VMF3o: Enhanced Tropospheric Mapping Functions for Optical Frequencies

Janina Boisits[1], Daniel Landskron[1], Krzysztof Sosnica[2], Mateusz Drozdzewski[2], Johannes Böhm[1]

[1] TU Wien, Department of Geodesy and Geoinformation, Research Area Higher Geodesy, Vienna, Austria; janina.boisits@geo.tuwien.ac.at
[2] Wroclaw University of Environmental and Life Sciences, Institute of Geodesy and Geoinformatics, Wroclaw, Poland

Introduction

Correcting delays caused by the neutral atmosphere is crucial for deriving high precision Satellite Laser Ranging (SLR) products. The current model recommended by the IERS Conventions 2010 (Petit & Luzum, 2010) for correcting these delays is computed by calculating the Zenith Total Delay (ZTD) following Mendes & Pavlis (2004) and the Mapping Function (MF) as stated in Mendes et al. (2002). For further advancing the accuracy level of SLR products, a new model implying vertical information of the neutral atmosphere and with the capability to account for azimuthal asymmetries is needed.

TU Wien is now working on new Mapping Functions incorporating the above-mentioned requirements. The Vienna Mapping Functions 3 – optical (VMF3o) are developed following the successful VMF3 for microwave techniques (Landskron & Böhm, 2018). The MF coefficients are estimated from ray-traced delays using numerical weather model data. So far, preliminary results are available and first tests have been carried out.

Optical ray-tracing with RADIATE

RADIATE (Hofmeister & Böhm, 2017) is a ray-tracing software developed at TU Wien and is part of the Vienna VLBI and Satellite Software VieVS (Böhm et al., 2018). The software computes ray-traced delays for any user-defined station (or grid point) and any specified azimuth and elevation angle based on Numerical Weather Models (NWM) from the European Centre for Medium-Range Weather Forecasts (ECMWF). The software uses a refined piecewise-linear ray-tracing approach. Thus, the vertical resolution of the input data is increased by interpolation using height dependent increments (see *Table 1*).

Vertical interpolation levels					
Interval [km]	0 – 2	2 – 6	6 – 16	16 – 36	36 – 84
Increments [m]	10	20	50	100	500

Table 1: Increments for interpolating NWM data. The closer the interval to the Earth surface, the smaller the interpolation increments.

In order to derive mapping factors for optical delays, a new module for ray-tracing at optical frequencies was implemented. Therefore, a 3D refractivity field is computed by evaluating the Mendes & Pavlis (2004) equations for hydrostatic refractivity N_h *(1)* and non-hydrostatic or wet refractivity N_w *(2)* at every interpolation level.

$$N_h = N_{gaxs} \frac{T_d}{P_d} Z_d R_d \rho \quad \textbf{(1)}$$

$$N_w = N_{gws} \frac{\rho_w}{\rho_{ws}} - N_{gaxs} \frac{T_d}{P_d} \frac{Z_d}{Z} \frac{e}{T} \varepsilon \quad \textbf{(2)}$$

N_{gaxs} Group refractive index dry air P_d Pressure dry air e WV pressure moist air

N_{gws}	Group refractive index WV	T_d	Temperature dry air	T	Temperature moist air
Z	Compressibility factor moist air	ρ	Density moist air	ε	M_w/M_d
Z_d	Compressibility factor dry air	ρ_w	Density WV	M_w	Molar mass WV
R_d	Specific gas constant dry air	ρ_{ws}	Density pure standard WV	M_d	Molar mass dry air

To test the effects of different wavelengths, two sets of ray-traced delays were computed using a wavelength of 532 nm and 1064 nm. Comparing these two sets, the differences in the resulting mapping factors at 15° elevation angle are negligible. Even at 5° elevation angle, the differences are small compared to the variations over different azimuth angles, causing variation in STD below centimetre-level. Thus, the wavelength used for ray-tracing is fixed to 532 nm.

Estimating coefficients of VMF3o

The model parameters of VMF3o include the coefficients a_h, b_h, and c_h of the hydrostatic MF, as well as a_w, b_w, and c_w of the wet MF, both given as continuous fraction function. For estimating b and c coefficients, monthly mean NWM data of the years 2001 to 2010 was used. The ray-traced delays were calculated on a global grid with a resolution of 5° x 5°, for eight different azimuth and four different elevation angles per grid point. The MF coefficients were then estimated in a least squares adjustment assuming horizontal symmetry. Eventually, the resulting time series of b and c coefficients at every grid point were approximated by spherical harmonics (SH) expanded up to degree and order 12 with annual and semi-annual signals.

In contrast to b and c coefficients, which comprise only long-term variations of the atmosphere, a coefficients are estimated every 6h. Ray-traced delays are computed on a 1° x 1° grid at 5° elevation angle. The coefficients are then calculated by inverting the MF. The results, meaning a_h, a_w, as well as Zenith Hydrostatic Delay (ZHD) and Zenith Wet Delay (ZWD) derived from ray-tracing, are saved and published in so-called VMF3o-files with each file containing grid-data for one epoch.

First tests and results

First tests were carried out using data for the year 2005 and observations to LAGEOS-1 and LAGEOS-2 satellites. The processing was carried out with the Bernese GNSS Software, version 5.3 dedicated for laser observation processing. The b and c coefficients, depending on time and station location, were calculated by evaluating the SH expansion. The a coefficients for each observation were imported from the VMF3o-files of the closest two epochs and then interpolated in time and space. Finally, a height reduction following Niell (1996) was applied.

For purposes of comparison and validation, three different sets of solutions were produced, each with a different approach to correct for the tropospheric delay:

- '*MP*': This is the standard solution where the correction was calculated following Mendes & Pavlis (2004) for zenith delays and Mendes et al. (2002) for the MF. In the following, it will serve as the reference solution.
- '*VMF3o*': For this solution, the VMF3o coefficients were used to calculate the hydrostatic and wet MF. ZHD and ZWD again were calculated following Mendes & Pavlis (2004), because the standard equations are considered to yield very accurate results, especially for the ZHD, using the meteorological data log from the site.

- '*VMF3ow*': The last solution is identical to *VMF3o* except for the ZWD. Instead of using the standard equation, the ZWD derived from ray-tracing is used to test the effect of the additional vertical information.

To get a first idea of the impact of the new MF coefficients, the time series of slant delays for each solution type were produced and compared. Generally, the differences of *VMF3o* and *VMF3ow* compared to *MP* are rather small at 20° elevation angle. At station 7839 (Graz), for example, *MP* minus *VMF3o* yields values between 0 mm and -3 mm indicating systematically larger STDs for the *VMF3o* solution (see *Fig. 1, left*). The differences *MP* minus *VMF3ow* at station 7839 range from -4 mm to +3 mm with much higher short-term variations resulting from the differences in ZWDs (see *Fig. 1, right*).

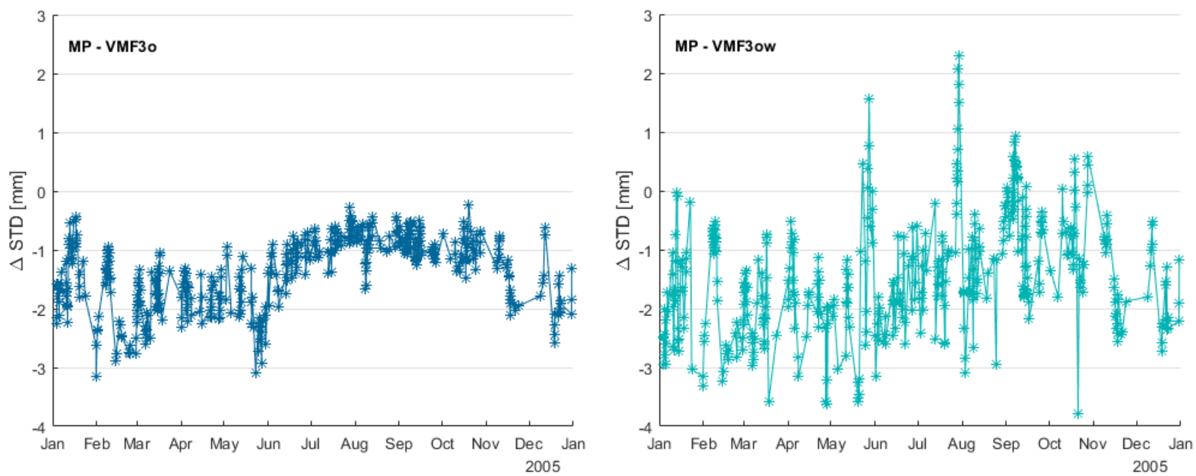

Fig. 1: Differences of Slant Total Delay (STD) at 20° elevation angle for the station 7839.
Left: solution *MP* minus *VMF3o*; right: solution *MP* minus *VMF3ow*.

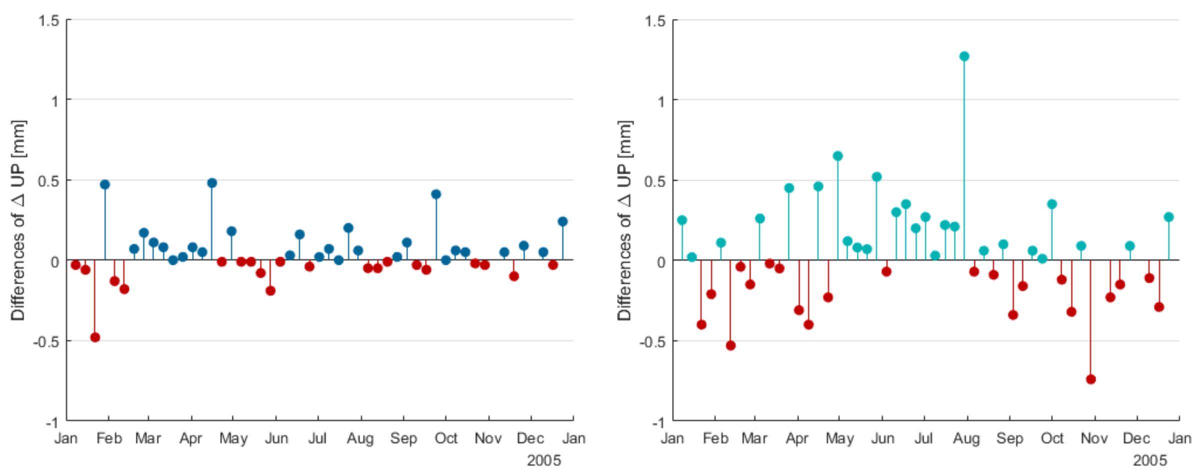

Fig. 2: Differences of the corrections in the vertical component (ΔUP) of the weekly solutions for the station 7839.
Negative values in red, positive values in blue/turquoise.
Left: solution *MP* minus *VMF3o*; right: solution *MP* minus *VMF3ow*.

When looking at the station coordinate repeatability, especially the corrections in the vertical component ΔUP of the weekly solutions seem to be systematically affected. Again, station 7839 serves as example. Comparing *VMF3o* to the reference solution results in a positive bias of the differences of ΔUP (see *Fig. 2, left*) meaning that the corrections of this solution are generally smaller. The differences of *MP* minus *VMF3ow* seem to follow an annual signal, which would be interesting for further investigations (see *Fig. 2, right*).

To get an idea of the accuracy of VMF3o the observation residuals were analyzed. Thus, the mean value of absolute observation residuals (MAR) was formed for every solution, respectively. Additionally, the observations were subdivided into 7 bins of elevation angles and the MAR calculated for each bin. When comparing these values, it becomes clear that *VMF3o* and even more so *VMF3ow* reduce the residuals especially at low elevation angles. Looking at the total MAR the *VMF3o* solution reduces the residuals at about half of the stations compared to the *MP* solution, whereas *VMF3ow* reduces the residuals at nearly two-thirds of the stations. These results suggest that aside from the notable effects of the new MF coefficients, the additional vertical information in ZWD derived from ray-tracing further improves SLR observation residuals.

Conclusion

The results of the first tests are very promising and VMF3o shows great potential to further advance the accuracy level of SLR observations and products. At the moment, grid interpolation and height extrapolation are considered a major error source in the current model. Thus, the next step is to perform the ray-tracing station-wise and provide MF coefficients for each SLR station. We expect this to introduce a significant improvement of the results. Additionally, the effect of using ZWDs derived from ray-traced delays will be further investigated. The final version of VMF3o will also include horizontal gradients to account for atmospheric asymmetries.

Preliminary VMF3o-files including a coefficients, zenith delays, and in near future also horizontal gradients can be downloaded at the following link:
vmf.geo.tuwien.ac.at/trop_products/SLR_prelim/

References

- **Böhm, J., Böhm, S., Boisits, J., Girdiuk, A., Gruber, J., Hellerschmied, A., Krasna, H., Landskron, D., Madzak, M., Mayer, D., McCallum, J., McCallum, L., Schartner, M. & Teke, K. (2018).** Vienna VLBI and Satellite Software (VieVS) for Geodesy and Astrometry. *Publications of the Astronomical Society of the Pacific,* ***Vol. 130(986)****, 044503; 1 - 6.*
- **Hofmeister, A. & Böhm, J. (2017).** Application of ray-traced tropospheric slant delays to geodetic VLBI analysis. *J Geod (2017) 91: 945.* https://doi.org/10.1007/s00190-017-1000-7
- **Landskron, D. & Böhm, J. (2018).** VMF3/GPT3: refined discrete and empirical troposphere mapping functions. *J Geod (2018) 92: 349.* https://doi.org/10.1007/s00190-017-1066-2
- **Mendes, V. & Pavlis, E. (2004).** High-accuracy zenith delay prediction at optical wavelengths. *Geophys. Res. Lett. 31, L14602*. https://doi.org/10.1029/2004GL020308
- **Mendes, V., Prates, G., Pavlis, E., Pavlis D. & Langley, R. (2002).** Improved mapping functions for atmospheric refraction correction in SLR. *Geophys. Res. Lett. 29(10)*. https://doi.org/10.1029/2001GL014394
- **Niell, A.E. (1996).** Global mapping functions for the atmosphere delay at radio wavelengths, *J. Geophys. Res. 101(B2):3227-3246*.
- **Petit, G. & Luzum, B. (eds.) (2010).** IERS Conventions 2010 (IERS Technical Note; 36). *Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodaesie. 179 pp., ISBN 3-89888-989-6.*