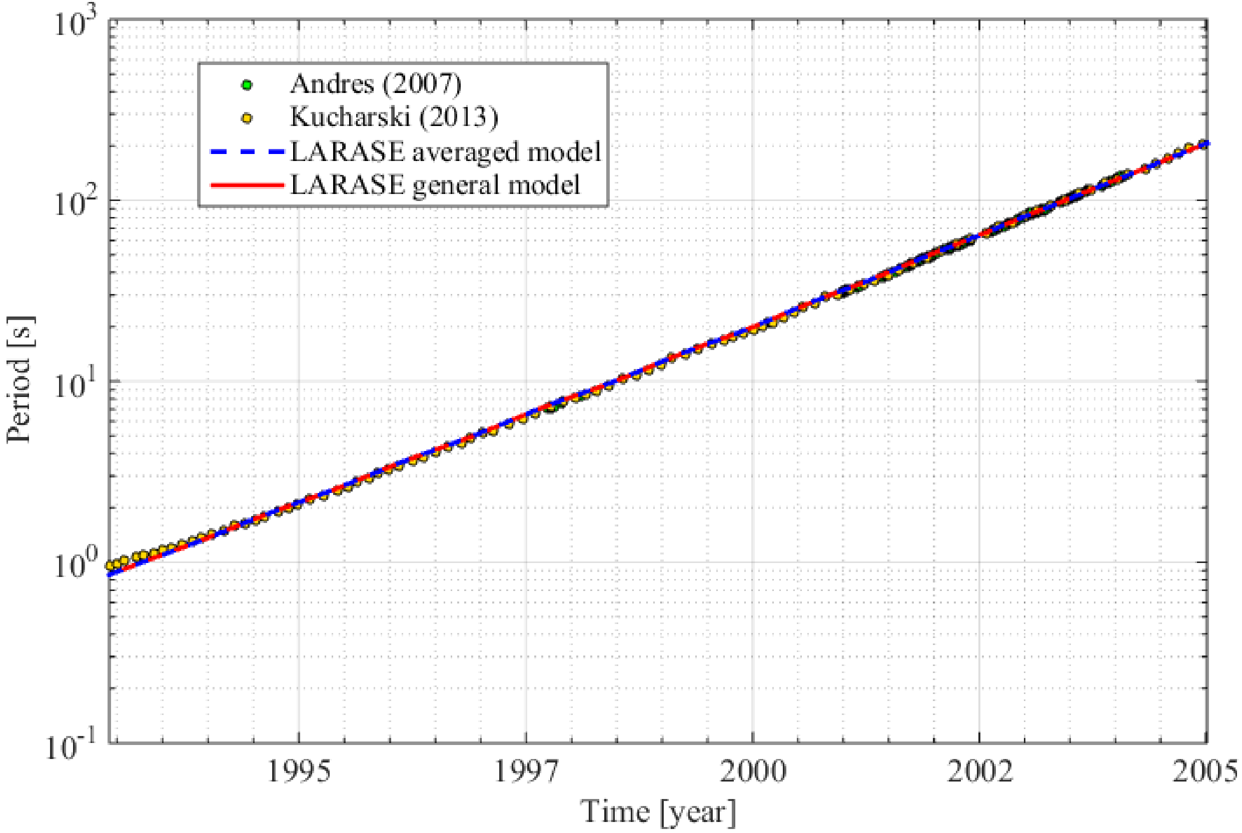


MODEL RESULTS LAGEOS II

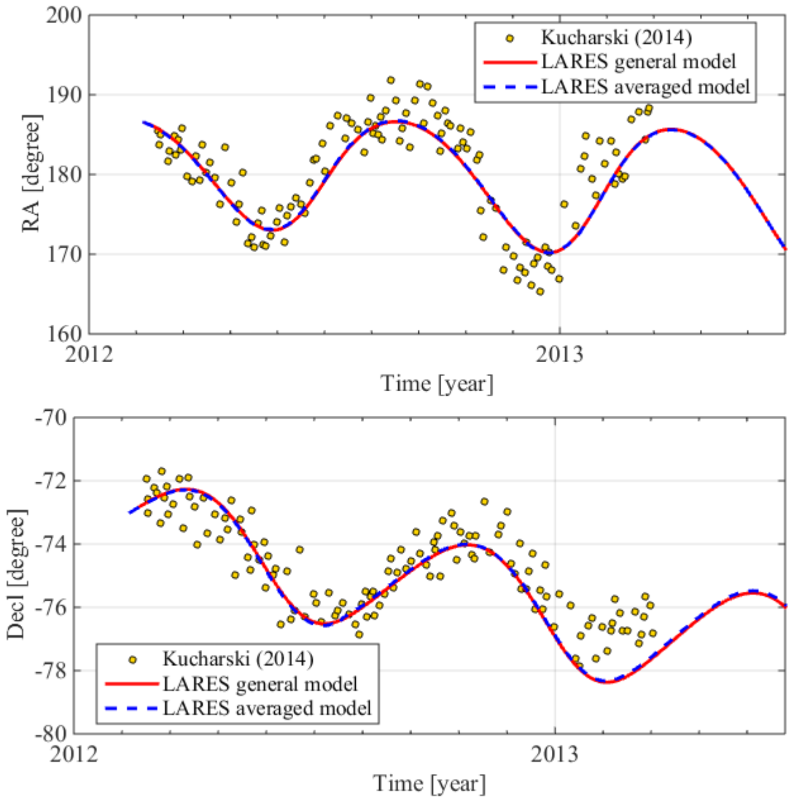




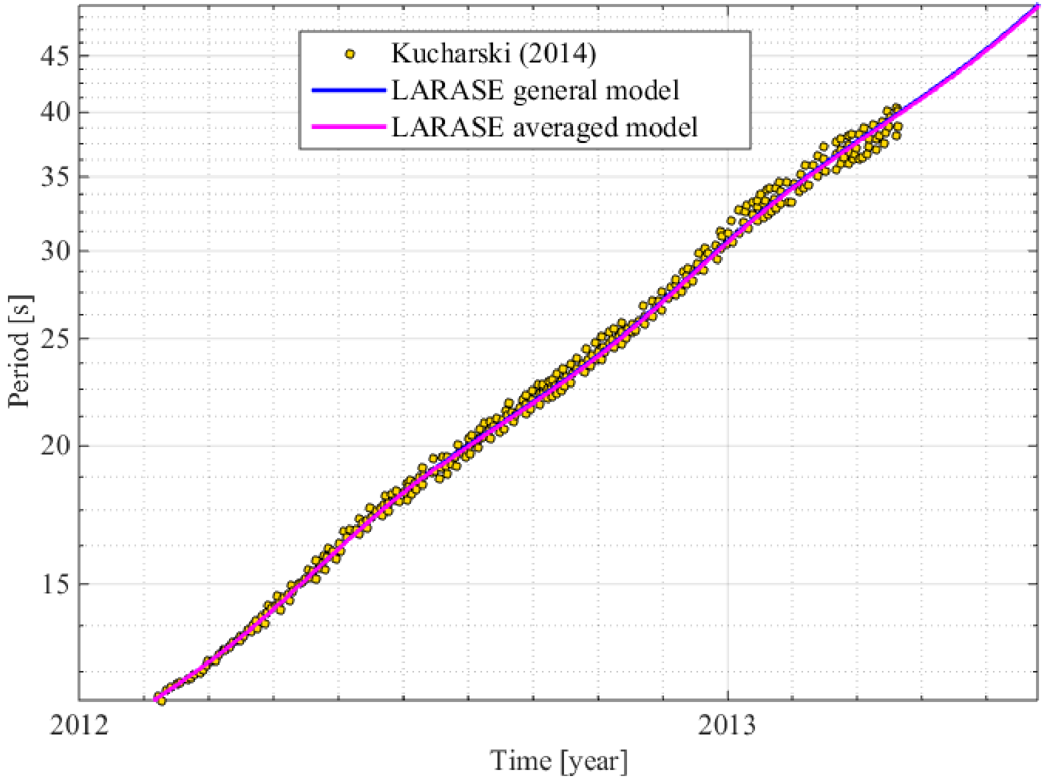
MODEL RESULTS LAGEOS II



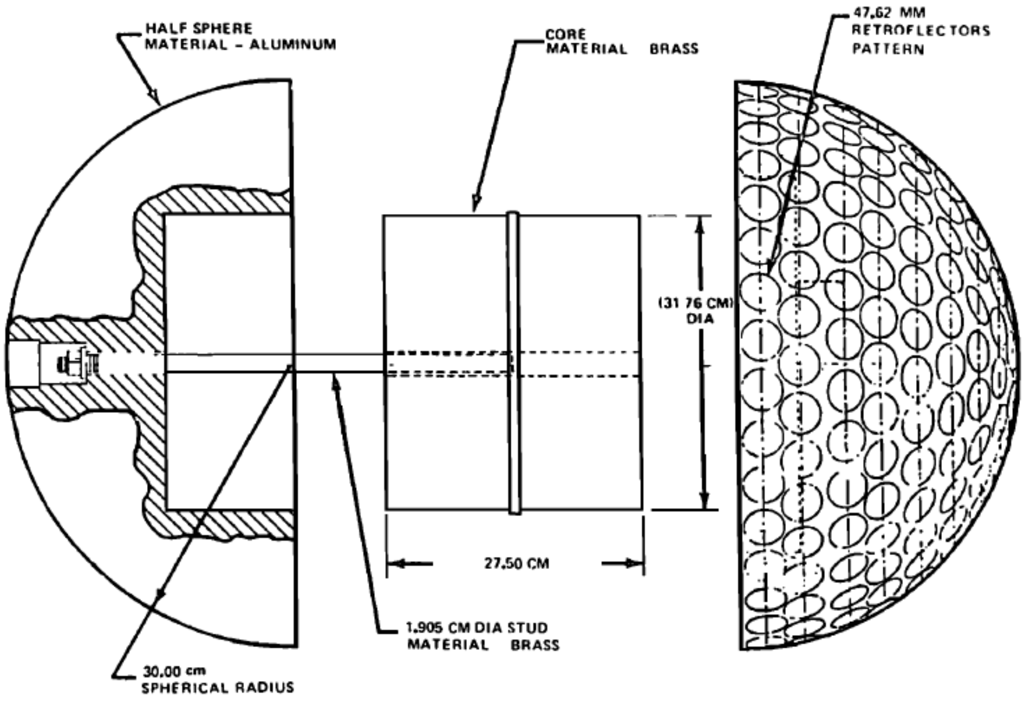
MODEL RESULTS LARES



MODEL RESULTS LARES



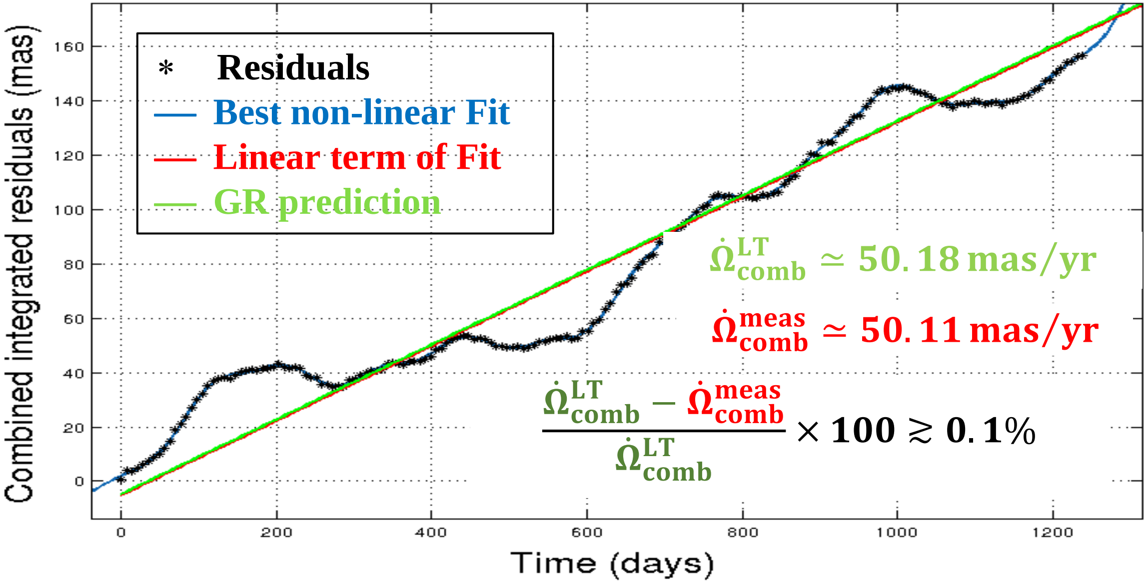
OLD DROWING


HALF SPHERE
MATERIAL – ALUMINUM
CORE
MATERIAL BRASS
47.62 MM
RETROFLECTORS
PATTERN
(31 76 CM)
DIA
27.50 CM
1.905 CM DIA STUD
MATERIAL BRASS
30.00 cm
SPHERICAL RADIUS

MEASUREMENTS OF RELATIVISTIC EFFECTS

- A very preliminary new measurement of the Lense-Thirring effect with the two LAGEOS and LARES (3.4 yr)

GGM05S $\dot{\Omega}_{\mathbf{comb}} = \dot{\Omega}^{\mathbf{res}}_{\mathbf{L1}} + \mathbf{k_1} \cdot \dot{\Omega}^{\mathbf{res}}_{\mathbf{L2}} + \mathbf{k_2} \cdot \dot{\Omega}^{\mathbf{res}}_{\mathbf{LR}}$



$\dot{\Omega}^{\mathrm{LT}}_{\mathrm{comb}} \simeq 50.18\ \mathrm{mas/yr}$

$\dot{\Omega}^{\mathrm{meas}}_{\mathrm{comb}} \simeq 50.11\ \mathrm{mas/yr}$

$$\frac{\dot{\Omega}^{\mathrm{LT}}_{\mathrm{comb}} - \dot{\Omega}^{\mathrm{meas}}_{\mathrm{comb}}}{\dot{\Omega}^{\mathrm{LT}}_{\mathrm{comb}}} \times 100 \gtrsim 0.1\%$$

We fitted also for a minimum of three to a maximum of twelve tidal waves (both solid and ocean):

$$\Omega^{Fit} = a + b \cdot t + \sum_{i=1}^{n} A_i \cdot \sin\left(\frac{2\pi}{P_i} \cdot t + \Phi_i\right)$$

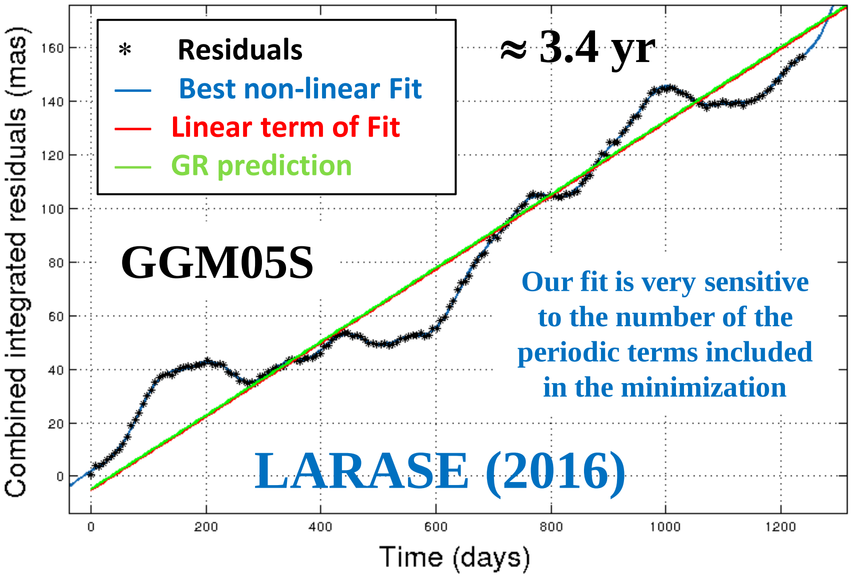
Indeed, tides mismodelling plus unmodelled nongravitational forces due to thermal effects may corrupt the measurement of the relativistic effect.

For instance, the (both solid and ocean) K1 tides have the same periods of the right ascension of the node of the satellites:

≈1044 days, ≈569 days and ≈224 days

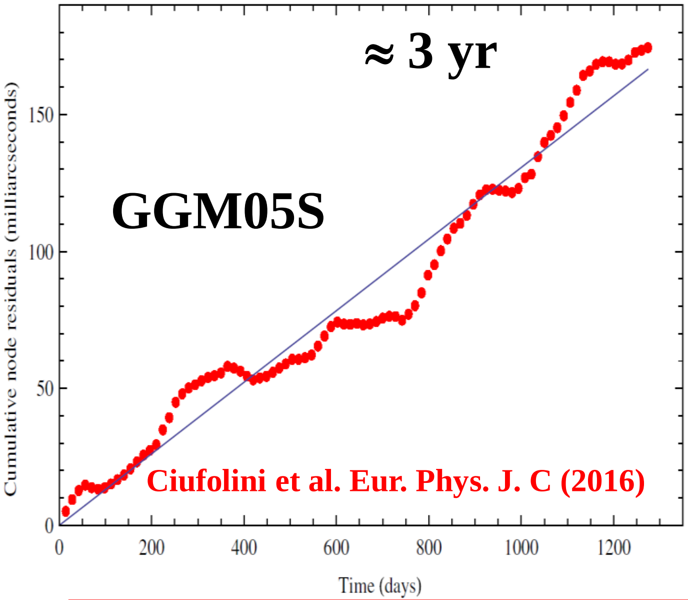
MEASUREMENTS OF RELATIVISTIC EFFECTS

- Comparison with a recent measurement



$$\mu = (0.999 \pm \varepsilon(fit)) \pm \varepsilon(sys)$$

Up to 9% from a sensitivity analysis of the main tidal waves



$$\mu = (0.994 \pm 0.002) \pm 0.05$$

0.2% formal error of the fit (1-sigma) plus 5% preliminary estimate of systematics (4% grav. + 1% non-grav.)