Local orbit uncertainty reduction in follow-up passes based on single-pass debris laser ranging

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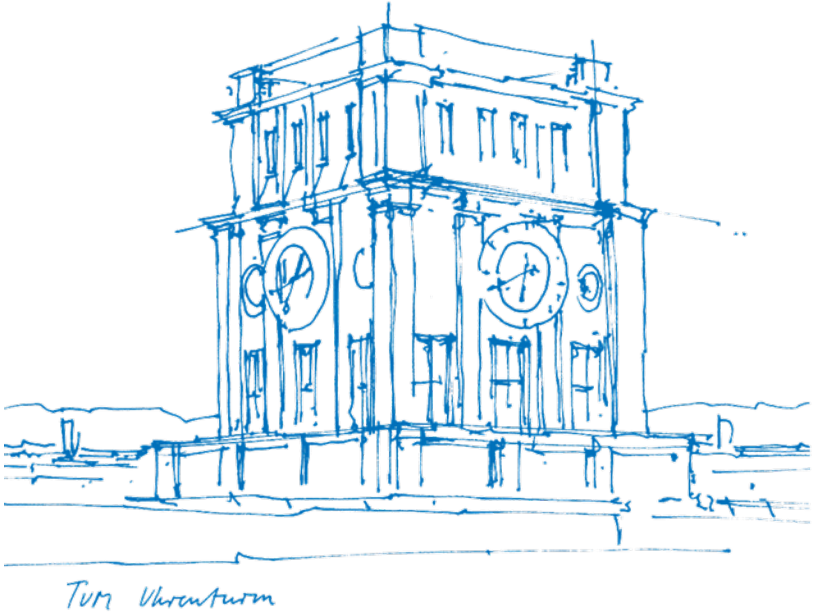


Challenges: Orbit determination of space debris with laser tracking

- Many objects, few stations → limited observation times
- **Inaccurate predictions** → no blind tracking possible
- **Terminator** (illumination) and weather (clouds) limitations
- Unfavourable observation geometries

➥ Sparse and ill-distributed tracking data

➥ Observability issues / overfitting

➥ Large estimation uncertainties

Data fusion with TLE and DISCOS catalog data

- Adds **constraints** for parts of the orbit that cannot be seen by tracking stations in close proximity
- May provide **additional a-priori information** (TLE orbits, mass and cross-section, ...)
- Can use a-priori orbit and uncertainties from **improved TLEs** (SP model fitted to TLE-derived pseudo-observations)

  ➥ Makes the solution parameters observable in case of unfavorable observation geometries

  ➥ Avoids overfitting, in particular regarding the ballistic coefficient in LEO

  ➥ Allows orbit determination even with **single-pass data** in the extreme case

What is the achievable orbit prediction uncertainty for single-pass debris laser ranging?

Force model paramaters

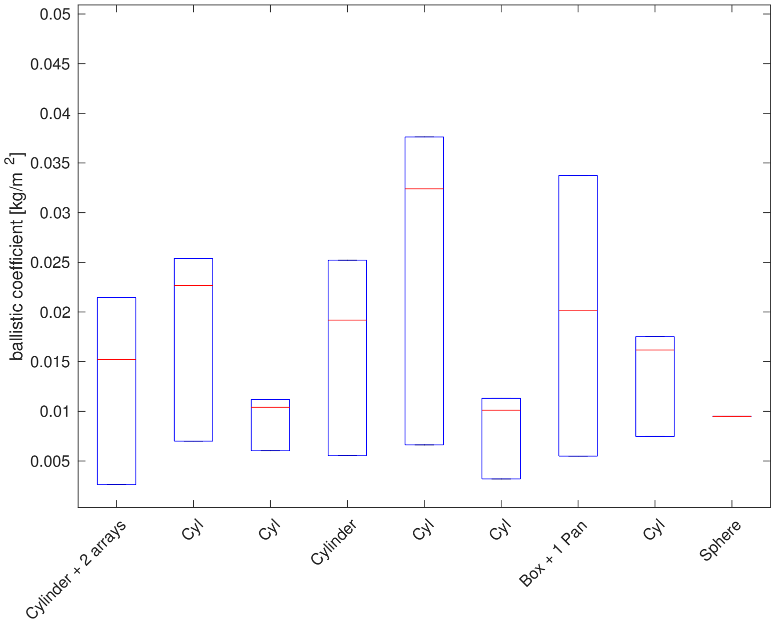
- For very low objects (<500km) good results are obtained if the ballistic coefficient $BC = C_D \frac{A}{m}$ is estimated using the rate of change of the semi-major axis from historical TLEs:

$$\left.\frac{da}{dt}\right|_{drag} = \frac{2a^2 v}{\mu} \dot{\mathbf{v}}_{drag} \cdot \mathbf{e}_v$$

Together with $\dot{\mathbf{v}}_{drag} = -\frac{1}{2}\rho BC \left|\mathbf{v} - \mathbf{V}\right|^2 \cdot \mathbf{e}_{v-V}$ we obtain:

$$BC = -\frac{\mu \Delta a_{t_1,drag}^{t_2}}{\sum_{t=t_1}^{t_2} a_t^2 v_t \rho_t \|\mathbf{v}_t - \mathbf{V}_t\|^2 \mathbf{e}_{v_t - V_t} \cdot \mathbf{e}_{v_t} \Delta t}$$

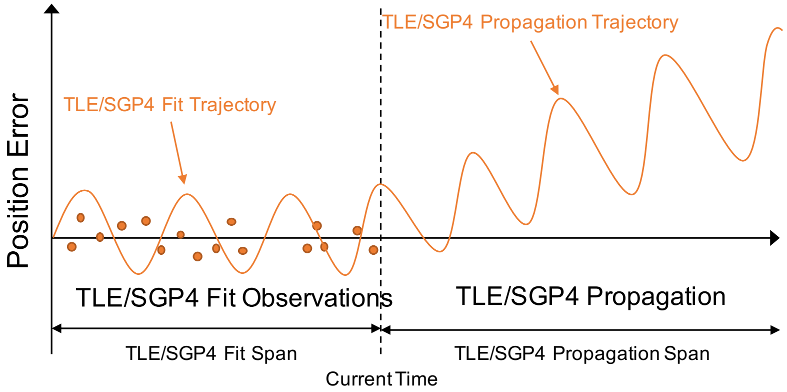
- Object properties (mass, shape, size) from DISCOS - ESA's database and information system characterising objects in space
- The ballistic coefficient and its uncertainty is derived from an object's mass, minimum/average/maximum cross-section and an estimated value for C_d

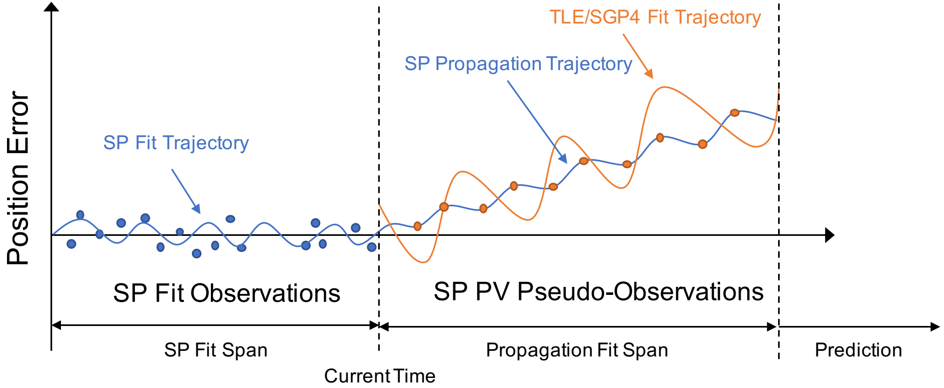


TLE generation process

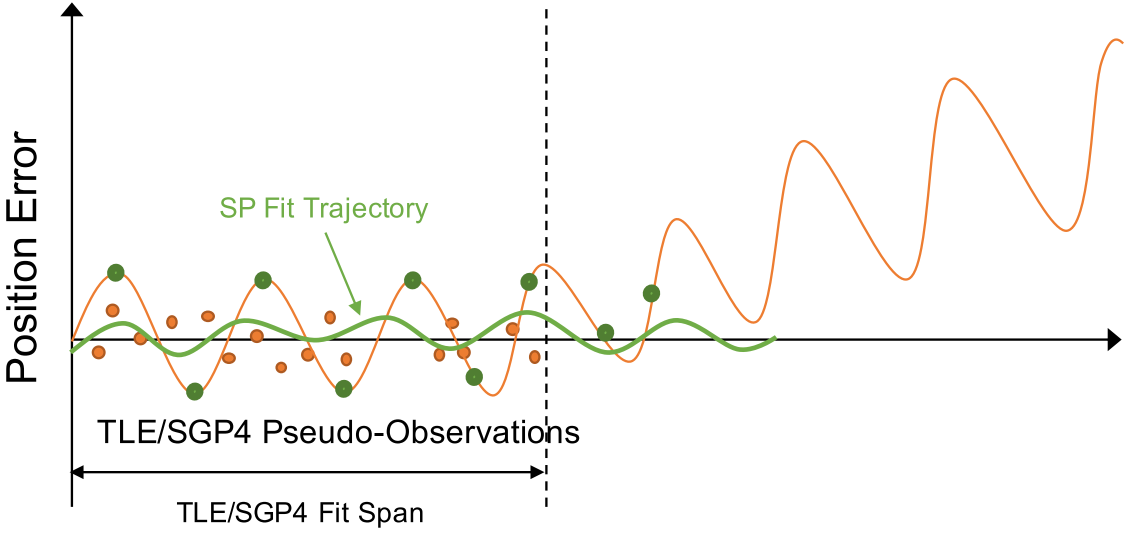
- Classical TLEs are generated by fitting an analytical model to real observations

- Since 2013 enhanced TLEs are generated by fitting an analytical model to orbit predictions computed using high-fidelity models



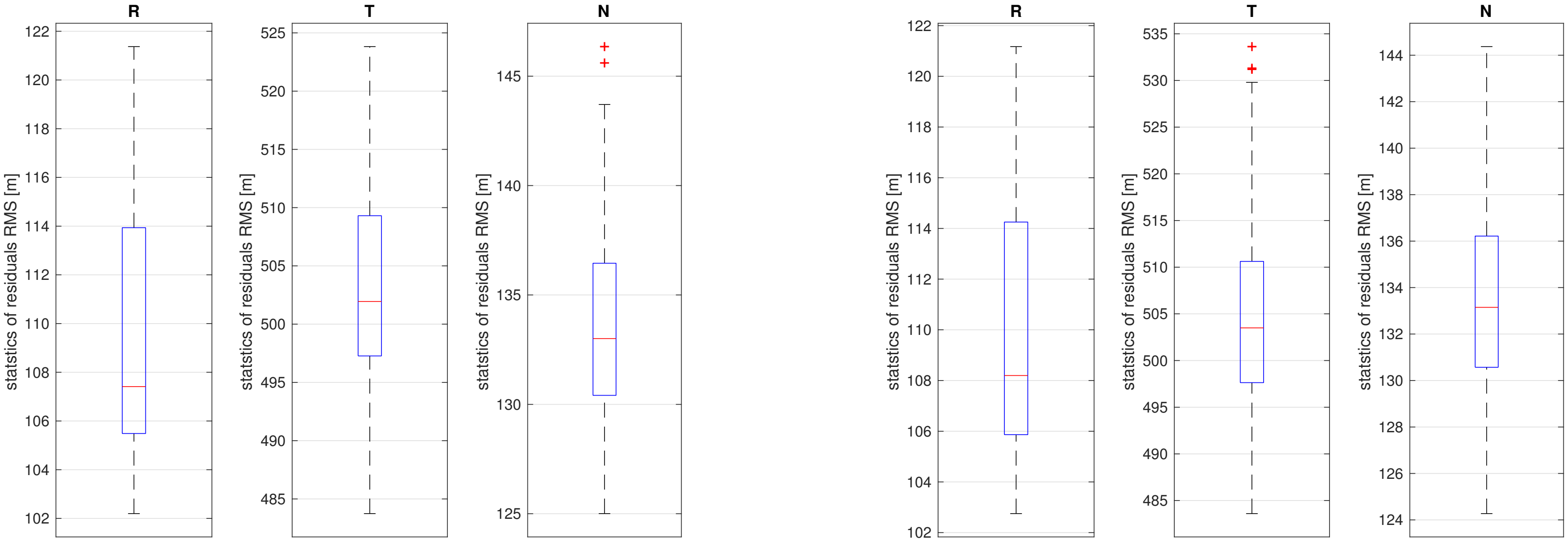


TLE improvement via SP model fitting



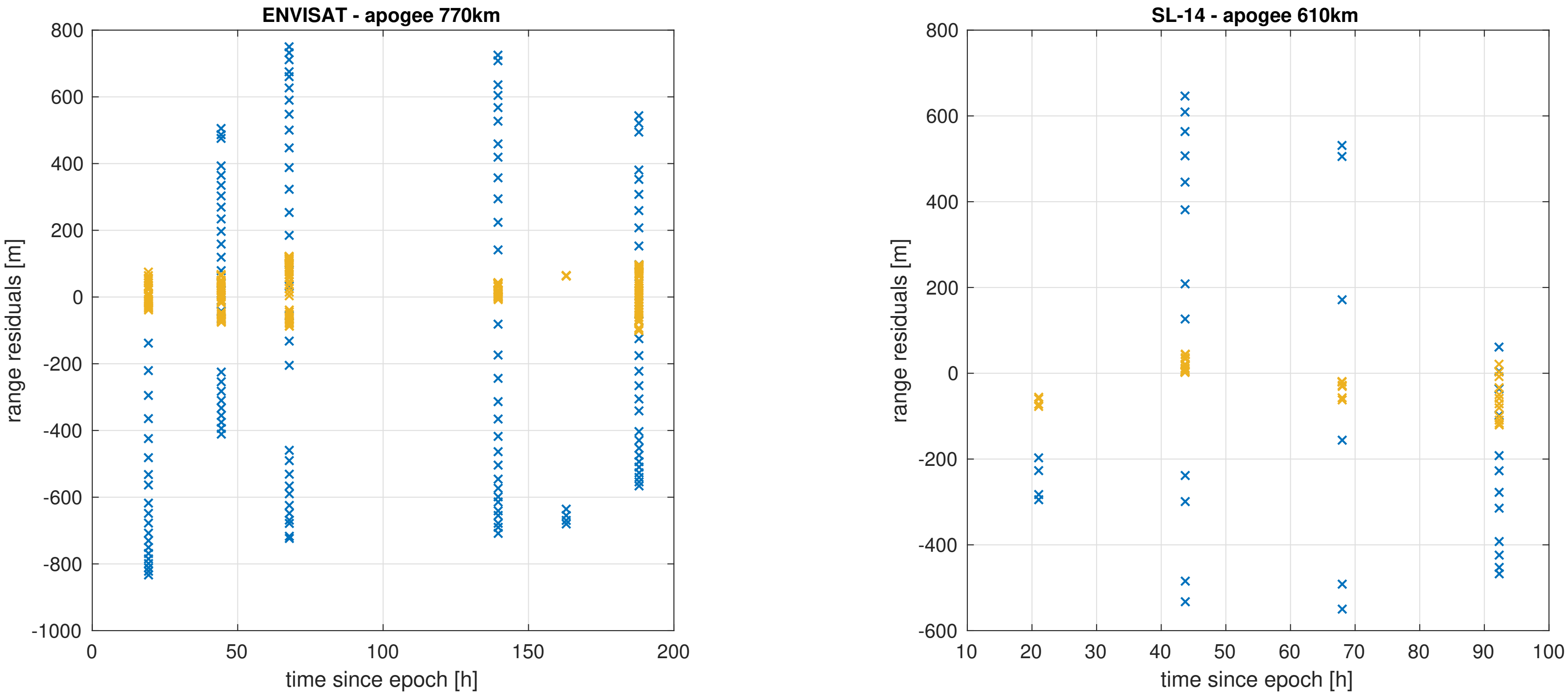
We fit a high-fidelity model to pseudo position-velocity observations from TLEs to obtain a physically more realistic (and improved) orbit

Statistics of TLE improvement post-fit residuals (RTN frame)



Zenith-2 second stages (left: 22566, right: 22220), altitude ≈ 800 km

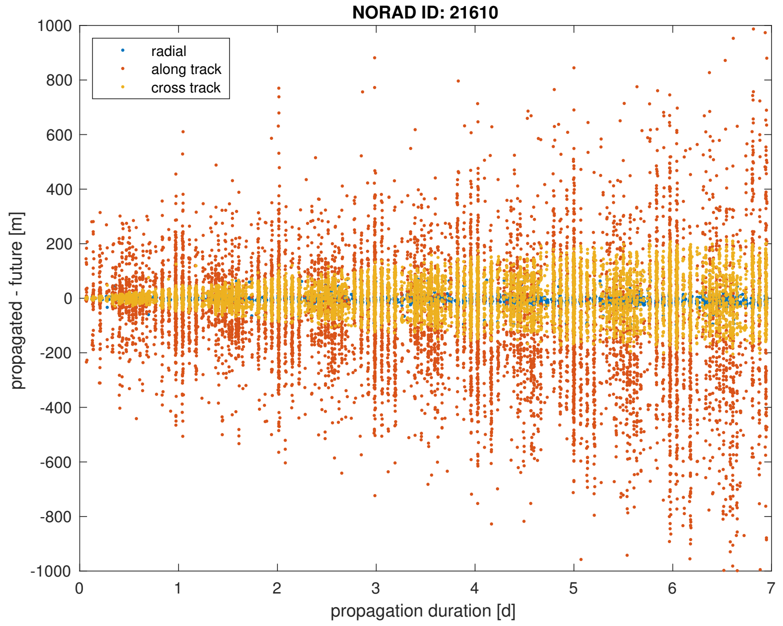
Validation using real leaser ranging data

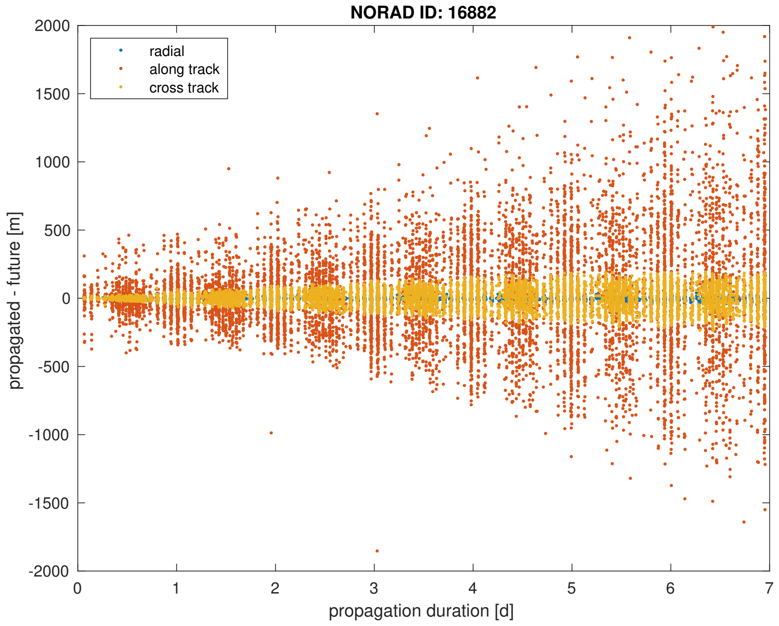


Uncertainty of improved TLEs

- Several approaches for TLE uncertainty estimation: TLE differencing, RMS after SP-model fitting, ...
  - TLE differencing generally yields non-Gaussian and biased differences that grow polynomially in time
  - OD post-fit residuals using TLEs as pseudo-observations neglect potential TLE biases
- Using an initial state from TLE improvement, we are interested in the uncertainty of this state and not of uncertainty of the underlying TLEs
- We therefore compute differences of TLE improvement solutions from different epochs linked by orbit propagation $\rightarrow$ these are the samples for covariance estimation

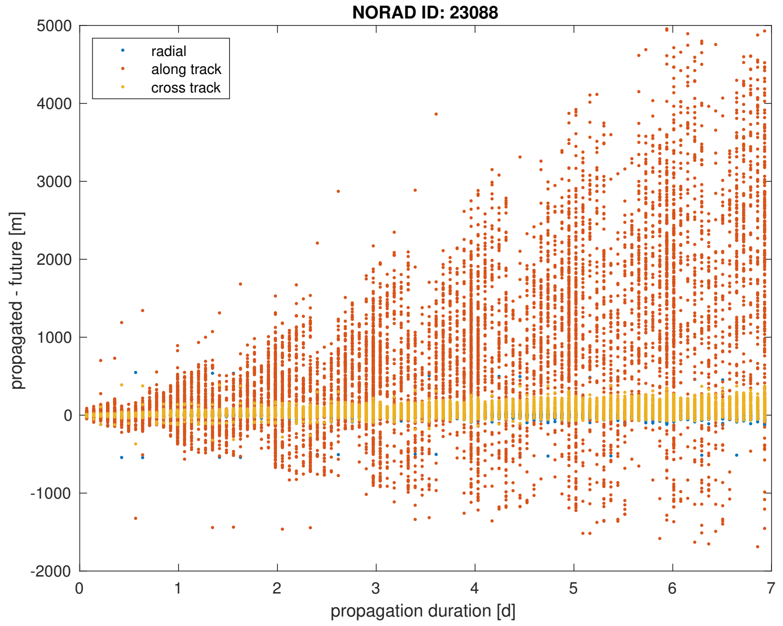
Differences of improved TLEs at different epochs via propagation (1)

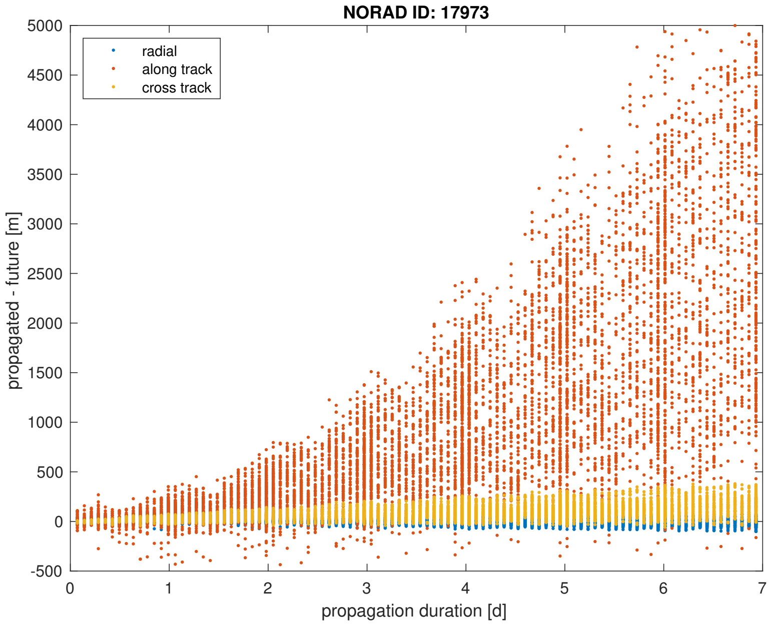




fitted 4 day arc, used BC from historical TLE time series

Differences of improved TLEs at different epochs via propagation (2)





fitted 4 day arc, used BC from historical TLE time series

Statistical model of differences of improved TLEs

- In a first-order Taylor series approximation the state-transition matrix $\Phi(t_2, t_1)$ maps deviations from the reference orbit from one TLE epoch to the next:

$$\Delta x(t_2) = \Phi(t_2, t_1)\Delta x(t_1)$$

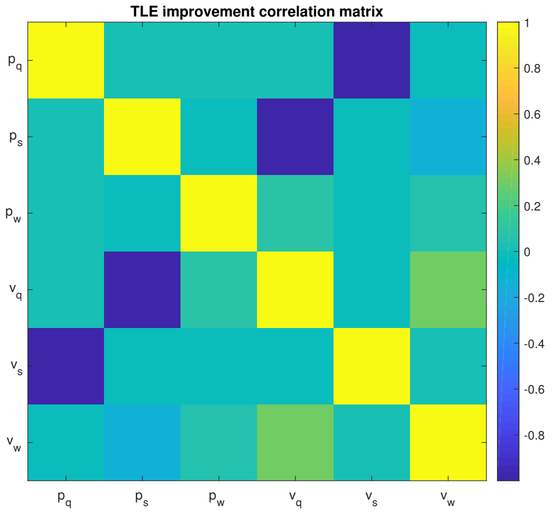
- The **covariance** $P(t_1)$ **is propagated linearly** via $P(t_2) = \Phi(t_2, t_1)P(t_1)\Phi(t_2, t_1)^T + Q(t_2, t_1)$. The additive process noise matrix $Q(t_2, t_1)$ accounts for integrated force model errors during propagation.

- Consequently, a sample $d_{j,i} = x(t_j) - \phi(x(t_i); t_j)$ is assumed to be drawn from the following distribution:

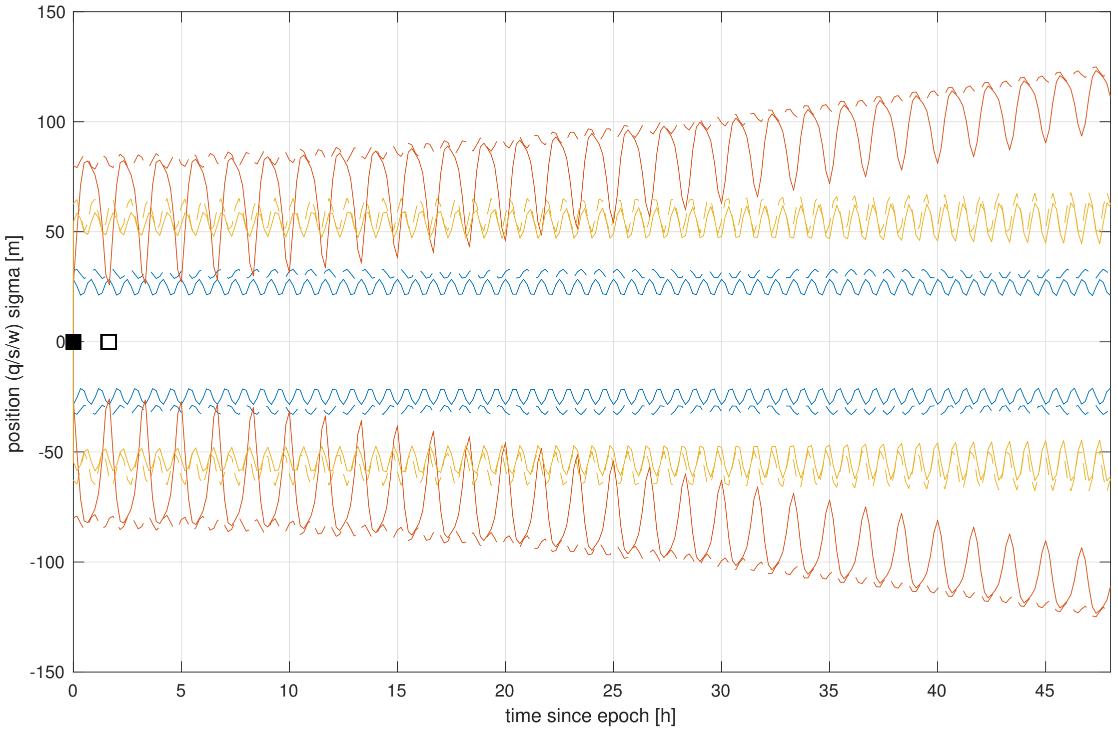
$$d_{j,i} \sim N\left(0, P(t_j) + \Phi(t_j, t_i)P_i\Phi(t_j, t_i)^T + Q(t_j, t_i)\right)$$

Correlation matrix of improved TLE

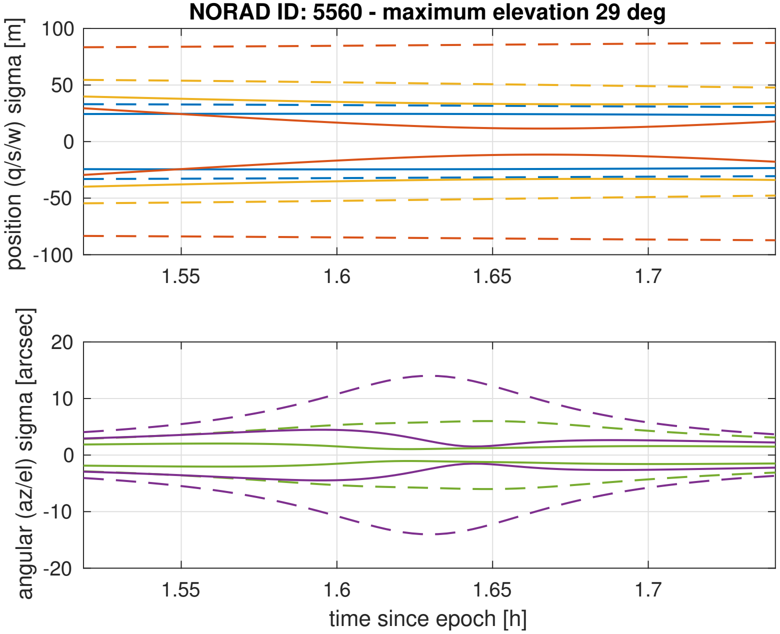
- Assuming $P(t_j) = P(t_i) = P$ **in the local orbital frame** we maximize the likelihood function $L = \sum \log f(d_{j,i})$ using an analytical expression for $Q(t_j, t_i)$.
- In doing so, we use the **correlation matrix** as obtained from an individual least-squares adjustment for TLE improvement and **estimate only the variances** using the differencing samples.

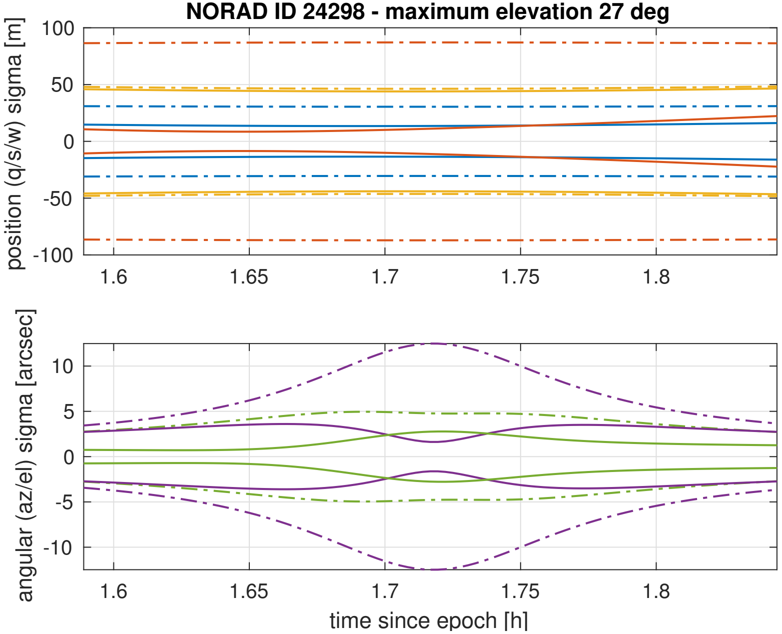


Results - Single-pass improvement of RTN uncertainties



Results - Improved TLE vs. improved TLE + laser





Results - Covariance ellipsoid vs. pass geometry

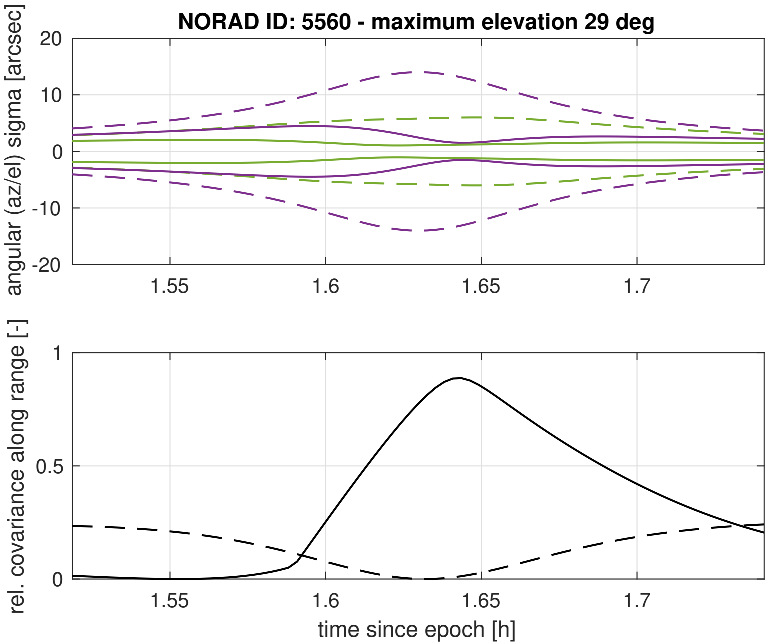
- Ranging along the principle direction of covariance ellipsoid...
  - provides maximum information regarding the orbit uncertainty
  - maximizes the likelihood of hitting the target in bind tracking
- Potential approaches to quantify this:
  - 1D: Based on projection of normalized view direction vector $\frac{\mathbf{v}_{view}}{\|\mathbf{v}_{view}\|}$ onto principal axis vector $\mathbf{u}_1$ of the covariance ellipsoid P_{pos}

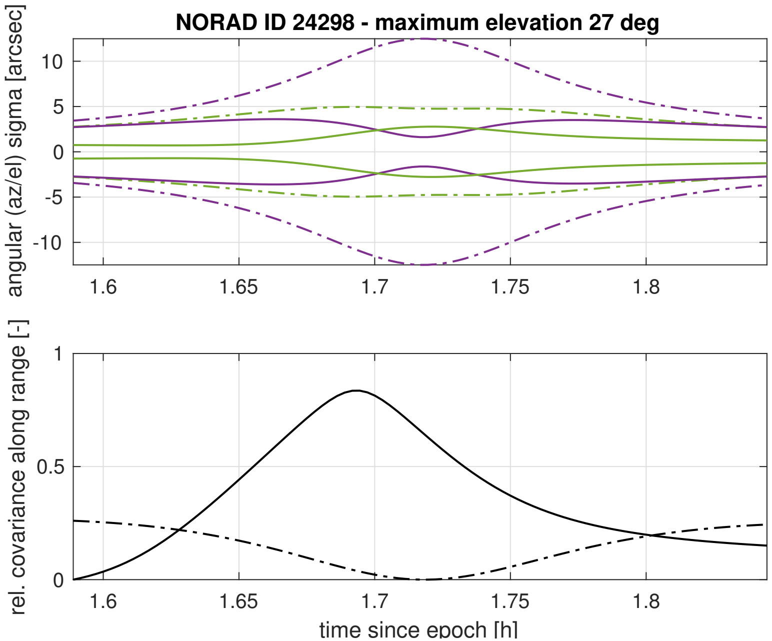
$$\left\langle \frac{\mathbf{v}_{view}}{\|\mathbf{v}_{view}\|}, \mathbf{u}_1 \right\rangle \tag{1}$$

  - 2D: Using area A_{view} of covariance ellipse projected onto view direction and area A_{12} of covariance ellipse along the two principal axes

$$1 - \frac{A_{view}}{A_{12}} \tag{2}$$

Results - Improved TLE vs. improved TLE + laser





Results - Quality testing via filter consistency

- Under the hypothesis that the filter is consistent, the **normalized innovation squared** at $t = t_k$

$$\varepsilon_\nu(k) = \nu(k)^T S(k)^{-1} \nu(k) = \nu(k)^T \left(H(k)P(k|k-1)H(k)^T + R \right)^{-1} \nu(k) \tag{3}$$

  has a **chi-squared distribution** with n_z degrees of freedom, where n_z is the dimension of the measurement, i.e. equal to one for range measurements.

- From N **independent samples** $\varepsilon_\nu(k)^i$ one calculates the average

$$\bar{\varepsilon}_\nu(k) = \frac{1}{N} \sum_{i=1}^{N} \varepsilon_\nu(k)^i \tag{4}$$

  which is then tested with acceptance region determined based on the fact that $N\bar{\varepsilon}_\nu(k)$ is chi-square distributed with Nn_z degrees of freedom.

Results - Comparisons and filter consistency

	max. cov. range [% of pass]	sig_max az/el [arcsec]	rel. estimation consistency	rel. prediction consistency
<u>**22566 (maximum elevation first / follow up pass: 22° / 53°)**</u>				
Impr. TLE	1	29 / 10	–	1.00
Impr. TLE + laser	48	17 / 9	0.92	0.95
<u>**23088 (maximum elevation first / follow up pass: 30° / 58°)**</u>				
Impr. TLE	3	34 / 12	–	1.00
Impr. TLE + laser	42	7 / 4	1.00	1.00
<u>**24298 (maximum elevation first / follow up pass: 44° / 27°)**</u>				
Impr. TLE	1	12 / 5	–	0.73
Impr. TLE + laser	42	4 / 3	1.00	1.00
<u>**27386 (maximum elevation first / follow up pass: 24° / 26°)**</u>				
Impr. TLE	1	13 / 6	–	1.00
Impr. TLE + laser	56	6 / 5	1.00	0.80
<u>**5560 (maximum elevation first / follow up pass: 27° / 29°)**</u>				
Impr. TLE	100	14 / 6	–	0.92
Impr. TLE + laser	57	4 / 2	0.88	1.00

Summary

- Extended Kalman Filter framework for data fusion of laser ranges with TLEs
- Force model parameters from historical TLEs and the DISCOS database
- Developed an uncertainty estimation method for improved TLEs
- Tested improved TLEs & TLE-laser data fusion based on real data
- Significant reduction of uncertainty for telescope pointing in follow-up passes
- Highest chance of "hitting"target shifts from edges to middle of a pass

Outlook

- Use predicted uncertainties to derive search strategies for blind tracking
- Blind tracking campaign?