# Relativity and Fundamental Physics

**Sergei Kopeikin**

**University of Missouri-Columbia**
**&**
**Siberian State University of Geosystems and Technology**

# Outline

- Where does fundamental physics starts from?
- Fundamental parameters
- Zoo of alternative theories of gravity
- Discovery of gravitational waves
- Experiments in the solar system
  - Time and clocks
  - Gravitomagnetic field
  - Laser ranging for testing big G
- Summary

**Modern theory of fundamental interactions relies heavily upon two main pillars both created by Albert Einstein – special and general theory of relativity.**

**Special relativity is a cornerstone of elementary particle physics and the quantum field theory.**

**General relativity is a metric-based theory of gravitational field.**

- **Understanding the nature of fundamental physical interaction is the ultimate goal of experimental physics.**

- **The most important but least understood is the gravitational interaction due to its weakness in the solar system – a primary experimental laboratory of gravitational physicists for several hundred years.**

- **We study gravity by observing orbital/rotational motion of celestial bodies with light rays and radio waves.**

- **Physical motions of the bodies and propagation of light are described by solutions of equations of motion which, in their own turn, depend on the solutions of equations of a gravity field theory**

- **The mathematical model of motion fits to observational data to determine various fundamental parameters characterizing the structure of spacetime (NB: most of the fitting parameters are not fundamental though).**

# Where does fundamental physics start from?

Physical fields = sections of fiber bundles

Volume of integration

Lagrangian

$$\boldsymbol{S} = \int d^4x\sqrt{-g}\, L\left(x, \psi_b, \psi_{b;\alpha} \,\middle|\, G, h, c, \beta, \gamma, \ldots\right)$$

The action

Path integral

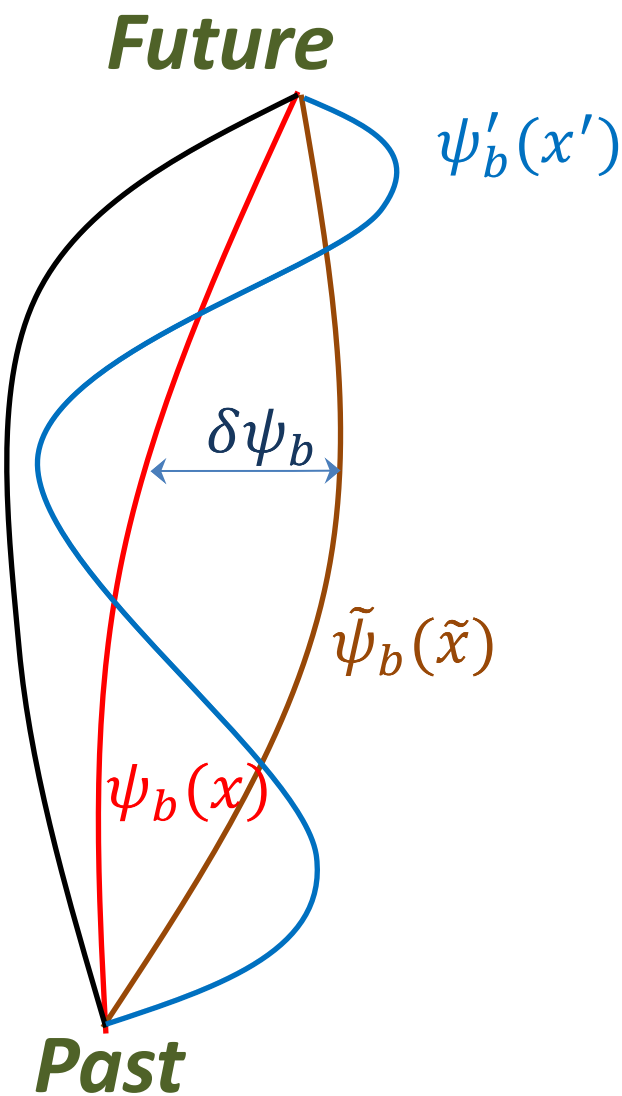
coordinates

the metric tensor

Covariant derivatives

Fundamental physical constants

# The principle of a stationary action




The path taken by the system has a stationary action ($\delta S = 0$) under small changes $\delta\psi_c(x)$ in the configuration of the system.

$$\delta\psi_b = \tilde{\psi}_b(\tilde{x}) - \psi_b(x)$$

Field equations/equations of motion

$$\frac{\delta(\sqrt{-g}L)}{\delta\psi_b} = 0$$

# Lagrangian

$$L = L_E(g_{\alpha\beta}; g_{\alpha\beta,\mu}) + L_M(\phi_b; \phi_{b;\alpha}) + L_I(\phi_b, \phi_{b;\alpha}; \psi_c, \phi_{c;\alpha})$$

Einstein's Lagrangian

Depends only on gravity variables – the metric tensor and its first derivatives

Gravitoelectric field
Gravitomagnetic field
Gravitational waves

Matter Lagrangian

Depends on both matter and gravity variables through the covariant derivative

The minimal coupling principle
The equivalence principle
Special Relativity principle

Lagrangian of Interaction of matter fields (PPN parameters)

# Parametrized post-Newtonian (PPN) Formalism

- A global barycentric coordinate system $x^\alpha = (ct, \vec{x})$ (BCRS)
- A metric tensor $g_{\mu\nu}(ct, \vec{x} \mid \gamma, \beta, \xi, \ldots)$ = gravitational field potentials: depends on 10 PPN parameters

$\gamma$ - curvature of space ( = 1 in GR)

$\beta$ - non-linearity of gravity ( = 1 in GR)

$\xi$ - preferred location effects (= 0 in GR)

$\alpha_1, \alpha_2, \alpha_3$ - preferred frame effects (= 0 in GR)

$\zeta_1, \zeta_2, \zeta_3, \zeta_4$ - violation of the linear momentum conservation (=0 in GR)

- Stress-energy tensor: a perfect fluid in most cases
- Stress-energy tensor is conserved ("comma goes to semicolon" rule)
- Test particles move along geodesics
- Maxwell equations obey the principle of equivalence ("comma goes to semicolon" rule)

# Example: PPN β and γ parameters as fundamental constants of Nature

$$S = \frac{c^3}{16\pi}\int_{R^4}\left[\underbrace{\phi R}_{\text{gravity}} + \underbrace{\theta(\phi)\frac{\phi^{,\alpha}\phi_{,\alpha}}{\phi}}_{\text{scalar}} + \underbrace{\Lambda(\psi)}_{\text{matter}}\right]\sqrt{-g}\,d^4x$$

$$\phi = \phi_0\left(1+\zeta\right) \qquad \theta(\phi) = \omega + \omega'\zeta + \frac{1}{2}\omega''\zeta^2 + \dots$$

$$G \equiv \frac{1}{\phi_0}$$

$$\gamma \equiv 1 - \frac{1}{\omega+2}$$

$$\beta \equiv 1 + \frac{\omega'}{4(2\omega+3)(\omega+2)^2}$$

$$U = \frac{GM}{r}$$

$$g_{ij} = \delta_{ij}\left[1 + \left(1+\gamma\right)\frac{U}{c^2}\right]$$

$$g_{00} = -1 + \frac{2U}{c^2} + \beta\frac{2U^2}{c^4}$$

# Fundamental parameters

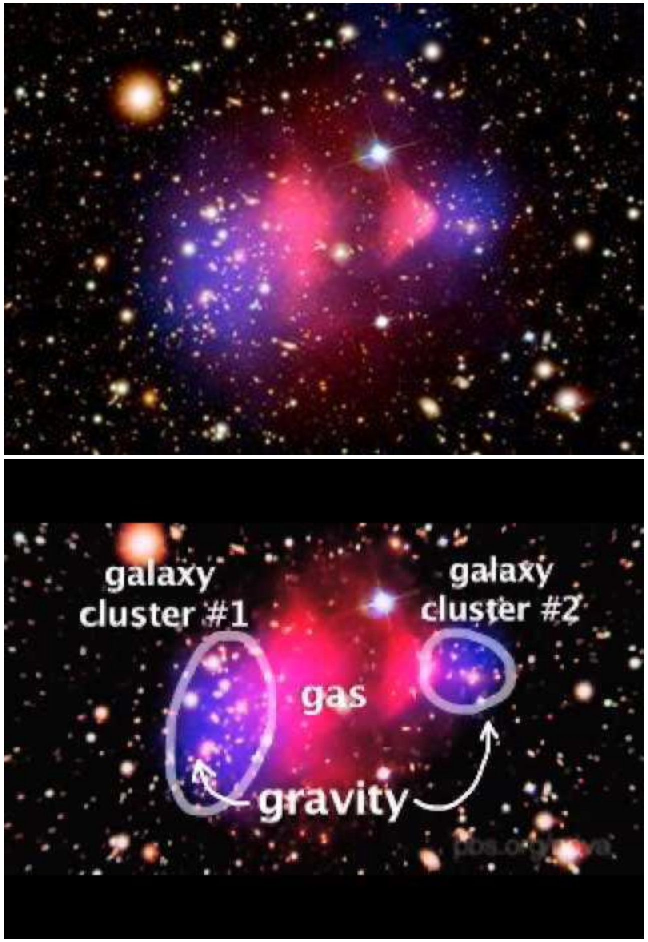
- **Fundamental parameters stay invariant (= keep the same numerical value) under the change of computational algorithm, coordinates, gauge conditions**

- **Measured value converges to a unique limit as the number of observations (normal points) increase.**
- **Examples:**
  - ***c*** ***electrodynamics;***
  - ***G, c*** ***general relativity*;**
  - ***$\beta$, $\gamma$*** ***scalar-tensor theory;***
  - ***Some of the post-Newtonian parameters or a gauge-invariant combination of the post-Newtonian parameters made up of the integrals of motion and/or adiabatic invariants.***

# Zoo of alternative gravity theories

- Alternative ("classic") theories of gravity with short-range forces
  - Scalar-tensor
  - Vector-tensor
  - Tensor-tensor
  - Non-symmetric connection (torsion)

  MOND, TeVeS (Milgrom, Bekenstein)
- Extra dimensions (Kaluza-Klein, etc.)
- Gauge theories on a fiber bundles
  - Standard Model Extension (SME)
- Super-gravity, M-theory
- Strings, p-branes
- Loop quantum gravity
- Dark matter, dark energy




The Bullet Cluster --
a harbor of dark matter

# Hierarchy of Relativistic Test Experiments

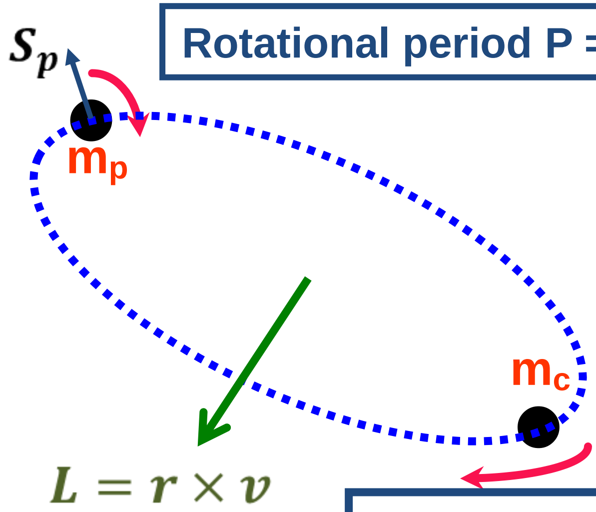
- Laboratory (torsion balance, atomic clocks, LHC,…)
- Earth-Moon System (weak-field tests: GNSS, GPB, SLR, LLR)
- Solar System (weak-field tests: deep-space spacecraft tracking, astrometry, VLBI, interplanetary ranging)
- Binary/Double Pulsars (strong field tests: pulsar timing)
- Gravitational Waves (strong-field tests: LIGO, VIRGO, PTA)
- Cosmology (strong-field tests: COBE, PLANCK, SKA,…)

# Gravitational Waves
## *the evidence through pulsar timing*

Hulse & Taylor binary pulsar

PSR 1913 + 16 -- discovered in 1974

Rotational period P = 1/17 sec



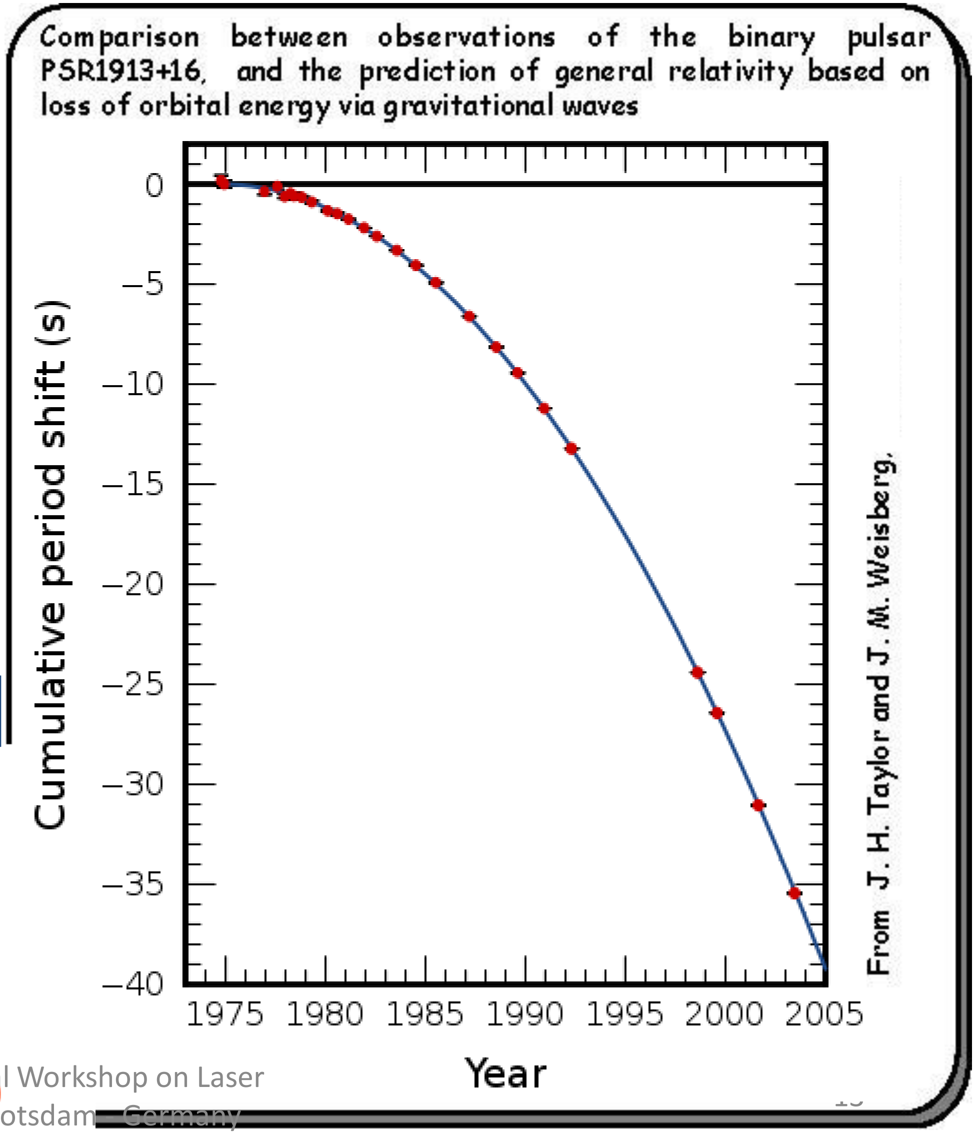

Orbital period $P_b$~ 8 hr

**Neutron Star Binary System**

- separated by $10^6$ miles
- $m_p = 1.4\ m_{\odot}$ ; $m_c = 1.36\ m_{\odot}$ ; $e = 0.617$

**Prediction from general relativity**

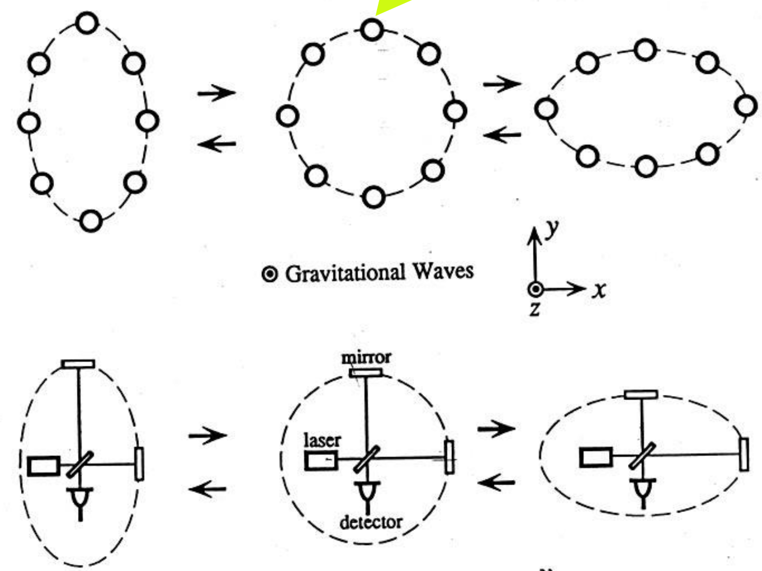
- the pulsar spirals in by 3 mm/orbit
- decrease of the orbital period (orbital decay)

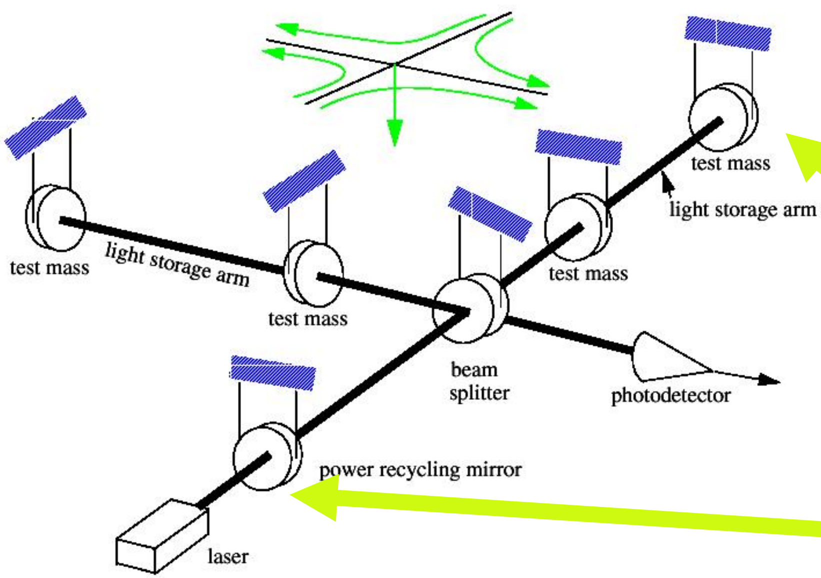


# The principle of detection of gravitational waves

**International network (LIGO, Virgo, GEO, TAMA, AIGO) of suspended mass Michelson-type interferometers on earth's surface detect signals from distant astrophysical sources**

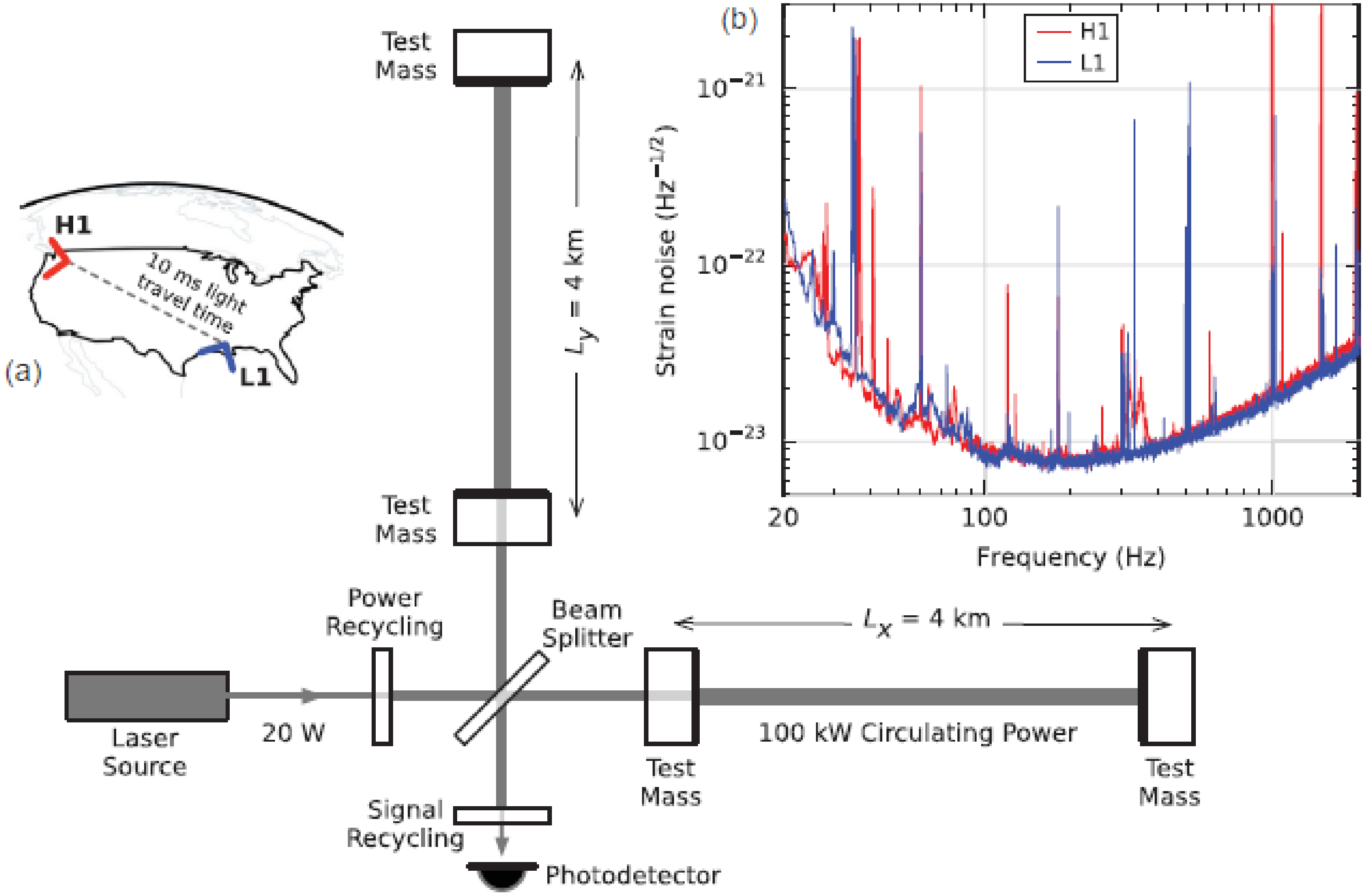
**free masses**



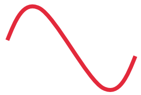
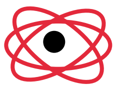
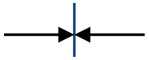




**suspended test masses**

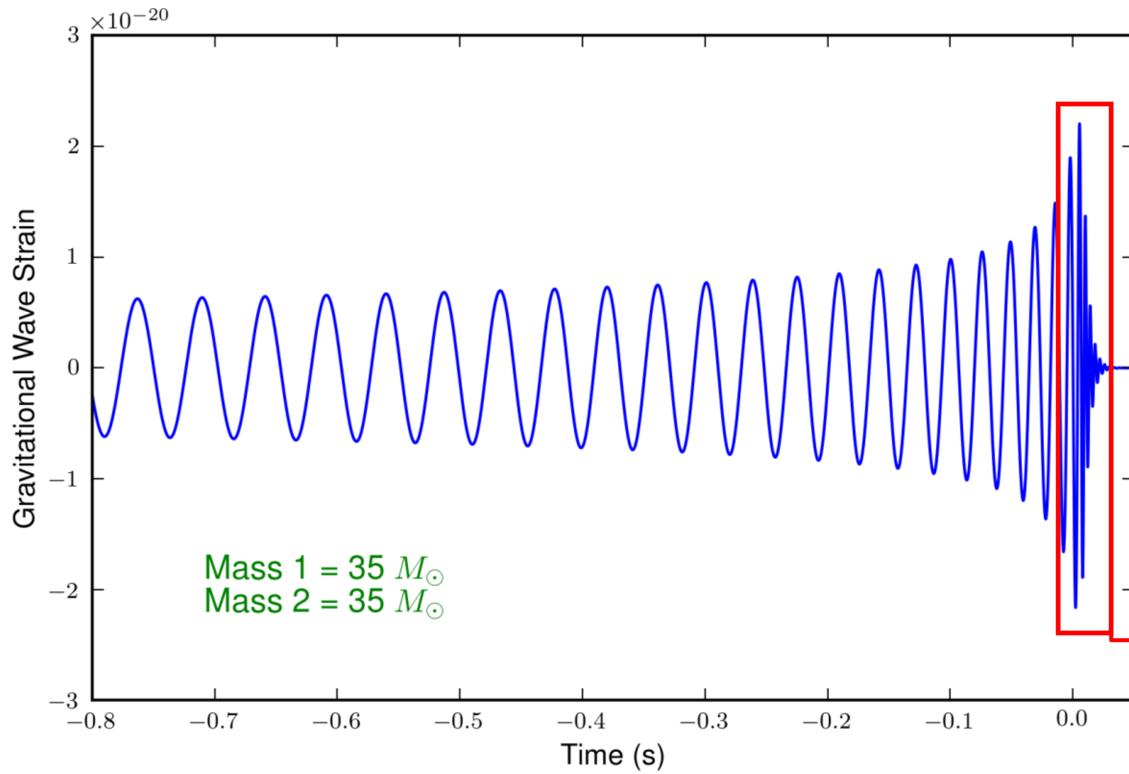
$$\frac{\Delta L}{L} = 10^{-23} \leftrightarrow \Delta L \simeq 10^{-20} \text{ meter!}$$

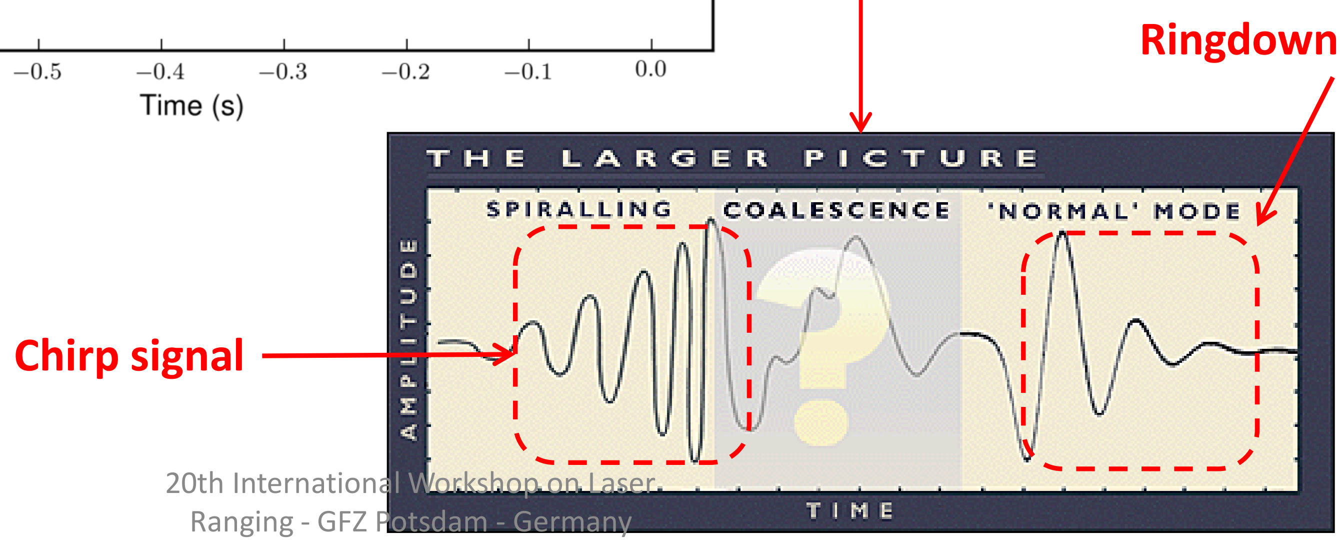


# How small is $10^{-20}$ meter?

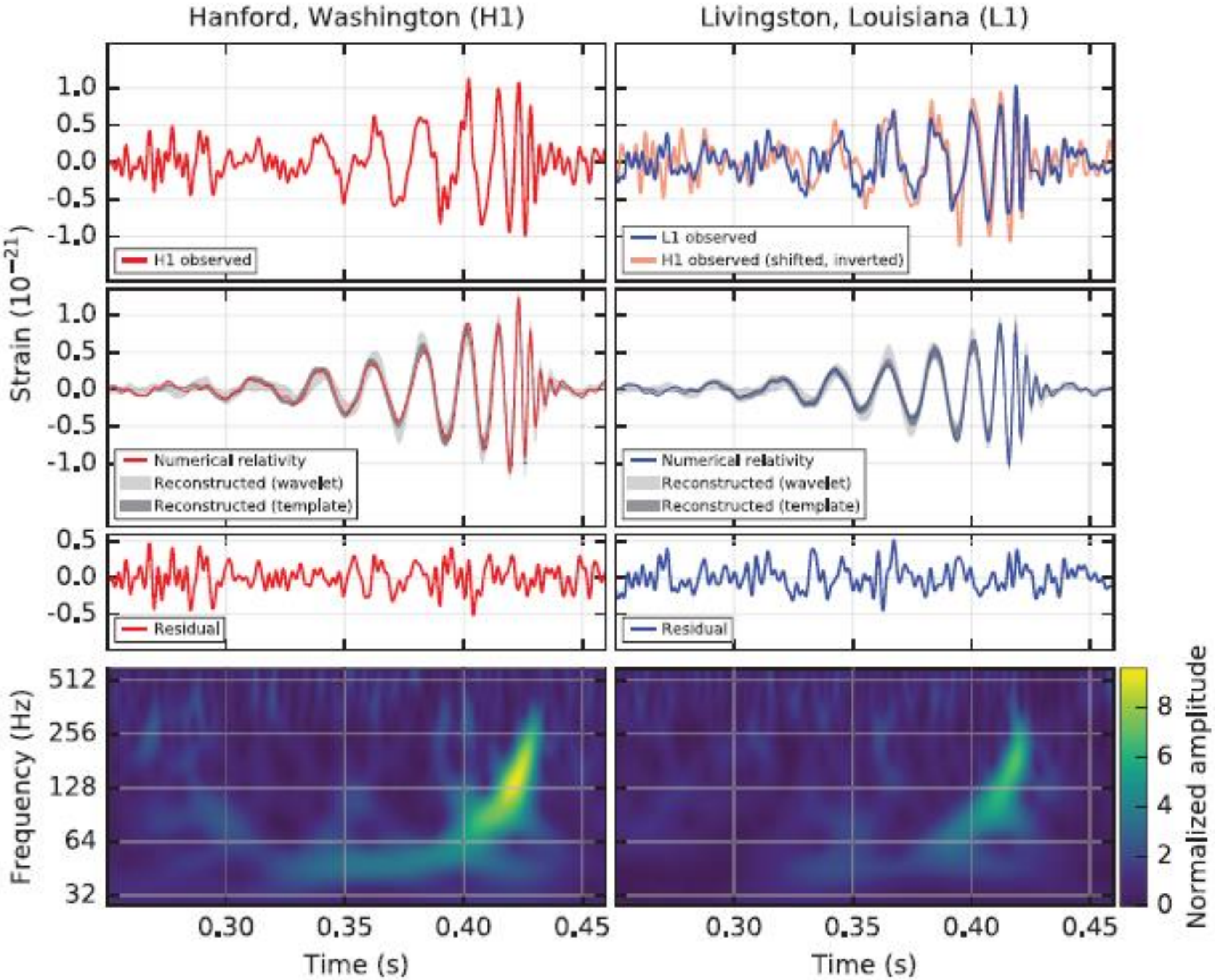
| | | |
|---|---|---|
| | | *One meter, about 40 inches* |
| ÷10,000 |  | *Human hair, about 100 microns* |
| ÷100 |  | *Wavelength of light, about 1 micron* |
| ÷10,000 |  | *Atomic diameter,* $10^{-10}$ *meter* |
| ÷10,000 |  | *Nuclear diameter,* $10^{-14}$ *meter* |
| ÷10 |  | *Proton/neutron,* $10^{-15}$ *meter* |
| ÷1,000 |  | *Electron,* $< 10^{-18}$ *meter* |
| ÷100 |  | *LIGO sensitivity,* $\sim 10^{-20}$ *meter* |

# GW Template of a Coalescing BH system



# Gravitational wave signal GW150914

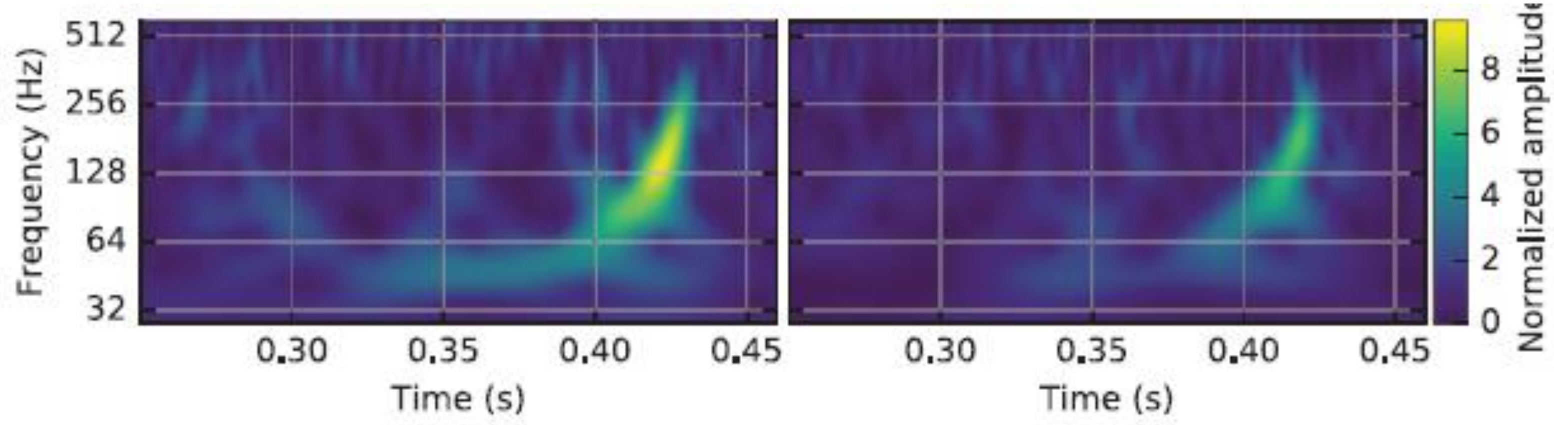


# GW150914: FACTSHEET

| | |
|---|---|
| observed by | LIGO L1, H1 |
| source type | black hole (BH) binary |
| date | 14 Sept 2015 |
| time | 09:50:45 UTC |
| likely distance | 0.75 to 1.9 Gly<br>230 to 570 Mpc |
| redshift | 0.054 to 0.136 |
| signal-to-noise ratio | 24 |
| false alarm prob. | < 1 in 5 million |
| false alarm rate | < 1 in 200,000 yr |
| Source Masses | M⊙ |
| total mass | 60 to 70 |
| primary BH | 32 to 41 |
| secondary BH | 25 to 33 |
| remnant BH | 58 to 67 |
| mass ratio | 0.6 to 1 |
| primary BH spin | < 0.7 |
| secondary BH spin | < 0.9 |
| remnant BH spin | 0.57 to 0.72 |
| signal arrival time delay | arrived in L1 7 ms before H1 |
| likely sky position | Southern Hemisphere |
| likely orientation | face-on/off |
| resolved to | ~600 sq. deg. |

| | |
|---|---|
| duration from 30 Hz | ~ 200 ms |
| # cycles from 30 Hz | ~10 |
| peak GW strain | $1 \times 10^{-21}$ |
| peak displacement of interferometers arms | ±0.002 fm |
| frequency/wavelength at peak GW strain | 150 Hz, 2000 km |
| peak speed of BHs | ~ 0.6 c |
| peak GW luminosity | $3.6 \times 10^{56}$ erg $s^{-1}$ |
| radiated GW energy | 2.5-3.5 M⊙ |
| remnant ringdown freq. | ~ 250 Hz |
| remnant damping time | ~ 4 ms |
| remnant size, area | 180 km, $3.5 \times 10^{5}$ $km^2$ |
| consistent with general relativity? | passes all tests performed |
| graviton mass bound | $< 1.2 \times 10^{-22}$ eV |
| coalescence rate of binary black holes | 2 to 400 $Gpc^{-3}$ $yr^{-1}$ |
| online trigger latency | ~ 3 min |
| # offline analysis pipelines | 5 |
| CPU hours consumed | ~ 50 million (=20,000 PCs run for 100 days) |
| papers on Feb 11, 2016 | 13 |
| # researchers | ~1000, 80 institutions in 15 countries |

# Public outreach:




**Guest | Feb 12, 2016 6:09 AM**

**"I feel that there is a very big blunder. The two diagrams show the recording of two sound signals from the collision of two black holes. However these sound signals can not propagate in vacuum.**

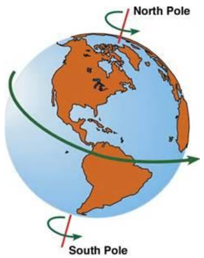
**How then the sound of the collision of the black holes came to Earth?!!! "**

# Relativity in Global Positioning System

- The combined effect of the second order Doppler shift (equivalent to time dilation) and gravitational red shift phenomena causes the GPS clock to run fast by 38 $\mu$s per day.
- **The residual orbital eccentricity causes a sinusoidal variation over one revolution between the time readings of the satellite clock and the time registered by a similar clock on the ground. This effect has typically a peak-to-peak amplitude of 60 - 90 ns.**
- The Sagnac effect – for a receiver at rest on the equator is 133 ns, it may be larger for moving receivers.
- **At the sub-nanosecond level additional corrections apply, including the contribution from Earth's oblateness, irregularity of the Earth's rotation, tidal effects, the Shapiro time delay, and other post Newtonian effects (ISSI Workshop 2015, Bern)**
- GREAT GR tests experiment (ZARM, SYRTE, ISLR) in progress from May 1, 2016 – the goal is to improve on the GP-A limit $1\times10^{-4}$ in measuring the gravitational red shift down to an uncertainty around $(3–4)\times10^{-5}$ after one year of integration of Galileo-201 data. ACES time transfer experiment (U. Schreiber et al, this workshop)

# Time Scales in Fundamental Physics

Kopeikin, PRD, **86**, 064004 (2012) "Celestial ephemerides in an expanding universe"

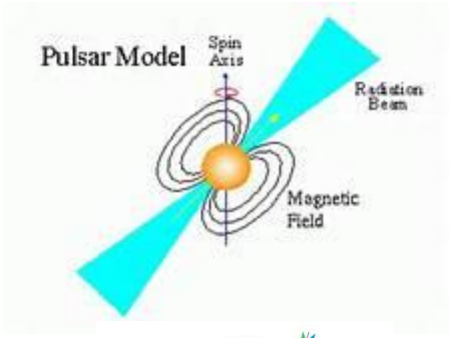


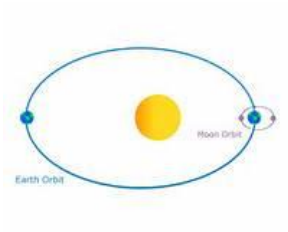

## Universal Time (UT)

$$\mathrm{UT} = \frac{\text{Inertia Tensor}}{\text{Spin}}$$

## Pulsar Time (PT)

$$\mathrm{PT} = k\,\frac{G^2 M^3}{c^4 \text{Spin}}$$



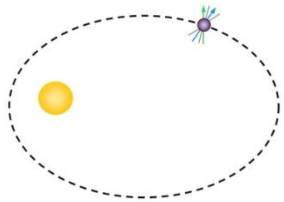



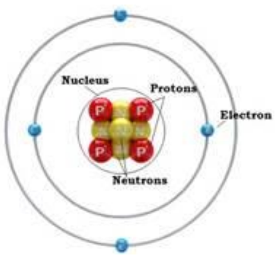

## Ephemeris Time (ET)

$$\mathrm{ET} = \frac{1}{2\pi}\sqrt{\frac{R^3}{GM}}$$

## Binary Pulsar Time (BPT)

$$\mathrm{BPT} = \frac{1}{2\pi}\sqrt{\frac{R^3}{GM}}$$



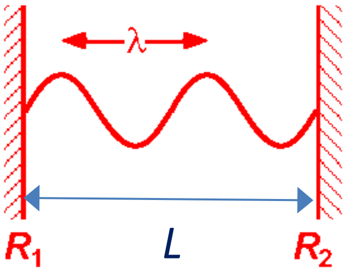



## Atomic Time (TAI)

$$\mathrm{TAI} = \frac{h^3 \varepsilon_0^2}{m_e e^4}$$

**"Time is merely an illusion" - A. Einstein**

**"Make time an observable" - U. Schreiber**



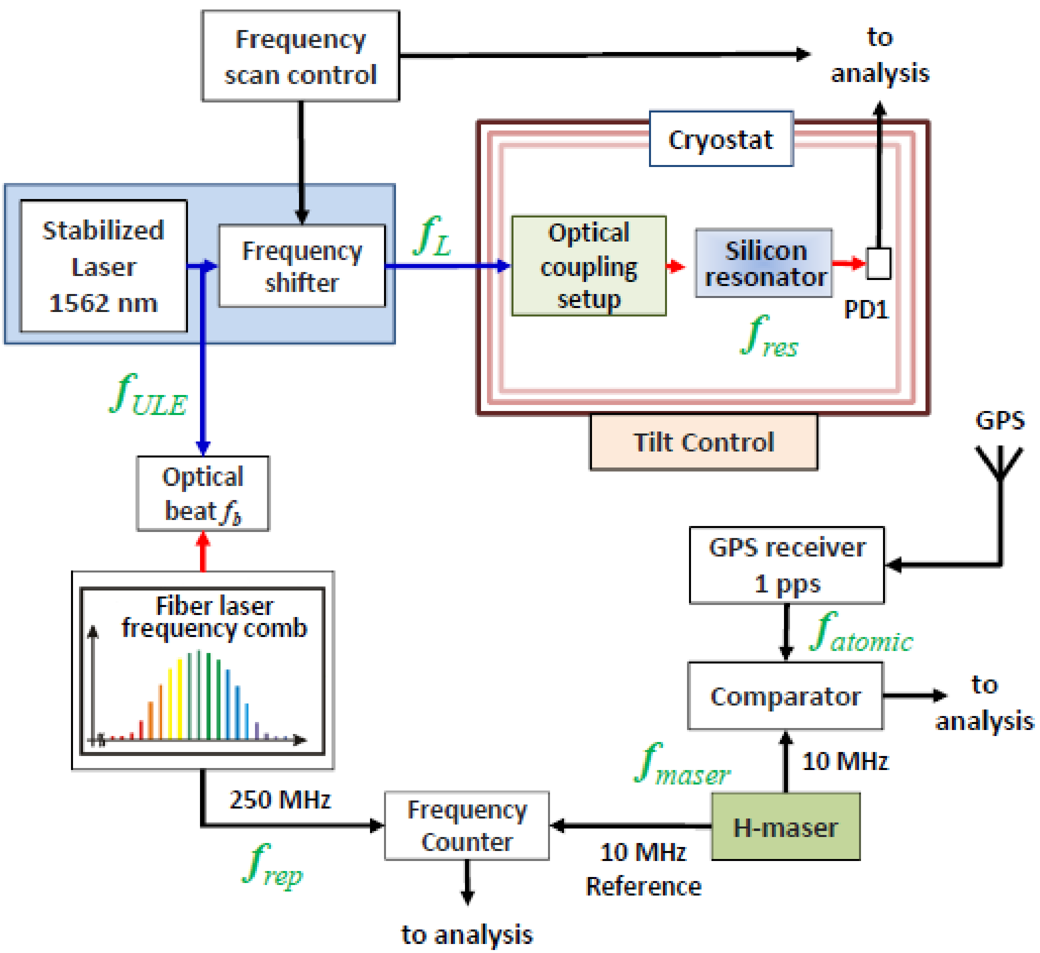

## Einstein's Time (optical cavity)

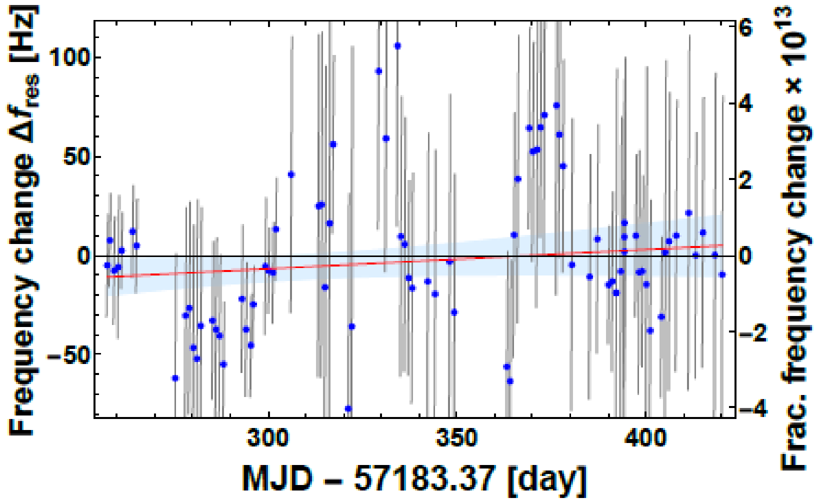
$$\mathrm{T} = \frac{L}{c}$$

Kopeikin, Gen. Rel. Grav., **47**, 5 (2015)
"Optical cavity resonator in an expanding universe"

# Optical cavity as a probe of the local Hubble expansion

S. Schiller et al., Univ. Düsseldorf (submitted)

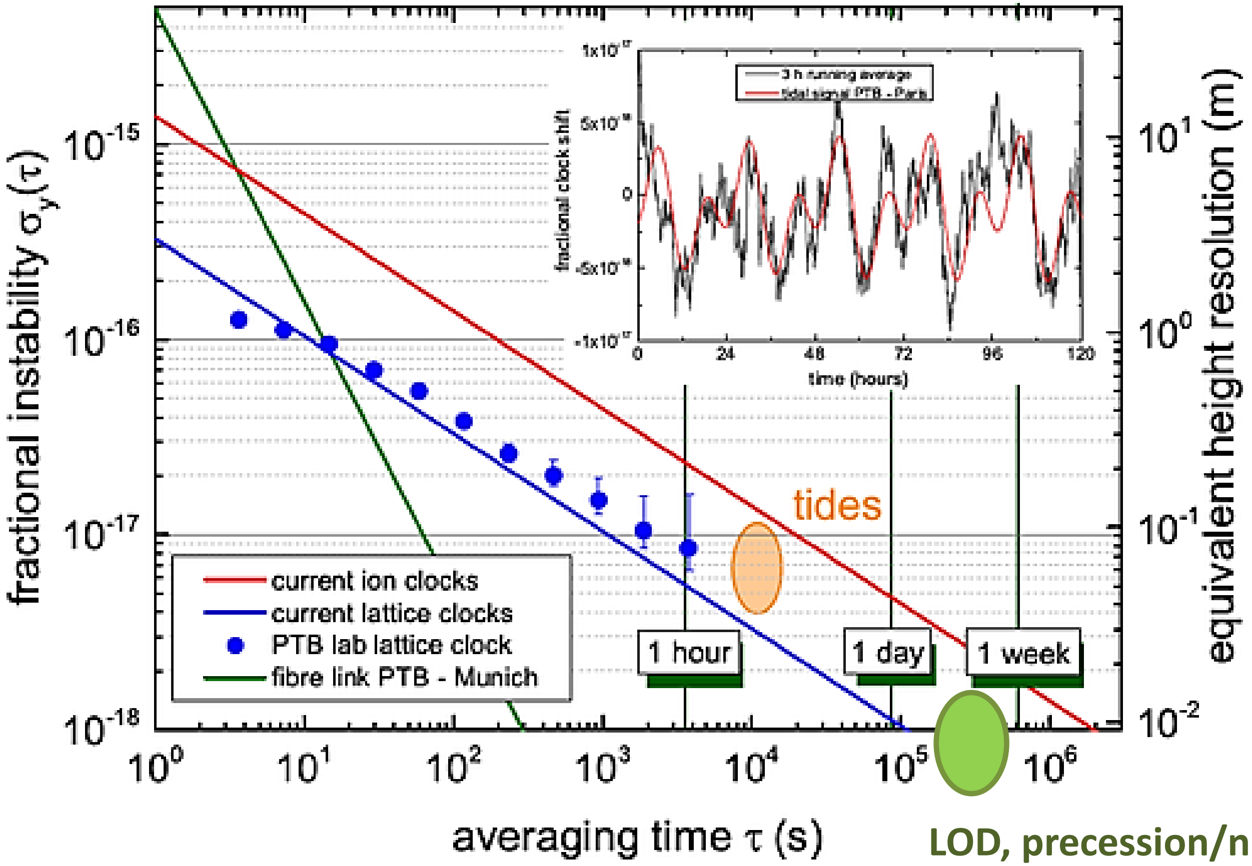


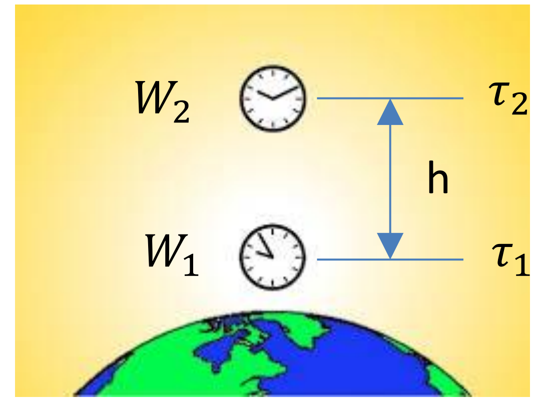

Resonator optical frequency variation $\Delta f_{res}$ corrected for $f_{maser}$ drift. Blue points are the measured values of $f_{res}$ The bars indicate the range twice the standard deviation. Red line: time-linear fit, exhibiting a drift rate $D_{res-maser} = 5.1 \times 10^{-21}/s$. Blue shaded area: $2\sigma$ uncertainty range of the time-linear fit. Zero ordinate value is defined as the mean of the data points.

# Optical clocks for relativistic geodesy.

http://www.geoq.uni-hannover.de/a03.html




LOD, precession/nutation




$$\Delta\tau = -\frac{W_2 - W_1}{c^2}\, t$$

# Optical clocks for TAI realization.

Fateev & Kopeikin: Measur. Tech., 58, 647 (2015)

$$\tau_i = \left(1 - \frac{W_i}{c^2}\right) t - \frac{1}{c^2} \int_{t_0}^{t} \left[\frac{1}{2}(\boldsymbol{\Omega}(\mathrm{t}) \times \boldsymbol{R}_i)^2 + (1 + k - h) U_{tide}(t)\right] dt$$

# Relativistic Geodesy: Altai Mountain Experiment

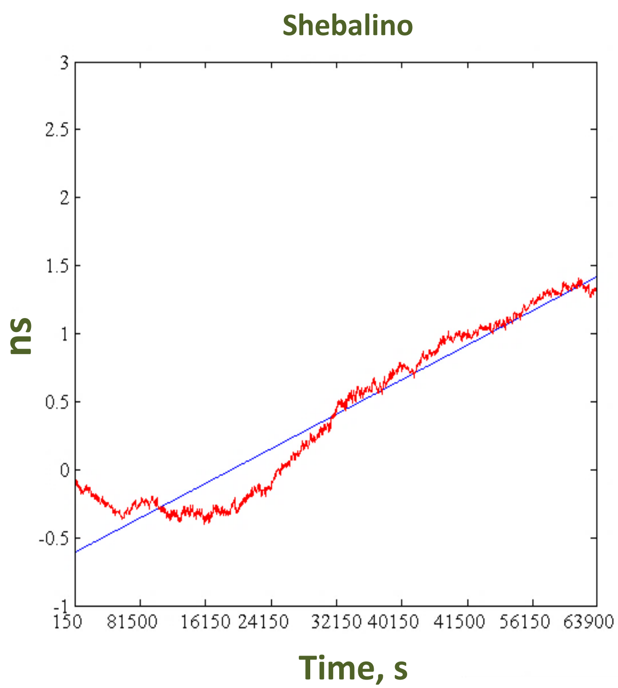
Kopeikin et al. Gravitation and Cosmology, **22,** 234 (2016)

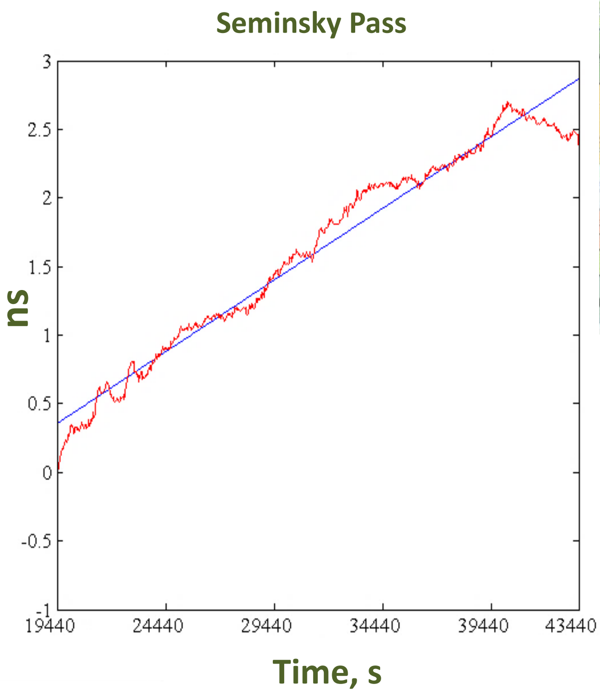
**Stationary:** **cesium clock with instability** $\sim 10^{-15}$

**Transportable:** **hydrogen clock with instability** $\sim 10^{-14}$

**Route: Novosibirsk -> Shebalino -> Seminsky Pass -> Novosibirsk** (Height difference **850** m)

**Time transfer : "Common View" GLONASS/GPS**





$$\left(\frac{\delta f}{f_0}\right)_{\mathrm{GNSS}} = 9.5\times10^{-14} \pm 1.5\times10^{-17}$$

$\Delta h$ =859 m

$$\left(\frac{\delta f}{f_0}\right)_{\mathrm{clock}} = 7.9\times10^{-14} \pm 7.3\times10^{-15}$$

$\Delta h = 725 \pm 64$ m

# Solar System Tests

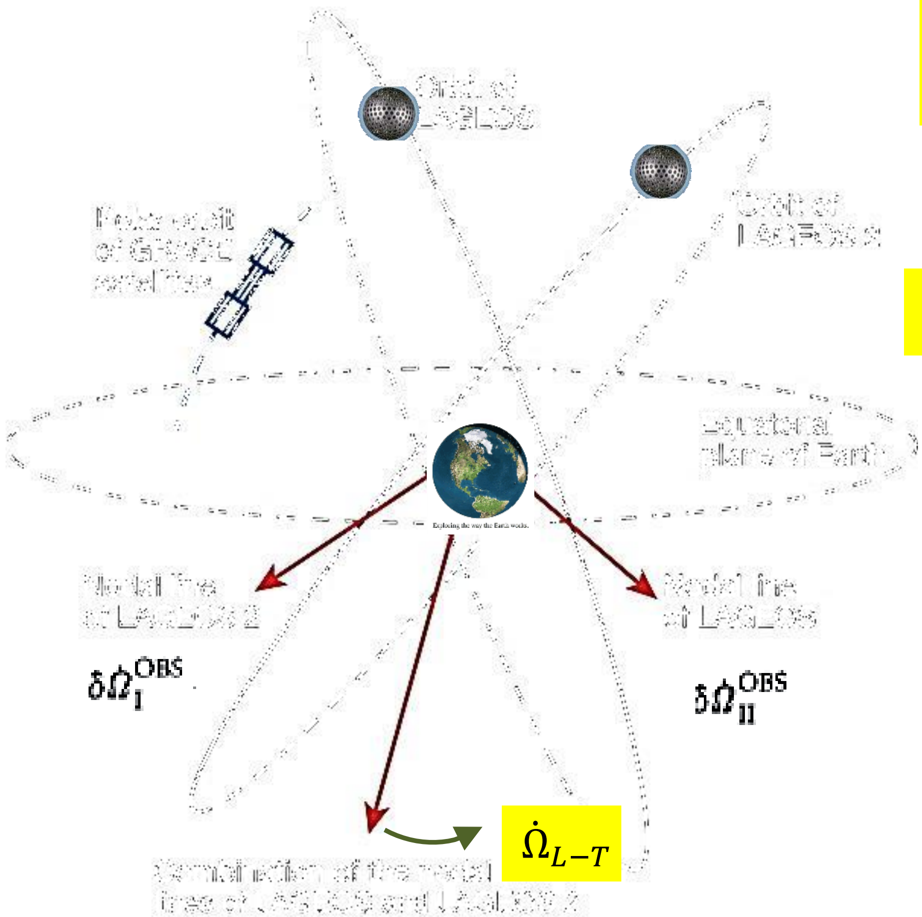
- **Advance of Perihelion**
- **Bending of Light**
- **Shapiro Time Delay**
- **Gravitomagnetic Field Measurement**

  – **The field induced by rotational mass current**
    - ***LAGEOS/LARES***
    - ***Gravity Probe B***

  – **The field induced by translational mass current**
    - ***Cassini***
    - ***VLBI Planetary Time Delay***

# LAGEOS/LARES: spin-orbital interaction

(*Ciufolini, PRL, **56**, 278, 1986*)




$$\dot{\Omega}_{L-T} = \frac{2S_{\oplus}}{a^{3}(1-e^{2})^{3/2}}$$

$$\dot{\Omega}_{L-T} = 31 \text{ mas yr}^{-1}$$

$$\dot{\Omega}_{L-T(total)} = 48 \text{ mas yr}^{-1}$$

Gravitomagnetic field measured with 10% error budget:
Ciufolini & Pavlis, Nature 2004

$$\delta\Omega_{I}^{OBS} + k\delta\Omega_{II}^{OBS} = \dot{\Omega}_{I}^{Lense-Thirring} + k\,\dot{\Omega}_{II}^{Lense-Thirring} \pm \sum_{2n\geq 4}\left(K_{I}^{2n}|\delta J_{2n}| + k\,K_{II}^{2n}|\delta J_{2n}|\right)$$

J2 perturbation is totally suppressed with *k* = 0.545

# A test of general relativity using the LARES and LAGEOS satellites and a GRACE Earth gravity model.

European Phys. J. C (2016) 76:120

(I. Ciufolini, A. Paolozzi, E. Pavlis, R. Koenig, J. Ries, et al)

**Table 1** Main characteristics and orbital parameters of the satellites used in the LARES experiment

| | LARES | LAGEOS | LAGEOS 2 | GRACE |
|---|---|---|---|---|
| Semimajor axis (km) | 7821 | 12270 | 12163 | 6856 |
| Eccentricity | 0.0008 | 0.0045 | 0.0135 | 0.005 |
| Inclination | 69.5° | 109.84° | 52.64° | 89° |
| Launch date | 13 Feb 2012 | 4 May 1976 | 22 Oct 1992 | 17 Mar 2002 |
| Mass (kg) | 386.8 | 406.965 | 405.38 | 432 |
| Number of CCRs | 92 | 426 | 426 | 4 |
| Diameter (cm) | 36.4 | 60 | 60 | |

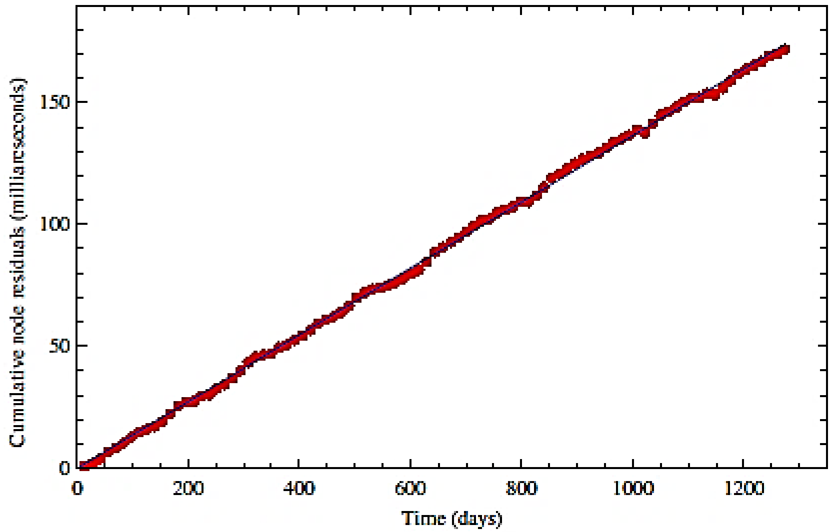


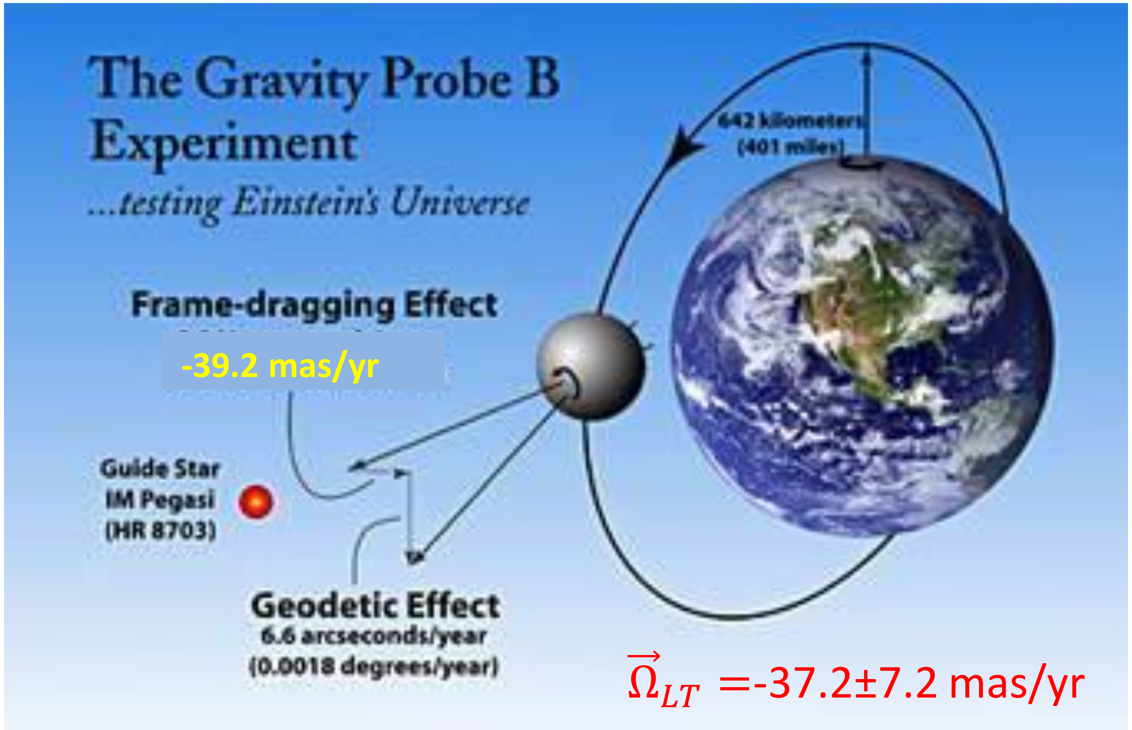

**Fig. 4** Fit of the cumulative combined nodal residuals of LARES, LAGEOS, and LAGEOS 2 with a linear regression plus six periodical terms corresponding to six main tidal perturbations observed in the orbital residuals

$$\boldsymbol{\mu = (0.994 \pm 0.002) \pm 0.05}\ \ \textbf{(experiment)}$$

$$\boldsymbol{\mu = 1}\ \ \textbf{(general relativity)}$$

# Gravity Probe B: spin-spin interaction

## Leonard I. Schiff (1960) – with R. Cannon and W. Fairbank




$$\frac{d\vec{S}}{d\tau} = \vec{\Omega} \times \vec{S}$$

$$\vec{\Omega} = \vec{\Omega}_S + \vec{\Omega}_{LT} + \vec{\Omega}_T$$

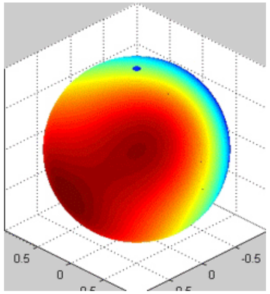
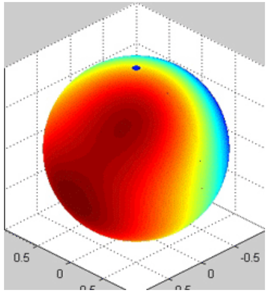
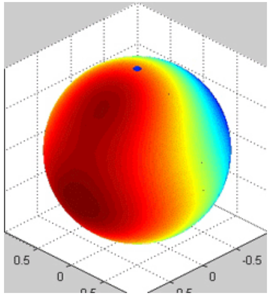
$$\vec{\Omega}_S = \left(\gamma + \frac{1}{2}\right)\frac{GM_\oplus}{c^2}\frac{\vec{r}\times\vec{v}}{r^3}$$

$$\vec{\Omega}_{LT} = -\frac{1}{2}\left(1+\gamma+\frac{1}{4}\alpha_1\right)\frac{GS_\oplus}{c^2}\frac{\vec{s}-3\vec{n}\left(\vec{n}\cdot\vec{s}\right)}{r^3}$$

$$\vec{\Omega}_T = \vec{v}\times\vec{A}$$

**Residual noise: GP-B Gyro #1 Polhode Motion (torque-free Euler-Poinsot precession)**

From website: http://einstein.stanford.edu/highlights/hl_polhode_story.html

Mission begins  =>  =>  Mission ends

# Gravitational Time Delay by a moving body

$$h_{00}=\frac{2GM}{|\mathbf{x}-\mathbf{z}(t)|}\qquad h_{ij}=\frac{2GM\,\delta_{ij}}{|\mathbf{x}-\mathbf{z}(t)|}\qquad h_{0i}=\frac{4GM}{|\mathbf{x}-\mathbf{z}(t)|}\left(\frac{\mathbf{v}}{c_g}\right)$$

photon: $\mathbf{x}\mapsto\mathbf{x}_N(t)=\mathbf{x}_0+c\mathbf{k}(t-t_0)$ massive body: $\mathbf{z}(t)=\mathbf{z}_0+\mathbf{v}(t-t_0)$

$$\Delta(t_1,t_0)=2\frac{GM}{c^3}\left(1-\frac{1}{c_g}\mathbf{k}\cdot\mathbf{v}\right)\ln\left[\frac{|\mathbf{x}_1-\mathbf{z}(s_1)|-\mathbf{k}\cdot\left(\mathbf{x}_1-\mathbf{z}(s_1)\right)}{|\mathbf{x}_0-\mathbf{z}(s_0)|-\mathbf{k}\cdot\left(\mathbf{x}_0-\mathbf{z}(s_0)\right)}\right]$$

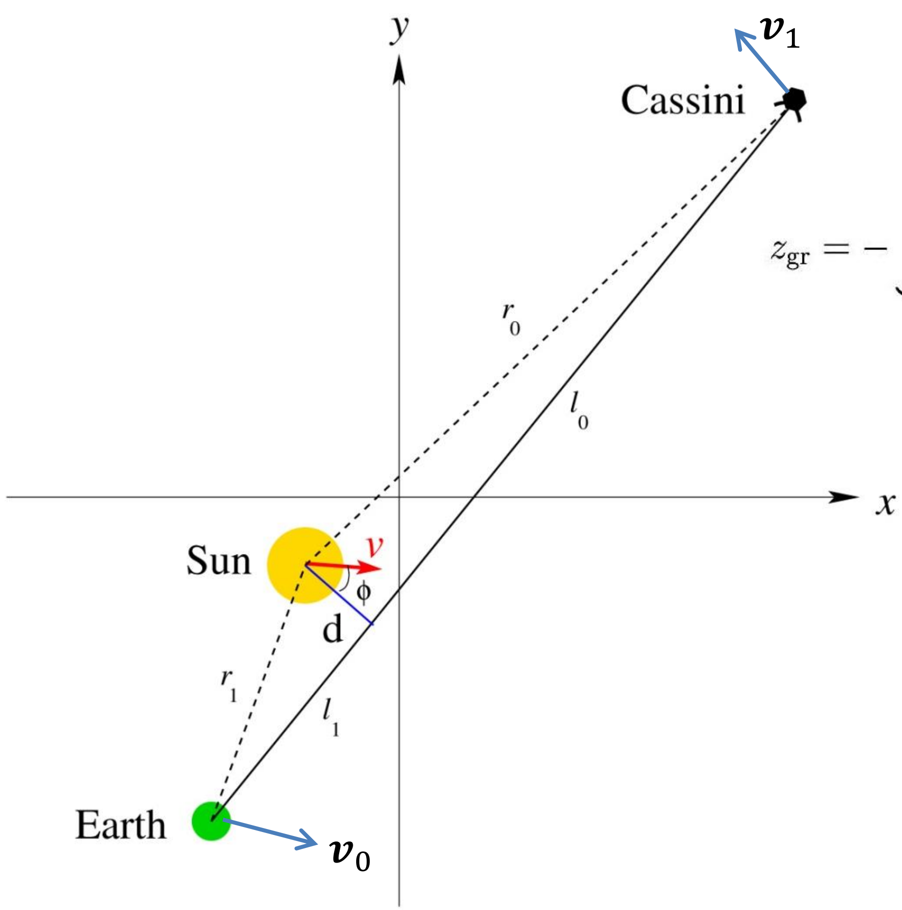
$$\mathbf{z}(s_1)=\mathbf{z}(t_1)-\frac{\mathbf{v}}{c_g}|\mathbf{x}_1-\mathbf{z}(t_1)|+O\left(\frac{v^2}{c_g^2}\right)\qquad \mathbf{z}(s_0)=\mathbf{z}(t_0)-\frac{\mathbf{v}}{c_g}|\mathbf{x}_0-\mathbf{z}(t_0)|+O\left(\frac{v^2}{c_g^2}\right)$$

Look like a retarded time

$$s_1=t_1-\frac{1}{c_g}|\mathbf{x}_1-\mathbf{z}(t_1)|\qquad s_0=t_0-\frac{1}{c_g}|\mathbf{x}_0-\mathbf{z}(t_0)|$$

# Gravitomagnetic Field in the Cassini Experiment

Kopeikin et al., Phys. Lett. A, **367**, 276 (2007)




Gravitomagnetic Doppler shift due to the orbital motion of the Sun

$$z_{gr} = \underbrace{-\frac{l_0}{cr}(\boldsymbol{v}_1\cdot\boldsymbol{\alpha}_B)}_{\text{observer shift } z_O} \underbrace{-\frac{l_1}{cr}(\boldsymbol{v}_0\cdot\boldsymbol{\alpha}_B)}_{\text{satellite shift } z_S} + \underbrace{\frac{1}{c_g}(\boldsymbol{v}_\odot\cdot\boldsymbol{\alpha}_B)}_{\text{gravimagnetic shift } z_{GM}}$$

$$\boldsymbol{\alpha}_B = \alpha_\odot \frac{1+\gamma}{2}\frac{R_\odot}{d}\hat{\boldsymbol{d}} \,,$$

$1.''7505$

Bertotti-less-Tortora, Nature, 2004

$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5}$

However, the gravitomagnetic field contribution has been never analyzed

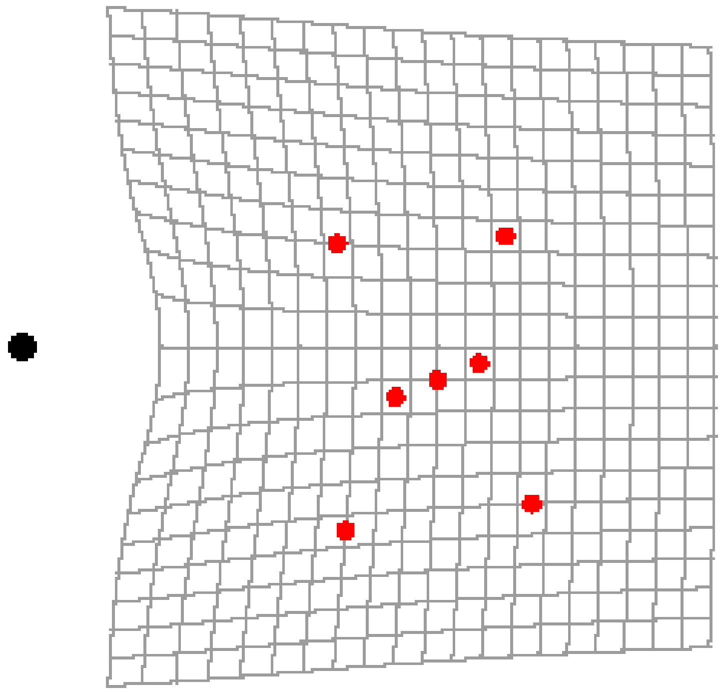
# The speed-of-gravity experiment (2002)

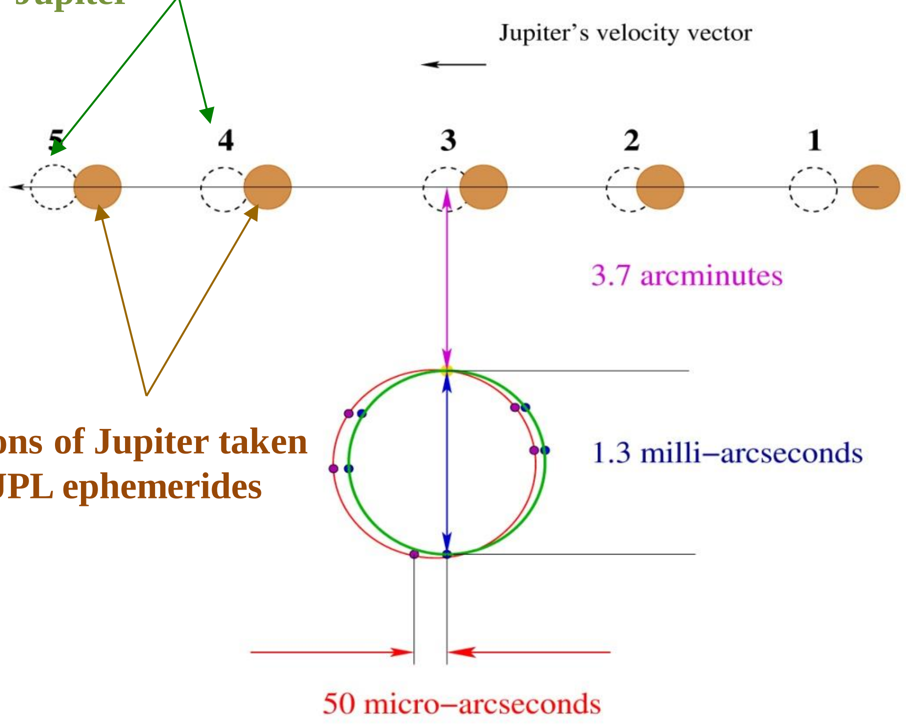
Edward B. Fomalont
Sergei M. Kopeikin

ApJ Lett, **556**, 1-5 (2001)
ApJ, **598**, 704-711(2003)


VLBA support team: NRAO and MPIfR (Bonn)

# The Jovian 2002 experiment



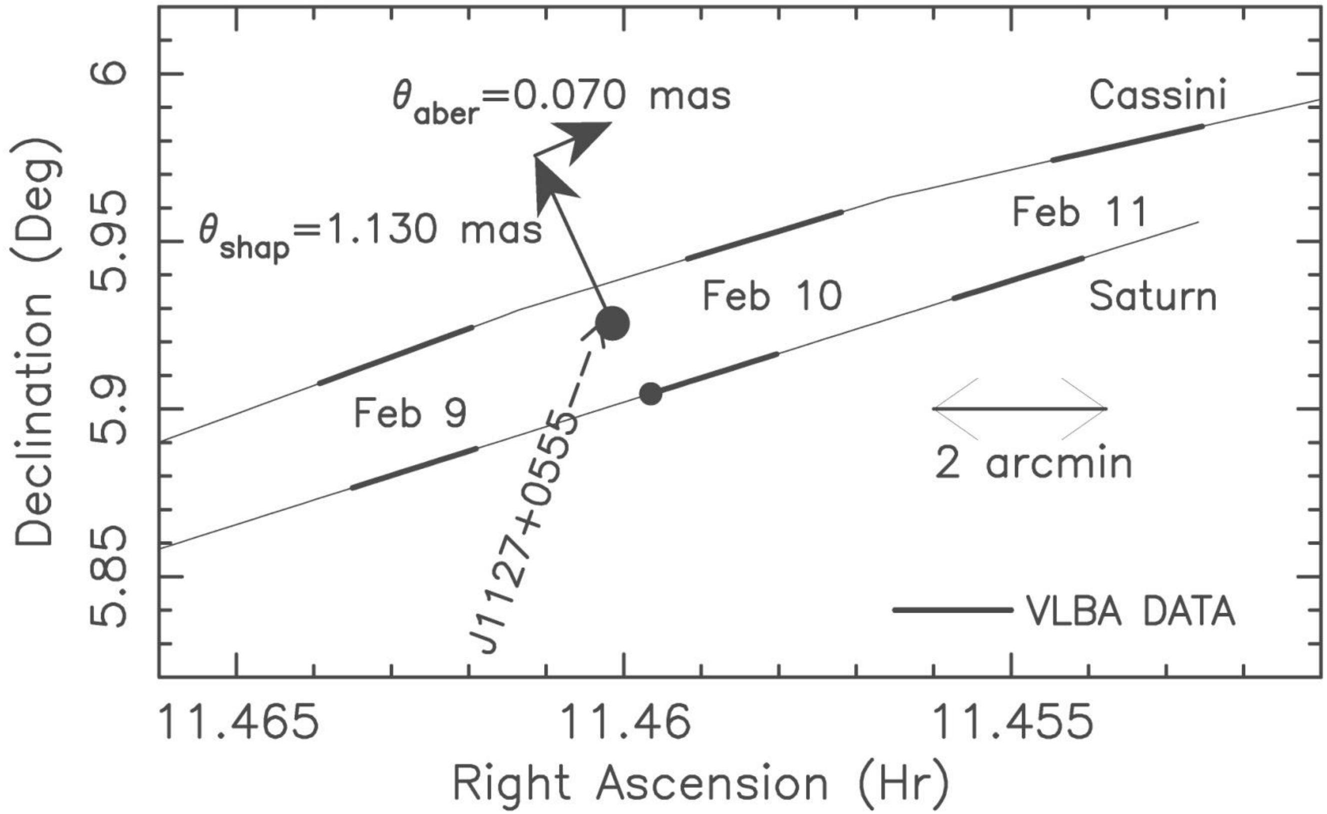



**10 microarcseconds** = the width of a typical strand of a human hair from a distance of 650 *miles*!!!

The retardation effect was measured with 20% of accuracy, thus, proving that the speed of gravity does not exceed the speed of light with 20% of accuracy.

# Light Deflection Experiment with Saturn and Cassini spacecraft

Fomalont, Kopeikin et al., Proc. IAU Symp. **261**, 291-295 (2009)

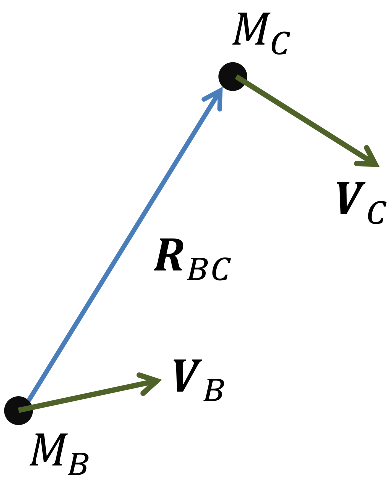


# Does LLR measure the gravitomagnetic field?

Kopeikin, PRL, **98,** 22, 229001 (2007)

Kopeikin & Yi, Cel. Mech. Dyn. Astr., **108**, 245-263 (2010)

$$a_B^i = \sum_{C\neq B}\left[\boldsymbol{E}^i_{BC} + \frac{2+2\gamma-2\lambda_C}{c}\left(\boldsymbol{v}_B\times\boldsymbol{H}_{BC}\right)^i - \frac{1+2\gamma-2\lambda_C}{c}\left(\boldsymbol{v}_C\times\boldsymbol{H}_{BC}\right)^i\right]$$




$$\boldsymbol{E}^i_{BC} = -\frac{GM_C}{R^3_{BC}}\boldsymbol{R}^i_{BC}\left(1+\frac{\mathcal{E}_{BC}}{c^2}\right)$$

**Gravitoelectric field**

$$\boldsymbol{H}^i_{BC} = -\frac{1}{c}\left(\boldsymbol{V}_{BC}\times\boldsymbol{E}_{BC}\right)^i$$

**Gravitomagnetic field**

$$\begin{aligned}\mathcal{E}_{BC} = {} & \left(-1-2\gamma+3\lambda_C\right)v_B^2 + \left(1+2\gamma-6\lambda_C\right)\left(\boldsymbol{v}_B\cdot\boldsymbol{v}_C\right) - \left(\gamma-3\lambda_C\right)v_C^2 - \frac{3}{2}\left(\boldsymbol{N}_{BC}\cdot\boldsymbol{v}_C\right)^2 \\ & -3\lambda_C\left(\boldsymbol{N}_{BC}\cdot\boldsymbol{V}_{BC}\right)^2 - \left(1+2\gamma+2\beta-2\lambda_B\right)\frac{GM_B}{R_{BC}} - 2\left(\gamma+\beta-\lambda_C\right)\frac{GM_C}{R_{BC}} - \\ & -\sum_{D\neq B,C} GM_D R^3_{BC}\left[\frac{1-2\beta+2\lambda_D}{R_{CD}R^3_{BC}} - \frac{2(\gamma+\beta)-\lambda_D}{R_{BD}R^3_{BC}} - \frac{2(\gamma+1)}{R_{BC}R^3_{CD}} - \frac{\lambda_C}{R_{BC}R^3_{BD}} + \frac{\lambda_C}{R_{CD}R^3_{BD}} + \frac{3+4\gamma-2\lambda_D}{2R_{BD}R^3_{CD}}\right] \\ & +\sum_{D\neq B,C} GM_D\left(\boldsymbol{R}_{BC}\cdot\boldsymbol{R}_{BD}\right)\left[\frac{1+2\lambda_C}{2R^3_{CD}} - \frac{\lambda_C}{R^3_{BD}} + \frac{3\lambda_D}{R_{BD}R^2_{BC}} - \frac{3\lambda_D}{R_{CD}R^2_{BC}}\right]\end{aligned}$$

# Ranging Time Delay

$$t_2 - t_1 = \frac{R_{12}}{c} + \sum_B 2\frac{GM_B}{c^3} \ln\left[\frac{R_{1B} + R_{2B} + R_{12}}{R_{1B} + R_{2B} - R_{12}}\right]$$

$$+ \sum_B \boldsymbol{\lambda_B} \frac{GM_B}{c^3} \frac{(R_{1B} - R_{2B})^2 - R_{12}^2}{2R_{1B}R_{2B}R_{12}} (R_{1B} + R_{2B})$$

$$+ \sum_B \boldsymbol{\alpha_B} \frac{GM_B}{c^4} \left[\frac{\boldsymbol{v}_B \cdot \boldsymbol{R}_{1B}}{R_{1B}} - \frac{\boldsymbol{v}_B \cdot \boldsymbol{R}_{2B}}{R_{2B}}\right]$$

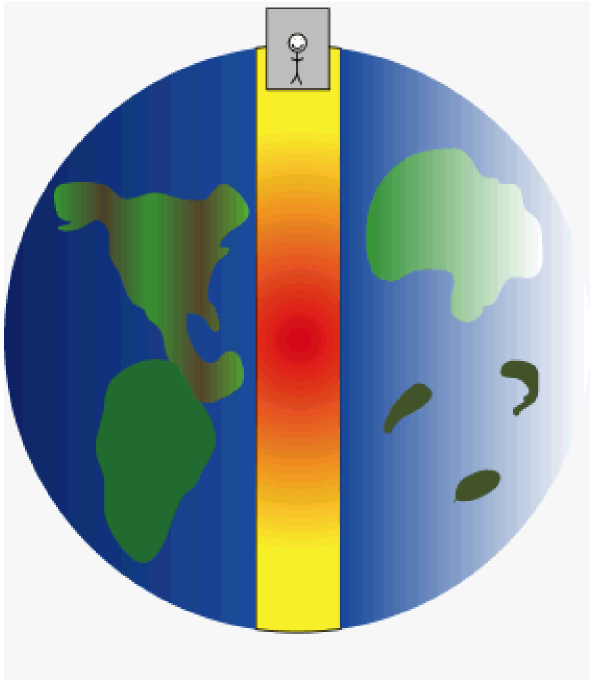
# Deep space experiment to measure G

M.R. Feldman, J.D. Anderson, G. Schubert, V. Trimble, S. Kopeikin, C. Laemmerzahl

*CQG* **33** (2016) 125013 arXiv:1605.02126 [gr-qc]

- Measure *G* with relative uncertainty surpassing 10 parts per million
  - National Science Foundation solicitation NSF 16-520
- Perform in isolated environment with minute and accountable number of forces
  - Relative vacuum of space would work
- Lifetime on the order of years to test reality of a periodic signature

# How to produce lifetime of years?



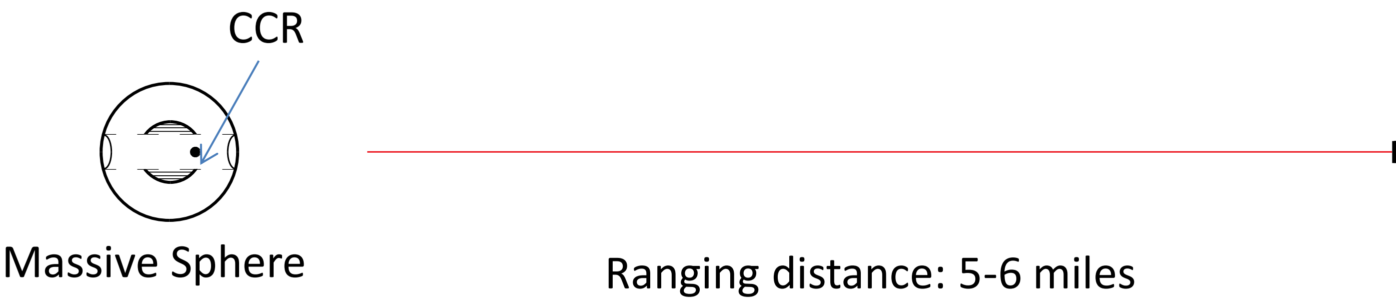
Gif courtesy: By Gotant6884 (http://www.gnu.org/copyleft/fdl.html)

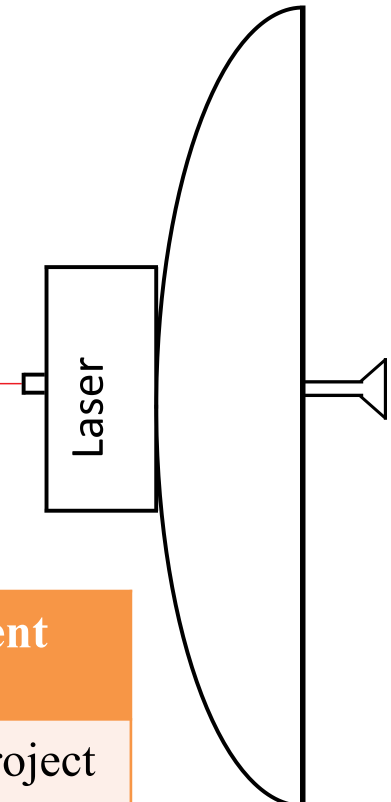
- *Gravity train mechanism*
  - Originally a thought experiment
    - Drill hole through center of Earth to other side
    - Unrealistically approximating Earth as uniform solid, observer inside the hole experiences simple harmonic motion along diameter of tunnel
    - Period of oscillation:

$$T = 2p\sqrt{\frac{R^3}{MG}} \longrightarrow G = 4\pi^2 \frac{R^6}{MT^2}$$

  - Using this mechanism but with much smaller object of known mass and radius, can produce an experiment on the order of years
    - $G$ determinations result if one can accurately measure period of oscillator

# Deep space experiment to measure G




Ranging distance: 5-6 miles

$$\frac{\Delta G}{G} = 3\frac{\Delta R}{R} + \frac{\Delta M}{M} + 2\frac{\Delta T}{T}$$

| $Q$ | Uncertainty | Experiment |
|---|---|---|
| $R$ | 7.3e-09 | Avogadro Project |
| $M$ | 5.0e-09 | Avogadro Project |
| $T$ | 1.8e-08 | Femtosecond laser |
| $G$ | 6.3e-08 | |

63 ppb uncertainty: potential three orders of magnitude improvement v.s. previous $G$ experiments

# Summary

- **Solar system tests continue to be competitive with pulsar timing and gravitational wave detectors in testing fundamental gravitational physics**
- **SLR continues to improve the accuracy in testing gravitomagnetic field with LARES/LAGEOS**
- **Light-ray deflection experiments are sensitive to the time-dependent component of the gravitational field of moving planets and Sun. Interplanetary laser ranging may improve testing of the “speed-of-gravity” effect by 10-100 times**
- **Relativistic geodesy with optical clocks opens a new window to a cm-precise normal height system on the global scale.**
- **Laser ranging systems for spacecraft in deep space are invaluable for future tests of general relativity and determination of fundamantal constants like big G.**
- **Much better theoretical model of the orbital/rotational motion of the Moon is required for providing an unambiguous testing relativistic theory of gravity.**