# Histogram calculations for Laser Ranging 

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SAO developed a program TRANSFR that calculates the far field cross section and far field centroid matrices for a retroreflector array. The program also records the cross section and position along the line of sight for each active cube corner. This is used for plotting the histogram of a retroreflector array.

## ILRS method of calculating histograms

The ILRS method uses only the radius of a spherical satellite and the type of cube corner. The satellite is assumed to be uniformly covered with cube corners.

$$
I \propto a^{n} e
$$

The intensity I of each cube corner is proportional to a power $n$ of the active reflecting area $a$ times the reflection coefficient $e$.

## Average far field cross section matrices for Lageos

Linear polarization


Circular polarization


The circles are at the minimum and maximum velocity aberration. The laser receiver is between these two circles at 32 and 40 microradians. The average cross section between the two circles is used as the signal from each cube corner when computing histograms.

## SAO Histogram - Lageos

The calculation uses the positions of all the cube corners



The histogram is obtained by binning data from 2500 incidence angles. The data is the cross section and position along the line of sight for the active cube corners at each incidence angle. Bin size $=2 \mathrm{~mm}$.

Histograms computed using the ILRS method


Histograms for exponents $n=1.0,1.1,1.2,1.3,1.4,1.5,1.6,1.7$ (red). The top line is for $n=1.0$ and the bottom is for $n=1.7$. The black line is the SAO histogram. The same data set is used for both the SAO and ILRS histograms.

## Tail of the histogram



The top red line is for $n=1.0$. The bottom is for $n=$ 1.7. The red curve for $n=1.4$ coincides with the black line for the SAO histogram.

## CoM Values

| $n$ | CoM (m) |
| :--- | :--- |
|  |  |
| 1.0 | 0.239378 |
| 1.1 | 0.240248 |
| 1.2 | 0.241030 |
| 1.3 | 0.241736 |
| 1.4 | 0.242377 |
| 1.5 | 0.242962 |
| 1.6 | 0.243499 |
| 1.7 | 0.243994 |



Red $=$ ILRS CoM vs exponent $n, \quad$ Green $=$ SAO CoM $=.2425 \mathrm{~m}$ The exponents determined experimentally by ILRS for Lageos-1/2 vary from $n=1.1$ to 1.25

